DATING BUSINESS CYCLES IN LITHUANIA
BY SIMPLE UNIVARIATE METHODS

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Abstract. In this paper, we use three basic univariate techniques, namely, BBQ algorithm, time series filtering, and Markov-switching models, to date and characterize Lithuanian business cycles from 1995 to 2010. We find that economic growth in Lithuania was relatively balanced after the Russian Crisis until late 2006. After that, the economy experienced an extreme, although relatively brief, period of an overheated economic climate before plunging into a very deep recession at the end of 2008. Using the BBQ algorithm, we provide some simple comparisons of the two recessions as well as international data obtained in other studies. Our Markov-switching regression exercise, confirming the findings above, additionally indicates that recessionary periods may have shocks with non-finite variances and economically significant permanent effects on output.

Key words: BBQ algorithm, Hodrick-Prescott filter, Markov-switching models, business cycle, Lithuanian economy

Introduction

Analysis of business cycle dynamics is arguably one of the most difficult problems faced by the contemporary macroeconomist. While Lucas (2003) famously argued that the “central problem of macroeconomics” had been solved, the recent downturn has brought the understanding of aggregate economic fluctuations and optimal policy responses to an uneasy spotlight. Opinions of economists have often been non-conforming, which can be taken as some indication of the difficulty of the problem at hand. For instance, Blanchard et al. (2010) argue that macroeconomic policy failures did not trigger the crisis, although they were exposed as a result. In contrast, Taylor (2009 a, b) shows that loose monetary policy was crucial in creating the boom and recession that followed; also, he claims that rules-based economic policy should be preferred to discretionary spending.

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1 For a historical perspective on the crisis, see Bordo, 2008.
in the recent context. Krugman (2009) writes that a flawed understanding of the (macro) economy is to be blamed and puts forward a case for a Keynesian-style intervention. To cite but two policy-relevant puzzles on the effects of fiscal policy, Mountford and Uhlig (2009), using the US data in a vector autoregression (VAR) framework, demonstrate that deficit-financed tax cuts have a larger positive effect on output than deficit-financed fiscal expansions (for a recent survey on the theory and empirics of discretionary fiscal policy, see Hebous, 2010); Blanchard and Perotti (2002), again using structural VARs, find that increases in both government spending and taxation have negative effects on investment, which is difficult to explain in a Keynesian framework. Thus, there are still-significant disagreements on the business cycle, and the problem clearly does not seem to have become a trivial one.

Turning to recent papers on economic fluctuations in Lithuania, Kuodis and Ramanauskas (2009) present a chronologic account of the development of the Great Recession in this country; they conclude that “retrospectively, it is clear that the economic overheating was almost predestined” and provide a number of economic policy failures deemed responsible. A study conducted by the World Bank (Mitra et al., 2010) stresses the importance of prudential fiscal policy in the run-up to the crisis; Landesmann (2010) argues that the growth model in the Baltic States was based on a high level of external liberalization, which left the countries vulnerable in face of a negative shock. Furthermore, Böwer et al. (2010) report the results of a simulation of the DG ECFIN QUEST III dynamic stochastic general equilibrium (DSGE) model (Ratto et al., 2008), which shows that the evolution of the main aggregated macroeconomic time series for the Baltic region after 2001 cannot be modelled realistically on the basis of technological progress alone: financial disturbances must be added as well. The authors provide empirical evidence that the Baltic States started overheating as early as 2000.

Although, as seen above, different aspects of business cycles in Lithuania have been studied, to the best of our knowledge, there has been no systematic treatment on dating and characterising recessions and expansions yet. While policy-makers and popular media often rely on heuristics such as Okun’s rule, measuring the business cycle transparently is hardly an easy task; many methods have been proposed (for a largely non-technical review, see Harding, Pagan, 2006). Our goal in this paper, thus, is to date the Lithuanian business cycle since 1995 to 2010, using simple univariate techniques; we will put a special emphasis on the analysis of the Great Recession, not least because of its high importance to economic policy. We proceed as follows. first, we describe the data used and provide its basic descriptive measures; second, we use the BBQ algorithm

\[ \text{BBQ} \]

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2 In what proceeds, the term “the Great Recession” will be used to refer to the most recent crisis, as there seems to be no standard definition in the academia yet; we hope that the reader will not find this distracting.

3 This rule-of-thumb, popularized by Arthur Okun, says that a recession consists of at least two consecutive quarters of negative GDP growth (Hess, Iwata, 1997).
to date the cycle and measure depth of recessions; third, we provide the estimates of the output gap by time series filtering; fourth, we use the Markov-switching regression model to date the business cycle. Finally, we summarize the results, discuss their implications and point to areas for possible future research.

A very important note of caution must be given, however, before proceeding further. In an influential paper, Aguiar and Gopinath (2004) have argued that the main source of volatility in emerging markets is variation in trend growth; in other words, “the trend is the cycle”. If that is indeed the case for Lithuania, one must admit that our attempts at characterizing the cycle are, at least to some extent, futile. Yet even if the business cycle in Lithuania is mostly generated by exogenous factors, we believe that it is still useful to date the (not strictly “natural”) cycle and provide some of its characteristics, even at the cost of the lack of a sound theoretical ground.

1. Data and descriptive characteristics

In what follows, EUROSTAT data on Lithuanian GDP, adjusted by season and working days, expressed in millions of constant (2000) Litas for the period 1995Q1–2010Q2, will be used (see Fig. 1; QK stands for the \(k\)-th quarter)\(^4\); in all our subsequent work, unless stated otherwise, growth rates refer to simple (i.e. not logarithmic) quarter-on-quarter growth rates\(^5\).

At first, we split our sample into two parts, 1995Q1–2008Q1 (Period I) and 2008Q2–2010Q2 (Period II), which can be roughly interpreted as periods of “expansion” and “recession”, respectively, as the first negative quarter-on-quarter growth rate of the Great Recession appears in 2008Q2. We observe that from 1995 until the end of 2007 Lithuania enjoyed a period of strong growth with a median quarter-on-quarter growth rate of 1.72\(\%\) (Table 1); this translates into annualized growth of a little above 7\(\%\). Although Period I encompasses the Russian Crisis, the minimum quarterly growth rate is -1.68\(\%\), which is relatively high, when compared to the minimum value in Period 2, i.e. -13.56\(\%\). Indeed, growth rates were positive for all quarters in Period I except for 1995Q3, 1999Q1 and 1999Q3, the latter two being a result of the Russian Crisis (for a historical account on Lithuania’s economic performance during this period, see Kuodis, 2008).

One can also notice that the distribution of GDP growth rates in the first sub-sample is somewhat negatively skewed, i.e. more of the density mass is concentrated to the right of the mean. Also, the empirical distribution is platykurtic, i.e. the peak is wider as compared to the normal distribution. This is a fact worth noting since it could have been at the time (incorrectly) interpreted as an evidence in favour of the proposition that the

\(^4\) At the time of writing, data only up to the second quarter of 2010 was available.

\(^5\) It must be mentioned that such growth rates look “nicer” than respective year-on-year growth rates, i.e. there are more positive numbers during recession, compared to those calculated on a year-on-year basis; this is because of a favourable base period effect.
data-generating process (DGP) of GDP growth rates in Lithuania does not possess “fat tails” and / or that the possibility of outliers is very low, which now, with the benefit of hindsight, seems to be almost surely wrong.

The descriptive statistics of real GDP growth rates since 2008Q2 are markedly different (Table 2). Both the mean and the median are negative, with the median being not significantly different from zero. Furthermore, sample kurtosis is very high, which is a direct consequence of one negatively large outlier-like observation in 2009Q1; for the same reason, the left-hand side tail of the empirical density is much longer, which is also shown by the greater, in absolute sense, skewness statistic. Sample standard deviation (4.45%) is approximately 3.6 times higher than that of the previous period, again indicating a high economic turbulence. Of course, in order to date the business cycle in a rigorous fashion, one needs more transparent methods, which is what will concern us for the rest of the paper.

### 2. Dating the cycle by BBQ algorithm

In the following section, we use a simple non-parametric algorithm, introduced by Harding and Pagan (2002), to date the business cycle. Then, we provide the basic characteristics of the two recessions found in the series and compare them to international statistics obtained by Claessens et al. (2009) and Hong et al. (2010).

The BBQ algorithm, which is an extension of a procedure proposed by Bry and Boschan (1971) to quarterly data, is based on an intuitive idea of how a graph of the cycle should look like. More specifically, let \( y_t = \log(GDP_t), t = 1, ..., T \). Then we say that \( y_t^P \) is a peak that occurs at time \( t \) if

\[
\begin{align*}
    y_t^P > y_{t-1}^P, & \quad y_t^P > y_{t-2}^P, \\
    \text{and} & \quad y_t^P > y_{t+1}^P, & \quad y_t^P > y_{t+2}^P,
\end{align*}
\]

(1)

i.e. \( y_t^P \) is a local maximum. Analogously, we define \( y_t^T \) to be a trough that occurs at time \( t \) if

\[
\begin{align*}
    y_t^T < y_{t-1}^T, & \quad y_t^T < y_{t-2}^T, \\
    \text{and} & \quad y_t^T > y_{t+1}^T, & \quad y_t^T > y_{t+2}^T.
\end{align*}
\]
\[ y_t^T < y_{t-1}^T, \quad y_t^T < y_{t-2}^T \quad \text{and} \quad y_t^T < y_{t+1}^T, y_t^T < y_{t+2}^T, \]

so that \( y_t^T \) is a local minimum. We then additionally impose two censoring rules: we require (a) each phase (i.e. expansion / recession) to last at least two quarters and (b) each business cycle (i.e. expansion + recession) to last at least five quarters. In order to deal with endpoints of the sample, (for instance, at \( t = 1 \), \( y_{t-1} \) is not defined), we remove the corresponding inequalities. However, in the specific case of \( y_1 \), this may produce funny results as the algorithm will mostly always recognize \( y_1 \) as a trough, even though this will usually not make economic sense. Thus, to avoid such curiosities, we assume the first observation of the sample not to be a trough. Results of the classification by the algorithm are shown in Table 3.

As seen above, the BBQ classification is consistent with observers’ comments (e.g., Kuodis, 2008). To gain more information, however, following the methodology set out in Claessens et al. (2009), we define a few simple characteristics of the business cycle. First, let the amplitude of the recession be \( A = GDP_t^P - GDP_{t+k}^T \), where the duration of recession is equal to \( k \) quarters. Second, we use a measure of cumulative loss of a recession, which is defined as \( L^C = \sum_{i=t}^{t+k} (GDP_i - GDP_i^P) \). This statistics takes into account both the length and amplitude of the recession and, thus, can be seen as a proxy for the total welfare loss of the crisis (Claessens et al., 2009). We also use two relative measures to abstract from units of measurement. First, we express the amplitude in percentage terms, i.e. we calculate \( 100\% \times (GDP_t^P - GDP_{t+k}^T) / GDP_t^P \); we do the same for the cumulative loss: \( \sum_{i=t}^{t+k} (GDP_i - GDP_i^P) / GDP_t^P \). These measures, calculated for the Russian Crisis and the Great Recession, are presented in Table 4.

We can see that the most recent economic downturn is almost three times longer and more than ten times “deeper”, as measured by the amplitude, than the Russian Crisis. This is consistent with the evidence from Claessens et al. (2009) who find that the amplitude and duration are positively correlated. Furthermore, cumulative loss, at almost 20 billions of 2000 Litas, is roughly equal to the GDP produced during one quarter at the peak of the cycle in the case of the Great Recession. Alternatively, the cumulative loss

\[^6\text{The code of the algorithm, written in GNU R, is available from the author upon request.}\]
is equal to approximately 21% of Lithuania’s real yearly GDP in 2008. In contrast, the Russian Crisis was much milder by these measures. This is also in line with the study cited above, as the authors find that the recessions that follow busts of property prices and are accompanied by credit crunches are both longer and deeper than other recessions, as is the case with the Great Recession (Kuodis and Ramanautskas, 2009).

**TABLE 4. Some measures of Lithuanian real GDP performance during the Russian Crisis (1999Q1–1999Q3) and the Great Recession (2008Q2–2010Q1), respectively, median values of measures obtained by Claessens et al (2009) and Hong et al. (2010) for all recessions analyzed**

<table>
<thead>
<tr>
<th>Measure</th>
<th>Russian Crisis</th>
<th>Great Recession</th>
<th>Claessens et al. (OECD)</th>
<th>Hong et al. (Emerging Asia)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude (millions of LTL)</td>
<td>276.3</td>
<td>3,310.8</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Amplitude (%)</td>
<td>2.4</td>
<td>16.1</td>
<td>1.9</td>
<td>5.5</td>
</tr>
<tr>
<td>Duration (quarters)</td>
<td>3.0</td>
<td>8.0</td>
<td>3.0</td>
<td>6.4</td>
</tr>
<tr>
<td>Cumulative loss (millions of LTL)</td>
<td>633.9</td>
<td>1,7263.8</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Cumulative loss (%)</td>
<td>5.6</td>
<td>84.1</td>
<td>3.0</td>
<td>6.9</td>
</tr>
</tbody>
</table>

Source: Claessens et al. (2009), Hong et al. (2010), Eurostat, author’s calculations.

Finally, we compare these measures to the ones obtained by Claessens et al. (2009) who analyzed business cycles in a sample of OECD countries over 1960–2007, and Hong et al. (2010) who analyzed the emerging Asian economies over 1961–2007 (Table 4). Claessens et al. calculate that the median duration of a recession in OECD countries is three quarters, which is exactly equal to the length of the Russian Crisis; the median cumulative loss percentage for the analyzed OECD countries is approximately 3%, which is almost half as low as the respective number for the Russian Crisis. However, one can observe that the median cumulative loss and the duration of a recession in emerging Asian economies are significantly higher than those of the Russian Crisis. On the other hand, the statistics cited above are almost incomparable to the Great Recession in Lithuania. Even if one considers what Claessens et al. (2009) classify as “severe recessions” for which the median cumulative loss percentage is 10%, it is still a relatively modest number compared to 84.1% (the median cumulative loss of analogously defined severe recessions in the Asian economies is approximately 17%).

These comparisons seem to imply that the two recessions in Lithuania were harsher than median recessions in OECD countries, while the Great Recession was more severe.
than median recessions in emerging Asian markets. However, such inferences should be made carefully as the samples considered by both Claessens et al (2009) and Hong et al (2010) do not contain data after 2007Q4, possibly making their estimates smaller. Furthermore, although it may be tempting to conclude that the higher severity of recessions in Lithuania, as compared to OECD, is a sign of inferior economic policy, such volatility may arise due to other factors. For instance, empirical studies show that GDP growth rates of small states are, on average, more volatile than those of other states, which is largely due to small countries’ higher openness to trade (Easterly and Kraay, 2000). Therefore, a serious statistical analysis is needed to verify or falsify such a statement. We present some possible methods in Conclusions.

Finally, the measures calculated above should not by any means be taken as objective estimates of true costs of recessions in Lithuania; for instance, if the economy was very much overheated at \( y_t^P \), the cumulative loss of a recession may be artificially overestimated. On the other hand, natural growth of output is not taken into account in our measure of the cumulative loss, as is the case with non-strictly-economic consequences of a recession, such as effects on health or loss of human capital, too. Therefore, this simplistic statistics perhaps should only be used for a careful comparative analysis and not as absolute measures of the depth of the recessions analyzed.

3. Output gap estimation via filtering

In the following section, we use Hodrick-Prescott (HP) filter and two band-pass filters to date the business cycle and obtain quantitative estimates of the output gap; we use these estimates to compare depths of the Russian Crisis and the Great Recession. Also, we analyze how much the Lithuanian economy was overheated in the run-up to the recent downturn. Finally, we give an empirical illustration of how wrong the estimates of the HP filter may be if used to estimate the size of the output gap on a real-time basis.

The HP filter (Hodrick, Prescott, 1997) aims to decompose the original series \( y_t \) into its directly unobservable trend (growth) and cycle components, \( g_t \) and \( c_t \), respectively, by solving the optimization routine

\[
\min_{\hat{g}_t} \sum (y_t - \hat{g}_t)^2 + \lambda (\Delta^2 \hat{g}_t)^2
\]  

where \( \Delta \) is the difference operator, i.e. \( \Delta y_t = y_t - y_{t-1} \), \( \lambda \) is the smoothing parameter, and \( \hat{g}_t \) denotes an estimate of \( g_t \) on the basis of data from the sample. The standard value of \( \lambda = 1600 \) was used as suggested by Ravn and Uhlig (2002). Due to the fact that the HP filter performs weakly at endpoints of a series (Mise et al., 2005), the output gap estimates at the beginning and end of the sample should be taken with a considerable caution

\[\text{footnote}{\text{9}}\]

\[\text{footnote}{\text{9}}\] We use the mFilter package in GNU R for all filters in this paper.
One can see from Fig. 1 that the Lithuanian economy was performing below its potential for most of the time from 1999Q2 until 2006Q2, with a median output gap of -1.62%. This is in contradiction to, e.g., Kuodis and Ramanauskas (2009) who estimate that Lithuania was 2–2.5% above its potential over 2000–2007. Our new estimates, however, are in line with the anecdotal evidence that, although the economy experienced a strong growth since 2000, average Lithuanian citizens started experiencing the fruits of this growth only around 2006–2007. The HP identifies the period 2006Q2–2008Q1 as the time of expansion with a median output gap of +7.01%. At the peak of the business cycle in 2008Q1, the Lithuanian economy was performing almost +10% above its potential. Thus, the results imply that the Lithuanian economy experienced a relatively short period of a highly overheated economic climate.

The HP output gap estimates also allow one to compare the depth of the Russian Crisis to the Great Recession. We find that at the trough of the Russian Crisis in 1999Q4, the Lithuanian economy was -3.30% below its potential. The HP filter identifies 2009Q1 as the trough of the Great Recession with an output gap of -7.4%. This estimate is twice as large, in the absolute sense, as the respective number for the Russian Crisis. However, the HP estimates near the endpoints of the sample must be treated with some scepticism; thus, more data are needed to make more convincing comparisons.

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10 This may be illustrated by the Consumer Confidence Index: the Index fluctuated around zero in 2006 and turned strongly positive only in 2007.
It must be stressed that the HP filter may give unsatisfactory estimates in some cases; for instance, Cogley and Nason (1995) argue that the filter may generate spurious business cycle dynamics. In order to check whether our results are robust to the use of different filters, we used two band-pass filters, namely, Baxter–King (BK) filter (Baxter and King, 1999) and Christiano–Fitzgerald (CF) introduced in Christiano and Fitzgerald (2003). We find that the results are qualitatively similar and follow the same pattern outlined above; a more detailed discussion is delegated to Appendix.

Finally, for an empirical illustration of the difficulties that arise when one wishes to estimate output gaps on a real time basis using the HP filter, see Fig. 2\textsuperscript{11}. As in Section 1, we split the sample into two parts; here, we only consider data over 1995Q1–2008Q1. Proceeding analogously as before, we find that in 2008Q1 Lithuania was performing below its potential, whereas, based on the whole sample, it is estimated to have been +10% above its potential. The CF filter, used for the same subsample, identifies the economy to be approximately +1% above its potential in 2008Q1. Thus, the sign of output gap is estimated correctly, although the extent to which the economy is overheated is underestimated dramatically. All of this reflects how difficult it may have been to see that the economy was overheated using simple filtering techniques ex ante, i.e. when information on the whole period was unavailable.

\textbf{FIG. 2. Lithuanian real GDP in 2000 Litas, adjusted seasonally and by working days, smoothed series and output gap, 1995Q1–2008Q1 (HP filter with $\lambda = 1600$); values of real GDP and smoothed series are shown on the left y-axis, while values of the cyclical component are shown on the right y-axis}

\textit{Source: Eurostat, author’s calculations.}

\textsuperscript{11} This example was provided by Dr Virmantas Kvedaras.
4. Calculating regime probabilities by Markov-switching models

In the following section, we use a simple Markov-switching (MS) model to obtain regime probabilities for the Lithuanian real quarterly GDP series. Second, we provide a brief economic interpretation of the model. Third, we report our simulation results on permanent effects of recessions to output. Lastly, we compare how the dating of the business cycle by the model relates to our previous findings.

We use a simplified version of the model, proposed by Hamilton (1989)\(^{12}\). The model is known for its ability to date recessions objectively (Hamilton, 2005, 2011). Let \( g_t \) be the simple quarter-on-quarter growth rate of real GDP at period \( t \). Consider the following univariate model

\[
g_t = \mu_{s_t} + \sum_{i=1}^{l} \beta_{s_t} g_{t-i} + \epsilon_t
\]

(4)

where \( k \) is the number of lags of the GDP growth rate, \( \epsilon_t \sim i.i.d. F_s(t) \), here \( F_s(t) \) denotes the cumulative distribution function (possibly conditional on \( s_t \)) and \( \mu_{s_t}, \beta_{s_t} \) are functions of \( s_t \in \{1, 2, ..., n\} \), an unobserved state variable. It is assumed that \( s_t \) follows a Markov-chain process with the \( n \)-dimensional square transition matrix

\[
P = \begin{pmatrix}
  p_{11} & \cdots & p_{1n} \\
  \vdots & \ddots & \vdots \\
  p_{n1} & \cdots & p_{nn}
\end{pmatrix},
\]

(5)

where \( p_{ij} \) are probabilities that the system will switch from the \( i \)-th to the \( j \)-th state, given that the system is currently in the \( i \)-th state. By the definition of probability, one constrains \( \sum_j p_{ij} = 1 \quad \forall i = 1, n \). In our model, we assume that \( p_{ij} \) are time-invariant (i.e. they do not change as a function of \( t \)) as well as that they are duration-independent, i.e. the probability that the system will switch from the \( i \)-th to the \( j \)-th state does not depend on the number of periods the system has already been in the \( i \)-th state. Whereas the first property is not very unrealistic (time-varying probabilities are perhaps more appropriate in modelling series where the regime shifts are very frequent, e.g., interest rates or returns on securities; Gray, 1996; Hamilton, 2005), the second one is somewhat problematic. It is quite possible that the probability of leaving the state of recession is higher after a few months of recession than at the beginning of it because of, for instance, a higher likelihood of government intervention. For instance, Durland and McCurdy (1994) find that recessions in the US are duration-dependent, whereas expansions are not. However, as our main goal is to merely date the business cycle as opposed to, e.g., forecast into the

\[^{12}\text{Estimation is done using MS_Regress package for MatLab (Perlin, 2009).}\]
future, this slightly unrealistic feature is perhaps acceptable since, empirically, models with duration-invariant probabilities do a good job of dating the business cycle (Hamilton, 2011). For details on estimation of the model by maximum likelihood, we refer the reader to Perlin (2009) and Hamilton (2004). We use the likelihood ratio (LR) test to determine the number of regimes modified for Markov-switching models as in Altug and Bildirici (2010); we choose the number of lags (i.e. \( l \)) by using the information criteria proposed by Akaike (1974) and Schwarz (1978).

A priori, one may expect the innovations \( \varepsilon_t \) not to be distributed normally because of a possible occurrence of outlier-like observations in the sample (i.e. tails of the density function of innovations may be “fat”); this was confirmed by experimentation: under an assumption of normality, the dating of recessions was not consistent with our previous results or the economic sense. In some cases, the optimization algorithm that was used to maximize the likelihood function, failed to converge. Thus, we let \( \varepsilon_t \sim t_{\nu t} \), i.e. innovations are \( t \)-distributed with \( \nu \) degrees of freedom, where \( \nu \) depends on the state that the system is currently in\(^{13}\).

By experimentation, we saw that allowing \( \beta \)'s to vary with states does not help to date the recessions significantly\(^{14}\). Adding into consideration the fact that our sample is small (only 60 observations), we decide to allow only \( \mu_{st} \) to switch. Further, we use the LR test to test for two over three regimes; we strongly reject the null hypothesis of two regimes and thus use three regimes in the model. By minimising information criteria mentioned above, we chose \( k = 1 \). Results of the estimated model are shown in Table 5.

**Table 5.** Estimates of the Markov-switching model for Lithuanian real GDP growth rates in percentage points (see text)

<table>
<thead>
<tr>
<th>Dependent variable: ( g_t )</th>
<th>( \hat{\mu}_{st} )</th>
<th>( \hat{\sigma}_{st} )</th>
<th>( \hat{\beta}_1 )</th>
<th>AIC</th>
<th>BIC</th>
<th>Log-likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 1</td>
<td>-0.3263</td>
<td>1.8362*</td>
<td>-0.0543</td>
<td>2.0371 (+( \infty ))</td>
<td>240.6942</td>
<td>276.2981</td>
</tr>
<tr>
<td>(0.4713)</td>
<td>(1.0442)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>State 2</td>
<td>1.7250**</td>
<td>100.00</td>
<td>-0.0543</td>
<td>2.0371 (+( \infty ))</td>
<td>240.6942</td>
<td>276.2981</td>
</tr>
<tr>
<td>(0.7744)</td>
<td>(467.7770)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>State 3</td>
<td>1.9474***</td>
<td>100.00</td>
<td>2.0371 (+( \infty ))</td>
<td>240.6942</td>
<td>276.2981</td>
<td>-103.347</td>
</tr>
<tr>
<td>(0.3018)</td>
<td>(199.2710)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Numbers in parentheses are standard errors; *, **, *** denote significance at 0.10, 0.05 and 0.01 levels, respectively.

**Source:** Eurostat, author’s calculations.

\(^{13}\) It may be of interest to observe that, using data only up to 2009Q1, the model with normally distributed disturbances seems to do an adequate job at dating business cycles; its results closely match the ones presented below.

\(^{14}\) Also, the problem of convergence becomes significant if the number of parameters to be estimated is high (Perlin, 2009); in that case, one may not be sure whether the maximum found is the global one. Thus, we fit as parsimonious a model as it seems to be economically sensible.
Also, we obtain estimates of the transition matrix $P$:

$$
\hat{P} = \begin{pmatrix}
0.90^{**} & 0.10 & 0.00 \\
0.00 & 0.00 & 1.00^{**} \\
0.05 & 0.15^{**} & 0.80^{**}
\end{pmatrix},
$$

(6)

where asterixes indicate significance at different levels analogously as in Table 5. By looking at the coefficients for $\hat{\mu}_{x}$, one can interpret States 1, 2, and 3 as “Recession”, “Medium Growth”, and “High Growth”, respectively. The coefficients have their expected signs, except for $\hat{\beta}_1$, which is not even statistically significant. To test for a serial correlation, we use the correlogram of the residuals and calculate the Ljung–Box statistics; we find that we cannot reject the zero of non-serial correlation at standard confidence levels.

We can see that the estimated number of degrees of freedom in State 1 is less than 2; it is known that the variance of a random variable, following such a distribution, is infinite (e.g., Weisstein, 2011). Thus, the results imply that the variance of shocks to GDP growth rates in the case of recession may not be finite (cf. Section 1); in times of medium and high growth, the variance is, in contrast, finite. Economically, this would imply that a negative shock in a time of recession may be arbitrarily (negatively) large.

Furthermore, the estimated transition matrix shows that the system may enter the state of recession from the state of high growth only; however, one can leave it only via the state of medium growth. This seems consistent with the economic intuition, in the sense that recessions in Lithuania are usually an abrupt end to a period of a strong growth (i.e. rapid economic convergence). However, once the recession has ended, the economic growth does not “take off” immediately. Via Okun’s Law, this may provide some insights in explaining the phenomenon of jobless recovery (Okun, 1970; Groshen, Potter, 2003). Moreover, utilising formulas for the ergodic Markov chains, one can calculate the limiting probabilities $\pi_i, i = 1, 3$ that can be interpreted as the part of time that is spent in each of the state over a long run (Ross, 2000). We find that $\hat{\pi}_1 = 29.41\%$, $\hat{\pi}_2 = 11.76\%$, and $\hat{\pi}_3 = 58.82\%$. $\hat{\pi}_1$ seems to be relatively high; this, however, could result from the fact that the Great Recession had not yet ended at the time of writing, according to the model which may imply that the sample is somewhat “biased” to include more recessions than is usually the case. Turning back to Section 2, one sees that the BBQ algorithm identified only 17.74% of all sample periods as belonging to the state of recession. Thus, the MS model seems to be stricter in identifying recessions than the BBQ algorithm.

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15 We also tested this result (in a somewhat ad hoc fashion) using $\chi^2$ and the Kolmogorov–Smirnov goodness-of-fit tests. To do this, we classified residuals as belonging to State 1 when the probability of a system being in State 1 was higher than 0.5 and taking all the remaining residuals to form a separate group. We found that we could not reject the null hypothesis by any of the tests that the residuals from the latter-mentioned sub-sample come from $t$ distribution with 100 degrees of freedom at standard confidence levels. However, the null hypothesis that residuals from State 1 follow the $t$ distribution with 1.8362 degrees of freedom was rejected strongly by both tests. Nevertheless, as the number of such residuals is only 14, one perhaps cannot make very strong conclusions as the empirical distribution is highly irregular as well as bimodal.
It is known that if the true DGP of GDP growth rates is a Markov-switching one, then a recession has a permanent effect on output (Hamilton, 1989). Consider Fig. 3 in which the natural logarithm of Lithuania’s real GDP over 1995Q1–2003Q4 is plotted along the trend line estimated for 1995Q1–1998Q4; a straight line, in such log-levels case, implies that the output is growing at a constant quarterly rate. It is seen in the picture that once the Russian Crisis had ended, Lithuanian GDP did not return to its pre-crisis trend level, i.e. the recession seems to have had a “permanent” negative effect. Hamilton (1989) gives a closed-form formula solution to \( \lim_{n \to + \infty} E(GDP_{t+1} | S_t = 1) - E(GDP_{t+1} | S_t = 2) \) for his model specification, where \( S_t = 1 \) means that the economy is in a state of expansion and \( S_t = 2 \) that it is in a recession. However, we do not know any such formula for the exact specification used in this paper. For this reason, we use a simple Monte Carlo experiment to calculate the long-run effect of a recession and derive standard errors by bootstrapping. Our approach is as follows: first, we assume that the true DGP is as the one in our model; then, we simulate two times 10 000 realizations of the DGP process, when (a) the process starts in a state of recession (State 1 in our estimated model); (b) the process starts in a state of high growth (State 3 in our estimated model). We let the process continue for \( t = 40 \) quarters (i.e. 10 years) and calculate the terminal values \( GDP_{40} \) for both (a) and (b) cases, letting \( GDP0 = 100 \). Finally, we use all simulated data to calculate the mean and median percentage differences between the two values as well as obtain standard errors and 95% confidence intervals by bootstrapping, using 2000 replications\(^{16}\). We also calculate the long-term effect of a negative unitary shock in \( \varepsilon_t \) in an analogous manner, the only difference being the fact that the initial state is now chosen randomly, using the long-run probabilities shown above.

\[ \begin{align*}
\text{FIG. 3. Log-levels of real GDP over 1995Q1–2003Q4 and trend estimated by OLS for 1995Q1–1998Q4} \\
\text{Source: Eurostat, author’s calculations.}
\end{align*} \]

\(^{16}\) We use boot package in GNU R for bootstrapping standard errors and confidence intervals.
The results of our simulation are reported in Table 6. Both the mean and median are around -6%, which is a little higher than the results obtained by Hamilton (1989) on permanent effects of a recession to the US output. This means that if the DGP of GDP growth rates in Lithuania is adequately described by (4), a typical recession in Lithuania decreases the output by about 6% permanently. In contrast, the effects of a unitary negative shock are negligible; the estimated confidence intervals show that such effects are not statistically significant at \( \alpha = 0.05 \) for the mean. This is also largely consistent with Hamilton (1989) who finds that such effects are quantitatively of a much smaller magnitude. However, as Kim et al. (2005) show, the estimates of permanent effects of a recession are sensitive to different Markov-switching specifications. Thus, the measures provided in Table 6 should perhaps be considered as first approximations rather than the “true” values of permanent effects of recessions.

**Table 6. Estimates of simulated long-run effects of a recession and negative unitary shock on Lithuania’s real GDP (see text), with respective 95% confidence intervals (CI). Numbers in parentheses are bootstrapped standard errors based on 2.5th and 97.5th empirical quantiles.**

<table>
<thead>
<tr>
<th></th>
<th>Recession</th>
<th>Negative unitary shock in ( \varepsilon_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>-5.96% (0.11%)</td>
<td>-1.55% (2.45%)</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>-6.20% (0.10%)</td>
<td>-0.68% (0.20%)</td>
</tr>
<tr>
<td><strong>Mean CI</strong></td>
<td>(-6.17%, -5.75%)</td>
<td>(-7.24%, 2.64%)</td>
</tr>
<tr>
<td><strong>Median CI</strong></td>
<td>(-6.36%, -5.98%)</td>
<td>(-1.04%, -0.27%)</td>
</tr>
</tbody>
</table>

*Source: Eurostat, author’s calculations.*

Finally, the model can be used to obtain probabilities that the system is currently in a certain unobserved state. Such probabilities can then be used to date the cycle (Fig. 4; Hamilton, 1989). By using a simple 0.5 rule (Pelagatti, 2002), the model estimates both recessions to be longer than the BBQ algorithm: 1998Q4–1999Q4 for the Russian Crisis and since 2008Q2 (i.e. the downturn has not yet ended) for the Great Recession. The results are largely consistent with the output gaps estimated via filtering, although the MS model indicates that the period of high growth started in 2000. However, this may not be a contradiction at all. The MS model is showing effectively that the growth rates over 2000–2007 are of a “similarly” high magnitude. Yet, if the growth rate of natural output was decreasing during this period, the economy may have still become overheated. Such a proposition is not unrealistic; for instance, one of the implications of the famous Solow (1956) model is that for a country whose initial capital stock relative to labour force is smaller than the equilibrium ratio, its output will grow faster until reaching its steady level. The HP estimates show that the growth of natural output in Lithuania indeed reached its maximum in 2003Q3 and was decreasing afterwards (Fig. 4).
**Conclusions**

In this paper, we used three basic univariate methods to date the business cycle in Lithuania over 1995–2010, namely the BBQ algorithm, time series filtering and the Markov-switching regression models. We find that the results are largely consistent for all the techniques used (see Fig. 5). The simple BBQ algorithm, however, seems to be somewhat more robust as the filters cannot be used near endpoints of the sample and the Markov-switching models date the recessions correctly only as long as the theoretical DGP is a Markov-switching one or can be approximated closely by it, which is in accordance to previous studies (Harding and Pagan, 2002). On the other hand, the BBQ algorithm provides significantly less information than the other two techniques. Thus, the use of a spectrum of business cycle dating and measurement tools seems to be optimal in making economic policy decisions.

Our main empirical findings are as follows. First, we show quantitatively that Lithuania underwent two major recessions in the sample period; however, the Russian Crisis was much less severe than the Great Recessions by all measures used. We estimate the cumulative cost of the Great Recession to be approximately equal to 20 billion of 2000 Litas, which is approximately a quarter of the nation’s yearly GDP in 2008. Further, using the Hodrick–Prescott filter, we show that Lithuania was more or less on a balanced growth path from the end of the Russian Crisis until the end of 2006; afterwards, it experienced a brief period when its economy was highly overheated. Our dating of recessions is confirmed by the Markov-switching model; the estimates of regime probabilities imply, however, longer durations of recessions for both the Russian Crisis and
the Great Recession and a greater level of overheating. In addition, we find evidence that the variance of disturbances of the real GDP growth rates in a state of recession may not be finite. Based on the empirical transition matrix, we argue that recessions are usually entered from the state of high growth; however, once the recession has ended, the growth is slower before “taking off” again. Lastly, we show that a typical recession may be associated with a significant decrease in long-run output.

However, our paper has certain limitations. As Lucas (1977) wrote in his seminal article, a defining characteristic of the business cycle is co-movement among different macroeconomic time series. Our approach, being univariate, leaves this important issue aside. A simple way of taking the phenomenon into account may be using Markov-switching VARs to obtain regime probabilities (Hamilton, 2005). As for the BBQ algorithm and various filters, they are constrained, by their very nature, to the analysis of a single time series at a time; however, researchers may find the algorithm’s results useful in the analysis of how other aggregated time series behaved at the time and/or whether the dating of recessions by the BBQ on different macroeconomic aggregates is similar. Furthermore, we took a “black-box” approach to modelling, i.e. we did not explain on what factors the regimes actually depend. This is, however, of crucial importance to practical economic policy-making. In the Markov-switching framework, the issue could be addressed by introducting well-chosen exogenous variables. Another, perhaps a more appropriate ap-
proach may be to regress the cyclical component of real GDP, extracted by a certain time series filter, on such indicators as property prices, consumer price index, current account deficit / surplus, controls for fiscal policy, etc. for a panel of EU countries. This would also allow one to make inferences on why some countries suffered more in face of the Great Recession than others – again a point that was not adequately addressed in this paper. We believe that all of these areas may be fruitful for future research.

REFERENCES


APPENDIX: Output gap estimation by the Baxter–King and Christiano–Fitzgerald filters

In the appendix, we show that using BK and CF filters to estimate the output gaps produces the results that are qualitatively and quantitatively similar to the ones obtained in Section 3.

First, we use the BK filter (Baxter, King, 1999). The main difference of the BK filter, when contrasted to the HP, is that it is a band-pass filter, i.e. its goal is to extract the specific frequencies that are associated with business cycles (e.g., those ranging from 1 to 8 years). In the terminology used by Estrella (2007), the BK filter is a solution to the so-called frequency extraction problem, whereas the HP filter solves the signal extraction problem (i.e. how to extract the signal $c_t$ from a given time series $y_t$).

![Graph showing Lithuanian real GDP in millions of 2000 Litas, adjusted seasonally and by working days, smoothed series and output gap, 1998Q1–2007Q2 (BK filter with a lower and upper frequency bounds of 8 and 32 quarters, respectively)](image)

**FIG. 6.** Lithuanian real GDP in millions of 2000 Litas, adjusted seasonally and by working days, smoothed series and output gap, 1998Q1–2007Q2 (BK filter with a lower and upper frequency bounds of 8 and 32 quarters, respectively), values of real GDP and smoothed series are shown on the left y-axis, while values of the cyclical component are shown on the right y-axis.

*Source: Eurostat, author’s calculations.*

The results of the BK filter, with the lower and upper frequency bounds of 8 and 32 quarters, respectively, are shown in Fig. 6. Baxter and King (1999) argue that the bands should be 6 and 32 quarters for quarterly data, in light of the seminal study of the business cycle by Burns and Mitchell (1946). However, a slightly higher lower bound was chosen in this case as the GDP series of developing countries are usually more volatile than those of mature economies. We can see that the results are more or less in agreement with the previous results obtained with the HP filter. According to the estimates, the Lithuanian economy was operating below its potential in the period 1999Q2–2006Q2 with a median output gap of -0.61%, with the exception of six successive quarters from 2003Q2 to 2004Q3 when the cyclical component of GDP was slightly above zero. How-
ever, since the BK filter by its construction drops observations at the beginning and end of the sample, it is not possible to use it for the analysis of most recent GDP dynamics. To use a CF band-pass filter (Christiano, Fitzgerald, 2003), the data should be de-trended, as noted by Estrella (2007); i.e. a CF filter should be applied on $\hat{y}_t = y_t - b_0 - b_1 t$, where $b_0$, $b_1$ are coefficients obtained by ordinary least squares (OLS) regression. An attractive feature of the filter is that it can be used on a “real-time” basis as it does not remove some of the observations as the BK filter does and fares better near the end-points of the sample than the HP filter.

![Graph](image)

**Fig. 7.** Lithuanian real GDP in millions of 2000 Litas, adjusted seasonally and by working days, smoothed series and output gap, 1995Q1–2010Q2 (CF filter with lower and upper frequency bounds of 8 and 32 quarters, respectively), values of real GDP and smoothed series are shown on the left y-axis, while values of the cyclical component are shown on the right y-axis

*Source: Eurostat, author’s calculations.*

The CF filter identifies the period from 1999Q3–2006Q2 as a period of an underperforming economy, with the exception of three consecutive quarters starting in 2003Q2 when the growth was slightly greater than zero (Fig. 7). During the boom (i.e. after 2006Q2), the economy was, on average, +4.94% above its potential (median value) with a maximum value of +9.85%. However, the CF estimates of the recession are almost twice as severe as those of the HP: they identify 2009Q4 as the trough with an output gap of -13.56%. It is of some interest to note that in 2010Q4 the economy was still +7.35% below its potential according to the CF estimates (the analogous value obtained with the HP is slightly smaller, in absolute terms, at -6.52%).

Thus, the results are, overall, in agreement for all of the filters used; this can also be verified by looking at the correlation matrix calculated for the values of the cyclical
component of the filters used (Table 7). While the correlations are not perfect, they seem to be high enough to ensure the robustness of our qualitative conclusions.

**Table 7. Correlation matrix of the cyclical component obtained with the filters used. Sample period is 1995-I–2010-II for the correlation between the HP and the CF and 1998-I–2007-II otherwise.**

<table>
<thead>
<tr>
<th></th>
<th>HP</th>
<th>BK</th>
<th>CF</th>
</tr>
</thead>
<tbody>
<tr>
<td>HP</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BK</td>
<td>0.946476</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>CF</td>
<td>0.89305</td>
<td>0.876319</td>
<td>1</td>
</tr>
</tbody>
</table>

*Source: Eurostat, author’s calculations.*