KOMPIUTERINIS MODELIAVIMAS

On the experimental investigation of investment strategies in the real and virtual financial markets

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The optimal financial investment (Portfolio) problem was investigated by leading financial organizations and scientists. The aim of these works was to define the optimal diversification of the assets depending on the acceptable risk level.

The aim of the paper is to evaluate different investment strategies in the real and virtual financial markets. This aim is the new element of the proposed simulation system since optimization is performed in the space of investment strategies; both daily and long-term. A number of different investment strategies are presented, including the ones based on the Modern Portfolio Theory (MPT).

The simulated investment procedures include different prediction methods. The methods that minimize the mean absolute error (MAE) are added to the traditional ones that minimize the least squares error (MSE). The results of the virtual financial market are compared with historical data.

The model is designed as a tool to represent the behavior of an individual investor which wants to predict how the expected profit depends on different investment strategies using different forecasting methods of real and virtual stocks.

Introduction

The traditional approach is represented by the Modern Portfolio Theory (MPT). MPT was created by Harry Markowitz (1959) and William Forsyth Sharpe (1994). A number of investment organizations are making decisions using the software based on the theoretical results of Robert C. Merton and Myron S. Scholes (1972). The recent developments and applications of MPT are discussed in (Jack Clark Francis, 2013; Baker and Filbeck, 2013; Sortino, 2009). Some limitations of this theory have been noticed during the recent financial crisis when the investors experienced considerable losses (Krugman, 2009).

Financial market simulators are developed to satisfy the needs of small individual investors. The examples are the StockTrak global portfolio simulator, the MarketWatch, virtual stock exchange", and the „Stock Simulator" of Investopedia. Some banks offer their own investment simulators such as the Barclays Fantasy Investment Game. Users of these simulators working with „Virtual Stocks" are informed about the results. The graphical user interfaces are friendly. However, the theoretical basis of these models and computing algorithms remains unknown. So the users cannot grasp the reasons why they win and why they experience losses.

To present the individual stockholders with a tool where everything is open is one of the aims of the PORTFOLIO model introduced in this paper. To accomplish this task, we deviate from the traditional portfolio models since this model is for defining the best investment strategies but not just the best diversification of assets what is a new element of this work.

In contrast to well-known results, this work not only simulates traditional results of utility and portfolio theories, but also complements them by various investment procedures and renders a possibility to users to develop and implement their own investment strategies.

Thus, the model can be useful to studies, to scientific collaboration, and to stockholders who are solving optimal investment problems and regard risk in an individual way. The market prediction and portfolio optimization were regarded in most of the financial market research. The investigation of different investment strategies is the feature of this work.

In Mockus and Raudys (2010); Mockus (2012), a preliminary investigation of the virtual stock exchange of a single stock is discussed. The results of these papers helped to initiate this work that simulates the optimal investment problem in the multi-stock market. Therefore we just refer to paper of Mockus (2012) for a mathematical description of the singe stock model and describe mainly the new resultas of experimental investigation. However, some definitions and expressions that describe the buying-selling strategies and investors' profit are repeated for convenience of reading.

1. The PORTFOLIO Model of a Virtual Financial Market

1.1 Buying and Selling Strategies

We consider a virtual market of I major players $i = 1,...,I$ and $j = 1,...,J$ stocks. The following notation is used:

$z(t, j) = z(t,i, j)$ is the price of stock j at time t, predicted by the player i,

$Z(t, j)$ is the actual* price at time t,

$U(t, j) = U(t,i, j)$ is the actual profit accumulated at time t by the player i buying-selling stock j,

$\delta(t, j)$ is the dividend of stock j at time t,

$\alpha(t)$ is the yield at time t,

$\gamma(t)$ is the interest rate at time t,

$h(t) = \gamma(t) - \alpha(t)$ is the haircut**,

$\beta(t,i)$ is the relative stock price change at time t as predicted by the player i

$$\beta(t,i, j) = (z(t+1,i, j) - Z(t, j) \;/ Z(t, j), \quad (1)$$

Expected profitability*** (relative profit) $p(t,i, j)$ of an investment at time t depends on the predicted change of stock prices $\beta_i(t, j)$, dividends $\delta_i(t, j)$, the bank rate $\alpha(t)$, and haircut $h(t)$

$$\begin{aligned} p(t,i, j) &= \beta(t,i, j) - \alpha(t) + \delta(t) - h(t) = \\ &= \beta(t, j) + \delta(t) - \gamma(t). \end{aligned} \quad (2)$$

The aim is profit, thus a customer i will buy some amount $n_b(t,i, j) \geq n(t, j)$ of stocks j, if profitability is greater comparing with the relative transaction cost $\tau(t,n)$; $p(t,i, j) > \tau(t,n)$, and will sell stocks, if the relative loss (negative profitability $-p(t,i, j)$) is greater as compared with the transaction cost $p(t,i, j) < -\tau(t,n)$, or will do nothing, if $-\tau(t,n) \leq p(t,i, j) \leq \tau(t,n)$. Here the relative transaction cost is defined as the relation

$$\tau(t,n) = \frac{\tau_0}{n(t, j)Z(t, j)}, \quad (3)$$

* The term 'actual' means simulated

** In finance, a haircut is a part that is subtracted from the value of the assets that are being used as collateral. The size of the haircut reflects the perceived risk associated with holding the assets.

*** The term "profit" can define losses if negative terms prevail.

where τ_0 is the actual transaction cost and $n = n(t, j)$ is the number of transaction stocks. It follows from the equality $\tau(t,n) = p(t,i,j)$ that the minimal number of stocks to cover transaction expenses is

$$n(t, j) = \frac{\tau_0}{p(t,i,j)Z(t,j)}. \quad (4)$$

Therefore, the buying-selling strategy $S(t,i,j)$ of stock j by the customer i at time t in terms of profitability levels is as follows

$$S(t,i,j) = \begin{cases} \text{buy } n_b(t,i) \geq n(t,j) \text{ stocks,} \\ \qquad \text{if } p(t,i,j) \geq \tau(t,n) \text{ and } n \leq n_b^{\max} \\ \text{sell } n_s(t,i,j) \geq n(t,j) \text{ stocks,} \\ \qquad \text{if } p(t,i,j) \leq -\tau(t,n) \text{ and } n \leq n_s^{\max}, \\ \text{wait, if } |\, p(t,i,j)| \leq \tau(t,n^{\max}). \end{cases} \quad (5)$$

Here $n^{\max} = \max(n_b^{\max}, n_s^{\max})$, where $n_b^{\max}$ is the maximal number of stocks to buy, and $n_s^{\max}$ is the maximal number of stocks to sell.

If $n_b(t,i,j) = n_b^{\max}$ *and* $n_s(t,i,j) = n_s^{\max}$, (6)

then this buying/selling strategy reflects the behavior of risk-neutral stockholders which invest all available resources if the expected profitability is higher than the transaction cost. If the expected losses are greater, then all the stocks are sold. It means that stockholders may tolerate a considerable probability of losses, if the expected profits are positive. In this way, the maximal expected profit is provided. However, the probability to get losses instead of profits could be near to 0.5.

From expressions (1) and (2), the buying-selling strategy $S(t,i,j)$ in terms of stock price levels is

$$S(t,i,j) = \begin{cases} \text{buy } n_b(t,i,j) \geq n(t,j) \text{ stocks,} \\ \qquad \text{if } Z(t,j) \leq z_b(t,n,i,j) \text{ and } n \leq n_b^{\max}, \\ \text{sell } n_s(t,i,j) \geq n(t,j) \text{ stocks,} \\ \qquad \text{if } Z(t,j) \geq z_s(t,n,i,j) \text{ and } n \leq n_s^{\max}, \\ \text{wait, otherwise.} \end{cases} \quad (7)$$

Here the price level of the player i to buy at least $n = n(t,j)$ stocks at time t is

$$z_b(t,n,i,j) = z(t+1,i,j)/(1-\delta(t) + \\ +\alpha(t) + h(t) + \tau(t,n)). \quad (8)$$

The price level of the player i to sell at least $n = n(t,j)$ stocks at time t is

$$z_s(t,n,i,j) = z(t+1,i,j)/(1-\delta(t) + \\ +\alpha(t) + h(t) - \tau(t,n)), \quad (9)$$

where $z(t+1,i,j)$ is the stock j price predicted by the investor i at time $t+1$.

The market buying price at time t is the largest buying price of players $i = 1,...,I$

$$z_b(t,n) = z_b(t,n,i,j^{\max}),$$

where

$$i^{\max} = \arg\max_i z_b(t,n,i,j).$$

The market selling price at time t is the lowest selling price of players $i = 1,...,I$

$$z_s(t,n) = z_s(t,n,i,j^{\min}),$$

$$i^{\min} = \arg\min_i z_s(t,n,i,j).$$

1.2 Price Simulation

1.2.1 Buying-selling price

The market buying price of stock j at time t is the largest buying price of players $i = 1,...,I$

$$z_b(t,n,j) = z_b(t,n,i,j^{\max}),$$

where

$$i^{\max} = \arg\max_i z_b(t,n,i,j).$$

The market selling price at time t is the lowest selling price of players $i = 1,...,I$

$$z_s(t,n,j) = z_s(t,n,i,j^{\min}),$$

$$i^{\min} = \arg\min_i z_s(t,n,i,j).$$

The number of stocks j owned by the player i at time $t+1$ is

$$N(t+1,i,j) = \begin{cases} N(t,i,j) + n_b(t,n,i,j), \\ \qquad \text{if } Z(t,j) < z_b(t,n,j), \\ N(t,i,j) - n_s(t,n,i,j), \\ \qquad \text{if } Z(t,j) > z_s(t,n,j), \\ N(t,i,j), \text{ if no deal.} \end{cases} \quad (10)$$

Here $n_b(t,n,i,j)$ and $n_s(t,i,j)$ are the numbers of stocks j for buying and selling operations by the player i at time t.

1.3 Investors Profit

The product $N(0,i,j)\,Z(0,j)$ is the initial investment to buy $N(0,i,j)$ shares j using an investors' own capital at the initial price $Z(0,j)$. The initial funds to invest are $C_0(0,i)$ and the initial credit limit is $L(0,i)$. $L(t,i)$, $t=1,...,T$ is the credit available for a customer i at time t. The investors' own funds in cash $C_0(t,i)$ available for investing at time t are defined by the recurrent expression

$$C_0(t,i) = C_0(t-1,i) - \sum_j (N(t,i,j) - \\ - N(t-1,i,j)\ \ Z(t,j), \quad (11)$$

where $t=1,...,T$. Here the product

$(N(t,i,j) - N(t-1,i,j)\ \ Z(t,j)$

defines the money involved in buying-selling stocks.

Stocks are obtained using both investor's own money $C_0(t,i)$ and the funds $b(t,i)$ borrowed at the moment t. The borrowed sum of the stockholder i accumulated at time t is

$$B(t,i) = \sum_{s=1}^{t} b(s,i)), \quad (12)$$

The symbol $b(t,i)$ shows what the user i borrows at the moment t

Part of Profit by Stock j

In long-term investment strategies using the Sharpe ratio, the general profit should be divided between different stocks. Denote by $C_0(0,i,j) \le L(0,i,j)$ the initial funds to be invested in the stock $j=1,...,J$, where $L(0,i,j)$ is the initial credit limit for stock j and

$$\sum_j L(0,i,j) = L(0,i) \quad (13)$$

For example, the initial funds may be divided into equal parts

$$\sum_j C_0(0,i,j) = C_0(0,i)/J \quad (14)$$

The investors' own funds in cash $C_0(t,i,j)$, accumulated buying-selling stocks j and available for investing at time t in the stock j, are defined by the recurrent expression below

$$C_0(t,i,j) = C_0(t-1,i,j) - (N(t,i,j) - \\ - N(t-1,i,j)\ \ Z(t,j), \quad (15)$$

where $t=1,...,T$. Here the product

$(N(t,i,j) - N(t-1,i,j)\ \ Z(t,j)$

defines the money involved in buying-selling stocks. j

Stocks are obtained using both investors' own money $C_0(t,i,j)$ and the funds $b(t,i,j)$ borrowed at the moment t. The borrowed sum of the stockholder i for the stock j accumulated at time t is

$$B(t,i,j) = \sum_{s=1}^{t} b(s,i,j)), \quad (16)$$

where

$$\sum_j B(t,i,j) = B(t,i) \quad (17)$$

The symbol $b(t,i,j)$ shows what the user i borrows at the moment t for the stock j.

The general borrowing expenses for stock j are

$$B_{sum}(t,i,j) = B(t,i,j) + \sum_{s=1}^{t} B(s,i,j)\ \ \gamma(s,i)), \quad (18)$$

where the first term denotes the loan accumulated at time t, the second term shows the interest, and

$$\sum_j B_{sum}(t,i,j) = B_{sum}(t,i). \quad (19)$$

The investor i gets a profit as the difference between the income from selling and buying stocks $D(t,i,j)$ and expenses for the borrowing funds $B_{sum}(t,i,j)$

$$U(t,i,j) = C_0(t,i,j) + D(t,i,j) - B_{sum}(t,i,j), \quad (20)$$

where

$$D(t,i,j) = N(t,i,j)\,Z(t,j) - N(0,i,j)\,Z(0,j), \quad (21)$$

The investor's i profit from the stock j at the end of investment period is denoted as

$$U_{i.j} = U(T,i.j), \quad (22)$$

where

$$\sum_j U_{i.j} = U_i, \quad (23)$$

If for some reason equalities (17), (19), and (23) are violated, then the normalization of components may be applied to restore them.. the funds invested in buying stocks j at time $T.x_j =$ The bank profit expressions are the same as in the single stock market model of Mockus (2012).

1.4 Multi-Level Operations

1.4.1 Strategy No. 1, ordinary stockholder: selling and buying by profitability levels

In the opinion of some professional brokers we have interviewed, one needs at least three buying profitability levels $p_b(t,i,j,l),\ l=1,2,3$, where

$$p_b(t,i,j,l+1) > p_b(t,i,j,l),\ p_b(t,i,j,1)=\tau(t), \quad (24)$$

and three selling profitability levels

$p_s(t,i,j,l),\ l=1,2,3$, where

$$p_s(t,i,j,l+1) < p_s(t,i,j,l),\ p_s(t,i,j,1)=-\tau(t),$$
$$p_b(t,i,j,l) > p_s(t,i,j,1), \quad (25)$$

to explain the behavior of major stockholders. The level $l=1$ means to buy-sell the minimal number of stocks. The level $l=3$ means to buy-sell as much stocks as possible, and the level $l=2$ is in the middle. Details of multi-level operations are in Mockus (2012).

2. Price Prediction

In this model, two versions of Autoregressive (AR(p)) and two versions of Autoregressive Moving Average (ARMA(p,q)) models are considered for stock rate predictions. The first versions – AR(p) and ARMA(p,q) – use traditional least squared approach, the second ones – AR-ABS(p) and ARMA-ABS(p,q) – minimize the absolute errors. The development and implementation of ARMA-ABS(p,q) is a new feature of this work.

The actual price of a stock at time $t+1$ is defined as the price of the previous deal of major stockholders plus the truncated Gaussian noise that represents the remaining small stockholders. Thus, the actual stock j price at time $t+1$ determined by buying-selling operations of the stockholder i is as follows

$$Z(t+1,i,j)=\begin{cases}(1-a)z_b(t,i,j^{\max})+aZ(t,j^{\max})+\varepsilon(t+1), & \text{if } p(t,i,j^{\max})>0,\\ (1-a)z_s(t,i,j,1)+aZ(t,j)+\varepsilon(t+1,j), & \text{if } p(t,i,j)\le p_s(t,i,j,1)=-\tau(t,j),\\ & \text{and } p(t,i,j)>p_s(t,i,j,2),\\ (1-a)z_s(t,i,j,2)+aZ(t,j)+\varepsilon(t+1,j), & \text{if } p(t,i,j)\le p_s(t,i,j.2),\\ & \text{and } p(t,i.j)>p_s(t,i,j,3),\\ (1-a)z_s(t,i,j,3)+aZ(t,j)+\varepsilon(t+1,j), & \text{if } p(t,i,j)\le p_s(t,i,j,3),\\ Z(t,j)+\varepsilon(t+1,j), & \text{if no deal.}\end{cases} \quad (26)$$

Here a reflects the market inertia, where $a = 0$ means there is no inertia and $a = 1$ describes maximal inertia (no market reaction to the last deal).

2.0.2 Strategy No. 2, risk-aware stockholders: selling all unprofitable stocks – buying the best ones

First, the stockholder i sells all the non-profitable stocks

$$p_s(t,i,j) \le -\tau(t,i,j), \quad (27)$$

and then invests all the available funds to buy the most profitable stock. The stockholder i does not sell the stock j, if the expected loss is smaller than the transaction cost $|p(t,i,j)| < \tau(t,i,j)$.

This selling strategy reflects risk-aware users which keep some less profitable stocks to avoid possible losses if predictions happen to be wrong.

2.0.3 Strategy No. 3, risk-neutral stockholders: buying the best stocks and selling all the rest

The risk-neutral stockholders use all available resources to buy the stock $j^{\max}$ which provides the highest expected profit:

$$j^{\max} = \arg\max_j p(t,i,j), \quad (28)$$

2.0.4 Strategy No. 4, risk-averse stockholders: selling and buying in proportion to profitability

Denote by J_+ a set of stocks with a positive profitability and by J_- the stocks with a negative profitability. Denote $J_b = |J_+|$ and $J_s = |J_-|$.

$$j_+^{\max} = \arg\max_{j\in J_+} p(t,i,j), \quad (29)$$

and

$$j_-^{\min} = \arg\min_{j\in J_-} p(t,i,j). \quad (30)$$

First, we sell stocks in proportion to $l=1,...,j_-^{\min}$ selling profitability levels $p_s(t,i,l)=p(t,i,j=l)$,

$l = 1,..., j_-^{\min}$. Then we use all accumulated resources to buy stocks in proportion to $l = 1,..., j_+^{\max}$ profitability levels

$$p_b(t,i,l) = p(t,i, j = l),\ l = 1,..., j_-^{\max}.$$

Strategy No. 5, Long Term Investment

In the previous sections, we analyzed short-term investing by daily decisions using different investment strategies. So, the search was in the strategy space. In this section, we consider the maximization the Sharp Ratio (Sharpe (1994)).

In the long term investing models, the time series are split into learning and testing sets. In the learning stage, the mean and variance of portfolio $P(x)$ profit are estimated using the first part of observations $1 \le t_0 < T$, where $x = (x_j,\ j = 1,..., J)$. Usually t_0 is about $T/2$. and the initial funds are equally divided among the stocks, meaning that $x_j^0 = C_0(0)/J$. Here $C_0(0)$ denotes initial funds of a single user.

Note that in virtual markets, the stock prices are generated by the interaction of different virtual investors. The search for the optimal distribution of funds is performed by maximizing the Sharpe Ratio. During the testing stage the profits of optimized portfolios are calculated using the remaining observations $s\ :\ t_0 < s \le T$.

The data of the learning stage are used to estimate average deviations and variances. The sample mean of portfolio $P(x)$ that contains stocks with weights $x_j,\ j = 1,..., J$ is as follows

$$m^u(x) = \sum_j x_j / x_j^0 \frac{1}{t_0} \sum_{t=1}^{t_0} U(t, j). \qquad (31)$$

Here $U(t, j)$ follows from (20) by omitting the investor‘s index i. The estimator of variance of the portfolio $P(x)$ is

$$(s^u(x))^2 = \frac{1}{t_0 - 1} \sum_j \sum_k \sum_{t=1}^{t_0} x_j / x_j^0 \,(U(t, j) - m^{u_j})\, x_k / x_k^0\, (U(t,k) - m^{u_k}). \qquad (32)$$

We maximize the Sharpe ratio in this specific form

$$\max_x \frac{m^u(x)}{s^u(x)}, \qquad (33)$$

because the risk free asset is regarded separately.

3. Experimental Results

The general aim of the experiments is to evaluate the profitability of different investment strategies in both the virtual and real markets, which involve well-known companies. In the experiments, the profits and average prediction errors of eight players by four different investmrnt strategies using eight prediction models each: $AR(1)$, $AR(3)$, $AR(6)$, $AR(9)$, $AR-ABS(1)$, $AR-ABS(3)$, $AR-ABS(6)$, and $AR-ABS(9)$ were tested. The time period is 360 days (virtual working days). This represents approximately 18 months of real time. The average daily and final values are estimated by 100 samples. Four investment strategies were tested in both the virtual and real environment including eight stocks of major companies.

3.1 Historical and Virtual Data

In this section, different investment strategies are investigated using historical data obtained automatically using the Yahoo data base. The historical prices of the following eight stocks were used: Microsoft (MSFT), Apple (AAPL), Google (GOOG), Nokia (NOK), Toyota (TM), Bank-of-America (BAC), Boeing (BA), and Oracle (ORCL). The time series covers about two years of rapid financial development (2008–2009) and include stocks eight major financial and technological organizations.

According to Figure 2, the maximal profit of \$20,000 was achieved using the first strategy by the prediction model $AR(6)$. Figure 1 shows average profits of eight prediction models by the first strategy using virtual data.

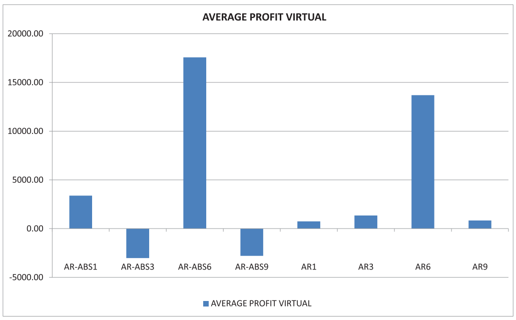
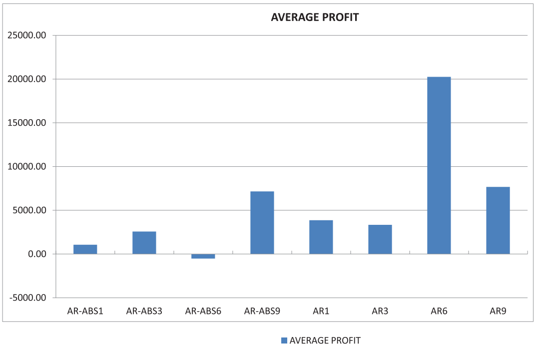
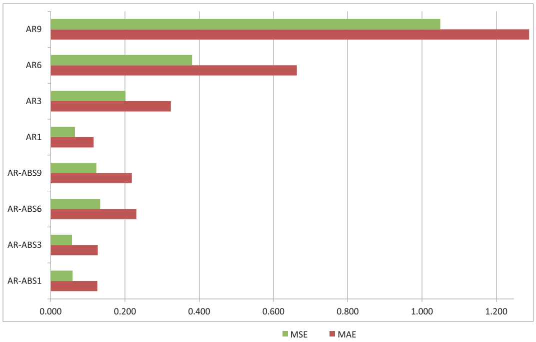


*Fig. 1. **Average profits of eight prediction models by the first strategy using virtual data***



F i g. 2. ***Average profits of eight prediction models by the first strategy using historical data***



F i g. 3. ***Aaverage prediction errors using different prediction models***

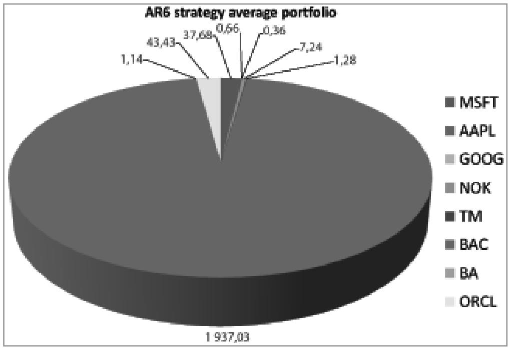
Figure 2 shows average profits of eight prediction models by the first strategy using historical data. Comparing Figures 1 and 2 shows that the profits and their distribution between different prediction models are different in real and virtual markets. The explanation is that the real data represent mainly the post-crisis period with gradual recovery of stock prices. In contrast, the virtual data were obtained assuming stable general conditions. However, a two year cycle of stock prices and profits was observed, in some experiments.

3.1.1 The Best Investment Strategy

Figure 3 shows average prediction errors using different prediction models. Figure 4 shows the most profitable portfolio defined using the first strategy and *AR*(6) prediction model. The dominant stock vas Banc-of-America (No.6), the second one was Oracle (No. 8) followed by Microsoft (No. 1).

The success of BAC can be explained by the rapid recovery of its stocks after the deep fall during depression.

Comparing average errors in Figures aapl-price.eps with profits in Figure prof-days-h1.eps we see that the minimal prediction error is provided by the model *AR*(1) and the maximal profits are achieved using the *AR*(6) model. This and other experiments indicate that minimal prediction errors do not necessarily provide maximal profits.



F i g. 4. ***The most profitable portfolio defined using the first strategy and AR(6) prediction model***

4. Concluding Remarks

The Game Theory is a suitable framework to model financial markets because the future market price of financial assets depends on predictions (and subsequent actions) of the market participants with conflicting interests.

The proposed financial market model PORTFOLIO is designed as a tool for simulating market processes in response to different changes of market parameters and for estimating the expected profits of different investment strategies using both the virtual and historical data. Convenient user interactions are provided by implementing the model as a Java applet and publishing it in an open web-site (Mockus, 2013).

A single-stock Stock Exchange Game Model (SEGM) was introduced in Mockus (2012) to simulate the behavior of several stockholders.

Since PORTFOLIO may be too simplistic for practical investing, it can serve as a useful tool for studies of market behavior by providing an easy way of simulating different scenarios of player strategies. For example, simulations can explain stock market reaction to deliberately set non-NE strategies of a major stockholder, such as manipulation of asset prices, designed to lower their value.

Thus, the PORTFOLIO model helps students of business informatics to understand better financial disasters that we are witnessing at present. The unexpected new result is the observation that the investment strategies using prediction models with minimal errors did not provide maximal profits.

REFERENCES

BAKER, HK; FILBECK, G. (2013). *Portfolio Theory and Management*. Oxford University Press, Oxford, UK.

BLACK, F.; SCHOLES, M. (1972). The pricing of options and corporate liabilities. *Journal of Political Economy, no.* 81, p. 637–654.

JACK CLARK FRANCIS, DK (2013). *Modern Portfolio Theory: Foundations, Analysis, and New Developments, + Website*. New York: Wiley.

KRUGMAN. P. (2009). How did economists get it so wrong? *The New York Times*, September 2.

MARKOWITZ, H. (1959). *Portfolio Selection: Efficient Diversification of Investments*. New York: Wiley.

MOCKUS, J. (2012). On simulation of the Nash equilibrium in the stock exchange context. *Informatica*, no. 23, p.77–104.

MOCKUS, J. (2013). PORTFOLIO. http://optimum2.mii.lt/igor/index.html/.

MOCKUS, J.; RAUDYS, A. (2010). On the efficient-market hypothesis and stock exchange game model. *Expert systems with applications*, vol. 37, p. 5673–5681.

SHARPE, WF. (1994). The Sharpe Ratio. *The Journal of Portfolio Management*, no. 21, p. 49–58.

SORTINO, F. (2009). *The Sortino Framework for Constructing Portfolios Focusing on Desired Target Return to Optimize Upside Potential Relative to Downside Risk*. Amsterdam: Elsevier Inc., Netherlands.

APIE EKSPERIMENTINĮ INVESTAVIMO STRATEGIJŲ TYRIMĄ REALIOSE IR VIRTUALIOSE FINANSŲ RINKOSE

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Santrauka

Darbo tikslas yra įvertinti įvairias investavimo strategijas pagal jų pelningumą realiose ir virtualiose finansų rinkose.

Darbe aprašytas atnaujintas finansų rinkos modelis, analizuojami eksperimentinių skaičiavimų rezultatai.