Development of a Mathematical Communication Curriculum

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The paper presents investigations and creation of a new Mathematical communication (MC) curriculum. The curriculum is intended to educate technologically well skilled mathematicians. In order to create the contemporary curriculum of an appropriate level, first of all, we considered the author's investigations on information and communication technologies (ICT) applicable to mathematics, as well as the relevant ICT and educational competence of mathematicians. The concept of MC is rather wide and can be interpreted in various aspects and considered at different levels. After investigation of its wide and diverse use we have drawn a conclusion that the curriculum "Mathematical communication", including all the main aspects of MC can realize the intention to educate contemporary innovative mathematicians and mathematics teachers. The curriculum was developed as a new branch of the study program "Mathematics and informatics", which was approbated, evaluated by external experts and was implemented at the Vilnius Pedagogical University in 2003. The new curriculum keeps the amount of mathematics subjects, the main structural elements and their amount in terms of credits of the study program.

1. Introduction

The development of information and communication technologies (ICT) has enriched ways, possibilities and importance of mathematical communication (MC) in mathematics education. Nevertheless, the investigations abroad (Cuban et al., 2001; Lavicza, 2006) and in Lithuania (Dagienė and Jasutienė, 2007) show that mathematics teachers do not use technological tools in a proper way in teaching mathematics. It is due to some factors: teaching traditions, attitude of authorities, technological abilities, but most of all, due to the competence of mathematics teachers. And it is especially slow the integration of technologies to teaching mathematics at the university level (Lavicza, 2006). The international investigation of technology use in mathematics teaching (Lavicza, 2007, 2008) emphasized the need to enhance innovations in mathematics teaching at all levels and investigated the factors that influence its integration into mathematics curricula. Personal characteristics and technological competence of teachers and lecturers have a direct influence on the quality of teaching. Therefore it is important to educate a proper technological competence of future mathematicians and mathematics teachers. The author summarized the investigations of

technological tools and their educational features applicable to mathematics, and characterized which abilities, knowledge and skills of application in teaching comprise the technological and educational ICT competence of a contemporary mathematician (Lipeikienė, 2008).

On the other hand, the concept of mathematical communication is very extensive. First of all, one can consider MC as mathematical information – mathematical contents or mathematical knowledge that has to be imparted or transmitted to students. Another aspect is the way for mathematics exploration. One more aspect of MC is mathematical documentation – presentation of mathematical content electronically. No less important is mathematical communication in the classroom: how to organize classroom activities with a view to promote communication. The MC concept also includes technological possibilities of visualization and exploration, transmission of mathematical knowledge and extending communication. The paper analyzes all the main aspects of mathematical communication concept's use and draws the conclusion that the curriculum "Mathematical communication", including all the main aspects of MC, can realize the intention to educate contemporary innovative mathematicians and mathematics teachers. The description of the curriculum is presented in the paper.

The ***main purpose of the investigation*** was to design a curriculum for education of mathematicians and mathematics teachers with a sound technological and educational competence.

The main questions of the investigation are: what are the main technological tools applicable to mathematics? What technological and educational competence of mathematicians is essential? What aspects does the MC concept consist of? Which subjects represent the aspects of MC and have to be included in the curriculum because of their connection with the essential items of the competence desirable?

Methods of investigation: comparative analysis of literature, proceedings of international conferences and mathematics curricula of the world universities; qualitative methods (observation and evaluation of the teaching process, reflexion, discussions with students).

2. Technological aspects of Mathematical communication.

One of the most important aspects of mathematical communication is technology use in mathematics (for example, Boas, 2005; AMTE position, 2006). The author has investigated CAS and the CAS teaching process for some years (Lipeikienė, 2002, 2004, 2005a, 2005b, 2007, 2008). In (Lipeikienė, 2008), the available technological tools applicable in mathematics exploration, communication and teaching, their development, variety, and innovations, are reviewed. Also, mathematical resources of the Internet, scientific works of the last 10 years, the contents of the international conferences meant for teaching mathematics with technology, were surveyed. Fig. 1 presents the main groups of up-to-date technological tools available for mathematics exploration, communication, and teaching. We do not refer here the basic computer literacy, information and social literacy, though they are important for teachers of all subjects, but well investigated.

Accordingly, the technological and educational competence of a mathematician and mathematics teacher consists of:

- knowledge, ability to evaluate and to use educational programs and computer algebra systems (CAS) in exploration and teaching;
- knowledge of visualization tools;
- web programming abilities;
- understanding of virtual learning environment essence and facilities;
- capability to evaluate and apply open source CAS;
- abilities not only to find, but also to create and disseminate educational information on the Internet.
- appropriate experiences in the use of a variety of technological tools to enhance their own learning of mathematics and the teaching of others.

To this end, *Information technologies for mathematics, Computer algebra systems, Virtual learning environments, Hypertext technologies, Programming methods, Applications of Computer graphics* and other courses were included to the curriculum. Technological subjects comprise 50 credits (about 75 ECTS credits).

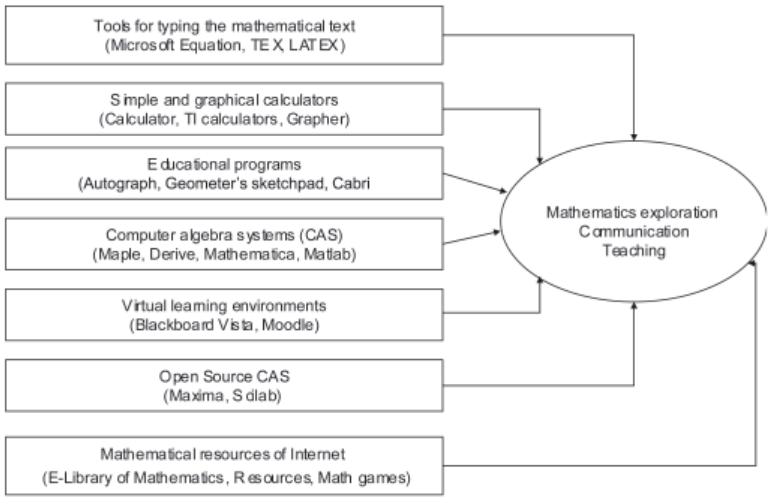

Fig. 1. ***Technological tools for mathematics***

3. Mathematical documentation

Mathematical documentation and transmission of mathematical content are connected to the technology specific to each period of civilization, starting from manuscripts to typographical press, from classical libraries to digital repositories (Rocha and Rodrigues, 2005). We are planning to teach abilities to present mathematical content electronically in the course of *Mathematical documentation*. The main document preparation systems and their development will be thought in the course. The course includes the evolution of documentation, the use of Microsoft equation, different versions of TeX and LaTeX. In addition, *Applications of computer graphics* and *Hypertext technologies* courses also give skills of innovative mathematical documentation.

Usually, the mathematical content has been entered using a keyboard or scanned from a typeset document, using special programs. With the spread of availability of hand-held pen-based devices there is an opportunity for a more interactive mathematical interface (CICM'09, 2009). For students it is useful to know ideas of mathematical formula recognition, interface of CAS design and character recognition. The useful subject of *Electronic publishing* is included into the curriculum as optional.

4. Mathematical communication as mathematics itself and mathematics exploration

One can consider MC also as mathematical information – mathematical contents or mathematical knowledge that has to be imparted or transmitted to students. Another aspect is the way of exploring mathematics. An example of MC as the mathematics itself is the international journal "Mathematical Communications" that publishes scientific articles from all fields of pure and applied mathematics (www.mathos.hr/mc/). Another journal "Communications in Mathematical Sciences" (www.intlpress.com/CMS/) covers modern applied mathematics in modelling, applied and stochastic analyses and numerical computations. An example of the aspect of mathematics exploration is the course of Mathematical Communication at the City University London that provides "an understanding of elementary set theory and logic, an appreciation of the notion of proof, and various methods of proof". The aim of the course is an appreciation of mathematical reasoning and the ways in which it is communicated. These aspects of mathematical communication in our curriculum are included in the subjects of *Discrete Mathematics and Logic* and other subjects of mathematics. They comprise the backbone of our curriculum of Mathematical communication. 60 credits (90 ECTS credits) are designated to the fundamentals of mathematics, including C*alculus, Algebra, Discrete mathematics*, etc.

5. Mathematical communication in the classroom

Some authors regard the mathematical communication in the classroom, ability to understand and to be understood as the essence of MC (Khaing, 2007). Teaching and learning through

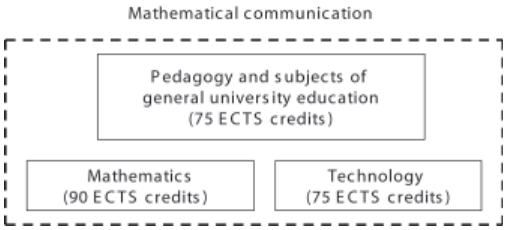

Fig. 2. ***The common structure of the Mathematical communication curriculum***

observation, manipulation, experimentation, organizing classroom activities with a view to promote the communication, asking the students to explain and justify their thinking, are of great importance. "Math is communication. You have to be able to communicate the concepts. You have to be able to communicate your thinking. Numbers are not enough for any good mathematician. You have to prove. You have to convince." (Clark, 2005). It is very important to promote activities that connect multiple representations (graphical, numerical, algebraic and verbal). The educated skills of oral and written MC are also important in the processes of social communication (Miyagui, 2007). Mathematicians should also understand negative aspects of technology use, when and how technology can advance teaching and when it can hinder the mathematical development. We include various didactic aspects in the courses *Didactics of mathematics* and *Communication teaching mathematics*. Fig. 2 describes the common structure of the Mathematical communication curriculum at the Vilnius Pedagogical University. After studies students receive a Bachelor's degree in mathematics and teacher qualification.

6. Summing-up and conclusion

Our investigations have showed that the concept of Mathematical communication is used in various aspects and levels, but four aspects are essential (sections 2–4). Fig. 3 presents the essential aspects of mathematical communication and the main subjects that cover particular subjects.

The curriculum including the main aspects of mathematical communication integrates all the important subjects that assure the proper innovative education of a contemporary mathematician and mathematics teachers.

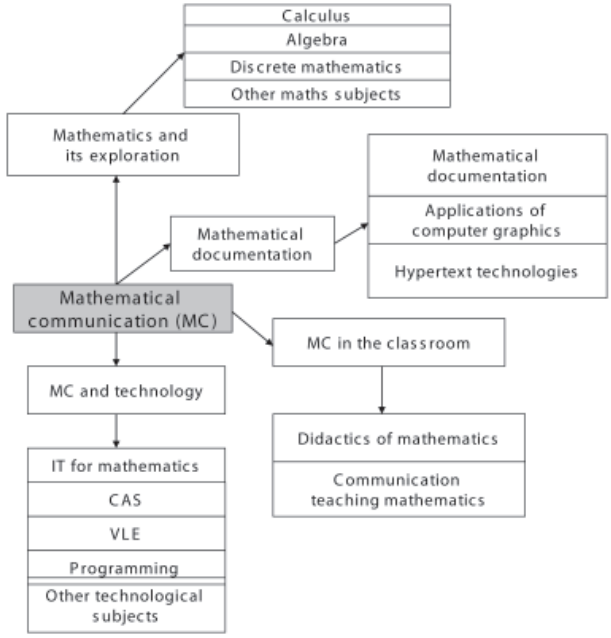

Fig. 3. ***The main aspects of mathematical communication and the subjects that cover them***

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MATEMATINĖS KOMUNIKACIJOS STUDIJŲ PROGRAMOS KŪRIMAS

Joana Lipeikienė

S u m m a r y

Straipsnyje aprašomi atlikti tyrimai kuriant naują matematinės komunikacijos studijų programą. Programa skirta technologiškai gerai išprususiems matematikams rengti. Siekiant sudaryti šiuolaikišką deramo lygio studijų programą, pirmiausia buvo atsižvelgiama į anksčiau autorės tirtų matematikai taikytinų informacinių ir komunikacinių technologijų apibendrinimą, padarytas išvadas apie reikiamą matematiko ir matematikos mokytojo technologinę ir edukacinę kompetenciją. Darbe atliktas matematinės komunikacijos sąvokos vartojimo įvairiomis prasmėmis tyrimas ir padaryta išvada, kad visus pagrindinius matematinės komunikacijos aspektus apimanti studijų programa gali būti šiuolaikiškų matematikų ir matematikos mokytojų ugdymo programa.