

# Distortions introduced by normalisation of values of criteria in multiple criteria methods of evaluation

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**Abstract.** Quantitative multiple criteria decision aid (MCDA) methods of evaluation gain increasing popularity among researchers. The idea of the methods is to comprise values of criteria characterising each object into a single non-dimensional cumulative criterion, which reflects attractiveness or position of the object in view of an objective chosen. Normalisation of weights is a compulsory procedure whenever criteria of different dimensions are present. There several methods of normalisation available. Nevertheless, each method may introduce distortions into transformed data. The paper is devoted to exploration of problems related to such distortions and reveals particular cases.

**Keywords:** multiple criteria decision aid methods, normalisation, distortions.

## Introduction

Multiple criteria decision aid (MCDA) methods of quantitative evaluation were introduced in the 80-s and are permanently gaining popularity among researchers as the most developing branch of operational research methods [7]. Such methods compete in popularity with statistical methods [1].

There are two following core ideas of multiple criteria decision aid (MCDA) methods [10]. (i) Values of criteria characterising each object are comprised into a single non-dimensional cumulative criterion reflecting attractiveness of the object in view of a chosen objective of research or evaluation. (ii) Differences of importance of criteria are reflected by assigning different weights of each criterion. Weights are obtained by eliciting opinions of decision-makers or principals on importance of selected criteria for the objective of the research chosen. While the first idea is used in all MCDA methods, the second idea is used in most of MCDA methods with few insignificant exceptions.

For the purpose of this paper we stress that the idea of weighing means that exclusively weights are meant to reflect differences of importance of criteria. This statement is either communicated to experts, who are asked to estimate the weights, or implicitly is understood by them. Also, we assume that evaluation is aimed at attaining objectivity [8].

Normalisation of weights is a compulsory procedure in most of MCDA methods, whenever criteria of different dimensions are present, which is the case in majority

of situations. There is a realm of methods of normalisation available. Nevertheless, magnitudes of criteria are accounted differently using different normalisation methods. Such an effect could be often treated as a distortion of data transformation, or illicit discrimination between criteria.

In this paper we show that instead of creating distortions for some criteria, a decision-maker should one way or another be invited to participate in making choices of normalisation methods, depending on character of criteria and data. Some other MCDA methods should be created, or modifications of existing ones attempted.

## 1 MCDA methodology and popular methods

Major steps of methodology of evaluation using MCDA methods is as follows [3].

(1) Creation of a set of criteria, which represents the process or object. This step is not formalised yet; criteria are usually chosen and outlined even without making references to the literature, where they have been found [2]. (2) Finding values of chosen criteria either using statistical data, or by eliciting estimations from experts, depending on availability of data. (3) Transformation and normalisation of data. While there are many methods of normalisation available, each method may have shortcomings in a particular case of data, and may introduce distortions into transformed data. The paper is devoted to exploring such distortions. (4) Estimation of weights of criteria in accordance with the objective of the research. (5) Analysis of available MCDA methods and choice of the appropriate one in accordance with particularities of data and the objective of the research. In more complete patterns of research several MCDA methods are applied; their particularities are discussed; correspondence between results is analysed. (6) In cases of large number of criteria a hierarchy structure is formed. Analysis of influence of values of criteria on the lower level to the higher level is performed. (7) Parameters for sophisticated MCDA methods, like PROMETHEE II, are estimated [4]. (8) Stability of results is evaluated.

Similar structure of MCDA evaluation methodology has been adopted by most of researchers; only insignificant variations or curtailing of the list of mentioned steps of the MCDA methodology can be observed. Nevertheless, a wide range of freedom is left to a researcher in making choices of criteria, of MCDA method or methods, methods of normalisation and transformation, estimation of weights of criteria. Such a freedom of choice should imply paying serious attention by researchers to the process of making choices among options available. Nevertheless, the most considerable shortcoming currently observed is often encountered absence of analysis of distortions introduced by transformations of data; only a limited number of researchers pay at least some attention to possible distortions introduced by each type of normalisation or transformation, and to their influences to the ultimate result. This paper is attempting to fill this gap.

It is worth to mention here that not all MCDA methods require normalisation. Nevertheless, such popular methods as SAW, TOPSIS, COPRAS, etc. require normalised data.

Denote the matrix of (not normalised) values of  $m$  criteria and  $n$  alternatives as  $R = \|r_{ij}\|$ ,  $i = 1, 2, \dots, m$ ;  $j = 1, 2, \dots, n$ .

The size of the matrix is  $m \times n$ ; values of each alternative are placed to columns while values of each criterion are found in corresponding rows. The matrix is often

called decision-matrix. For the purpose of this paper suppose that values of criteria are maximising only. Consequently, the largest value of a criterion is considered to be the best, while the smallest value is considered to be the worst. The great majority of MCDA methods deal with maximising criteria, consequently transformation of values of minimising criteria into maximising ones is required. Topics of transformation of values of minimising to maximising ones, as well as transformation of values of criteria containing negative numbers are not included to the scope of the paper. Different types of normalisation and possible sources of distortions introduced by such normalisations will be discussed in Section 3.

Denote transformed and normalised values of criteria as  $\tilde{r}_{ij}$ , which are all maximising and positive whenever the SAW and the COPRAS methods are used. In the SAW method minimising criteria must ultimately appear as maximising ones, after a transformation of values of minimising criteria into maximising ones. COPRAS method does not require transformation of minimising criteria into maximising ones. In the TOPSIS method minimising criteria can be used without transformation, even if the method requires normalisation of data. The method has a prominent feature as it deals well with negative numbers, and their transformation is not required.

## 2 Commonly used methods of normalisation of data

Each of the methods has its own set of plausible methods of normalisation. Since the TOPSIS method uses distances in the  $m$ -dimensional space, it has its own normalisation referred to as type 4, while for the SAW and the COPRAS methods normalisations 1–3, and 5 can be used. As there are a number of normalisation types available, only major, most influential methods of normalisation will be mentioned.

(1) Normalisation by comparison with the best value [9], formula (1)

$$\tilde{r}_{ij} = \frac{r_{ij}}{\max_j r_{ij}}. \quad (1)$$

(2) Classic normalisation, when sum of resulting values is equal to one [5], formula (2)

$$\tilde{r}_{ij} = \frac{r_{ij}}{\sum_{j=1}^m r_{ij}}. \quad (2)$$

(3) The normalisation, which assigns zero to the worst value of a criterion, and 1 (or 100%) to the best value [6], formula (3)

$$\tilde{r}_{ij} = \frac{r_{ij} - \min_j r_{ij}}{\max_j r_{ij} - \min_j r_{ij}}. \quad (3)$$

(4) Vector normalisation, formula (4)

$$\tilde{r}_{ij} = \frac{r_{ij}}{\sqrt{\sum_{j=1}^n r_{ij}^2}}. \quad (4)$$

(5) Statistical  $z$  score, which denotes the distance of the value  $r_{ij}$  of criterion to its mean  $r_i^0$  measured in standard deviations  $\sigma_i$ , formula (5)

$$\tilde{r}_{ij} = \exp^{-z^2/2}, \quad \text{where } z = \frac{(r_{ij} - r_i^0)}{\sigma_i}. \quad (5)$$

### 3 Cases of distortion

(1) *Linearity and dimensions.* All above-mentioned normalisation types (with the exception of the 5-th) are linear and single-dimensional. In other words, performing the normalisation function between the set of values of criteria  $R$  to the set of real numbers

$$f : R \rightarrow \mathbb{R}$$

is a mapping, which preserves both marginal values, and their ratios.

We pay attention that linearity may not be preserved in case if transformations of criteria with negative values, and minimising criteria to maximising ones, if they were necessary. Linearity of normalisation holds for types 1–4.

Even if in most cases such linear mappings account and well reflect perception of differences between values of criteria by experts and decision-makers, there are many cases, when perception should be expressed by a non-linear and/or multi-dimensional function. For example, the customer's utility function, which represents one's utility gained from consuming a basket of  $k$  goods is usually expressed in the following form, formula (6):

$$U = Ax_1^{s_1} x_2^{s_2} \dots x_k^{s_k}, \quad (6)$$

where  $A$  is a positive constant,  $x_i$  are variables, which represent quantities of goods in an evaluated basket of goods, and  $s_i \in \mathbb{R}$  are positive real numbers (often  $s_i = 1, \forall i$ ). Consequently, this case is multidimensional and non-linear, and multidimensional and non-linear function would be more plausible to use for normalisation in such cases. Alternatively, creation of new MCDA methods or modifications to existing ones should be considered.

(2) *Mapping to different intervals.* The normalisations 1–3 map values of criteria to different intervals, which may often have influence on the resulting cumulative criterion or may introduce distortions. We recall that only weights of criteria are supposed to reflect magnitudes of relative importance of criteria in accordance with the MCDA paradigm. Nevertheless, types of normalisations 1 and 2 are mapping values to the following intervals respectively, formulae (7), (8):

$$\left[ \frac{\min_j r_{ij}}{\max_j r_{ij}}, 1 \right], \quad \text{and} \quad (7)$$

$$\left[ \frac{\min_j r_{ij}}{\sum_{j=1}^m r_{ij}}, \frac{\max_j r_{ij}}{\sum_{j=1}^m r_{ij}} \right]. \quad (8)$$

Boundaries of the intervals, in general, are different for each criterion  $i$  (with the only exception of the boundary 1 in formula (7)). This means that values of different criteria are accounted differently irrespectively of weights, which indicates the discrepancy with the paradigm of MCDA methods that values of each criterion should be accounted differently only because of multiplying by different weights.

The above observation can be more clearly observed in case we take only the best and the worst values of criteria. For the 1-st and 2-nd types of normalisation the worst values will be accounted as  $\frac{\min_j r_{ij}}{\max_j r_{ij}}$  and  $\frac{\min_j r_{ij}}{\sum_{j=1}^m r_{ij}}$ , while the best values for the 2-nd type of normalisation will be accounted as  $\frac{\max_j r_{ij}}{\sum_{j=1}^m r_{ij}}$ , which in general depend on  $i$  or on the criterion accounted.

We observe that the type of normalisation 3 accounts the best and the worst values differently to types 1–2 as it maps all values of criteria to the same interval  $[0, 1]$  irrespectively of index of criterion  $i$ .

(3) *Cases with large values.* There are more cases, when due attention should be paid to the choice of a mapping for normalisation. Suppose, values of a criterion are large, and nevertheless, deviations are highly perceived, and should have a considerable influence on the result. Obviously, normalisation types 1 and 2 cannot be used in this case as such normalisations map values of such criteria to very small intervals (see formulae (7), (8)) thus making their differences insignificant. Consequently, a researcher should opt for the 3-rd type of normalisation.

(4) *Insignificant differences.* High possibility of distortions may be introduced using the 3-rd normalisation in such cases, when the range of values of a criterion is small. This type of transformation maps the range of values of every criterion into the same interval  $[0, 1]$ , therefore the same importance would be assigned to values of a criterion of a small and insignificant range as well as to values of other criterion, which deviations are more meaningful. We recall that in accordance with the MCDA paradigm difference of importance of criteria should be accounted by weighing only.

For example, take the criterion of weight in the task of choosing the right tablet computer to acquire, and the following weights expressed in grams for three alternative tablets to choose: 950, 960, 970. Such values are most probably perceived quite equally by a customer. Nevertheless, they would be transformed to the whole interval  $[0, 1]$ , and result of normalisation will be  $\{0; 0.5; 1\}$ , which implies large influence of only 10 grams difference.

The author proposes to nullify influence of such criterion for the alternatives, where differences of values of criterion are insignificant by assigning value of the criterion 0.5 for all alternatives in question. It is not always wise to remove the criterion entirely from the list, for example, in such cases, when evaluations are made for a few different periods, and the criterion may have more considerable influence for other cases.

(5) *Cases with a target value.* There are cases, when deviations from the desired value should be considered as reluctant. Perception by a customer of the same increment on the size of a house may be perceived positively at a small considered size of the house, and may be considered even negatively at large sizes not only because of increasing heating expenses, but because of many other factors as increasing movement time between rooms, more efforts required for cleaning, etc. Therefore, the 5-th type of normalisation should be considered for this case.

## 4 Conclusions

A specialist should be asked to choose a plausible normalisation method for particular data in order to avoid distortions. A methodology allowing a non-specialist decision-maker to make choices of normalisation should be created. MCDA methods, which deal with different types of normalisation simultaneously, should be created, or modifications of existing ones, all of which allow to use only one normalisation for all criteria, attempted.

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## REZIUMĖ

### Normalizavimo algoritmų, naudojamų daugiakriterijuose vertinimo metuose, įnešami išskrypimai

A. Podvezko

Kiekybiniai daugiakriteriniai vertinimo metodai vis labiau populiarėja tarp mokslininkų. Pagrindinė šių metodų idėja – apjungti charakterizuojančių kiekvieną objektą kriterijų reikšmes į vieną bedimensį kriterijų, kuris parodytų objekto patrauklumą, atsižvelgiant į vertinimo tikslą. Duomenų normalizavimas – įprasta procedūra, kai naudojami kriterijai įvairių dimensijų. Naudojami keli normalizavimo metodai. Tačiau, jie gali įnešti išskrypimus į transformuojamus duomenis. Šiame straipsnyje nagrinėjamos problemos, atsirandančios dėl šių išskrypimų, pateikiami pavyzdžiai.

*Raktiniai žodžiai:* daugiakriteriniai MCDA metodai, normalizavimas, išskrypimai.