

Algebraic and spectral analysis of local magnetic field intensity

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Abstract. One of the kind in Europe magnetometer is situated in central Lithuania and registers local magnetic field intensity. The sensitivity of the device is of pT (pico teslas) order. Magnetic field intensity is a real world sequence and as a result does not have a rank. Nevertheless, H -rank based techniques enable to analyse the data as algebraic sequences. The work also pays a considerable amount of attention to the spectral distribution of the signal. A specialized software is being developed due to the amount of the data and computational time costs for problems investigated.

Keywords: magnetic field intensity, spectral distribution of a signal, rank of a sequence.

Introduction

The Earth is surrounded by the magnetic field which is constantly changing due to various factors such as solar wind, lightning strikes, even industry related perturbations. Some of resonances in the frequency spectrum corresponds to the frequencies observed in human organism. Probably the most known typical resonant frequencies are 7.8, 14, 20, 26, 33, 39 and 45 Hz. Those are called Schumann resonances. These essentially are quasi-stationary waves between the surface of the Earth and the lower layer of the ionosphere (at about 50 km height from the surface). Lightnings are thought to be the most influential factors to the existence of Schumann resonances although the primary frequencies they produce are measured in orders of kHz. As mentioned before corresponding frequencies are observed in human organism. Those, for example, are neural oscillations or resonances in a heart rhythm under electromagnetic interference [4].

Determining possible links between the Earth's magnetic field and human's cardiovascular system is an intriguing statement and a challenging problem. The first step towards it is to analyze the data of local magnetic field of the Earth. This paper presents the overview of this data in respect of brief spectral and algebraic analysis. The obtaining of such a data is possible thanks to the magnetometer installed in the territory of the Institute of Animal Science of LUHS situated in Baisogala, Lithuania. The device is currently one of the kind in Europe. Another aim of this work is to develop the software for analysis of magnetic field data.

Table 1. Data file structure for the magnetometer generated data.

No.	Intensity in N/S direction	Intensity in E/W direction	Timestamp	Temp, C
1	166 993	186 117	1337 915 034	20
2	36 385	12 612	1337 915 034	21
...

1 Spectral analysis of the magnetometer data

1.1 Data formats

The magnetometer writes data to binary files in the format shown in Table 1.

Sampling frequency for the data is 130 Hz. Magnetic field intensities are registered in nT (nano teslas) in East–West and North–South directions.

1.2 Some elements from the signal theory

Consider magnetic field intensity $\{I_t\}_{t=0}^{N-1}$, t is discrete time variable. Due to the convenience of working with representable numbers the intensity is converted to pT in this work.

$$f(\omega) = \sum_{t=0}^{N-1} I_t \cdot e^{-\frac{2\pi it\omega}{N}}, \quad t \in \mathbb{Z}. \tag{1}$$

In order to transform $\{I_t\}_{t=0}^{N-1}$ to the frequency domain the discrete Fourier transform (DFT) (Eq. (1)) could be used [1]. The drawback of DFT is that one cannot observe the change in spectral density over time unless sequentially computing DFT. To achieve this the discrete time short time Fourier transform (STFT) is employed.

$$F(\tau, \omega) = \sum_{t=-\infty}^{\infty} I_t \cdot \xi(t - \tau) e^{-it\omega}, \quad t \in \mathbb{Z}. \tag{2}$$

STFT for $\{I_t\}_{t=0}^{N-1}$ is represented by Eq. (2). In fact this is essentially the analogue for Eq. (1) but applied to the function $I_t \cdot \xi(t - \tau)$. $\xi(t)$ is a so called windowing function which has a value close to 1 in a subdomain of t centered on 0 and a value close to 0 elsewhere. The units of $f(\omega)$ and $F(\tau, \omega)$ are pT·s.

$$S(\tau, \omega) = |F(\tau, \omega)|^2. \tag{3}$$

Spectrograms investigated in this work is simply the squared modulus of STFT (Eq. (3)). Originally units of a spectrogram would be $\text{pT}^2 \cdot s^2$. In this work spectrograms are dimensionless due to the normalization procedure applied to I_t before the application of STFT. $S(\tau, \omega)$ is often referenced as power spectral density. Thus the value of $S(\tau, \omega)$ is interpreted as signal power at the time interval $\Delta\tau$ and at the frequency range $\Delta\omega$.

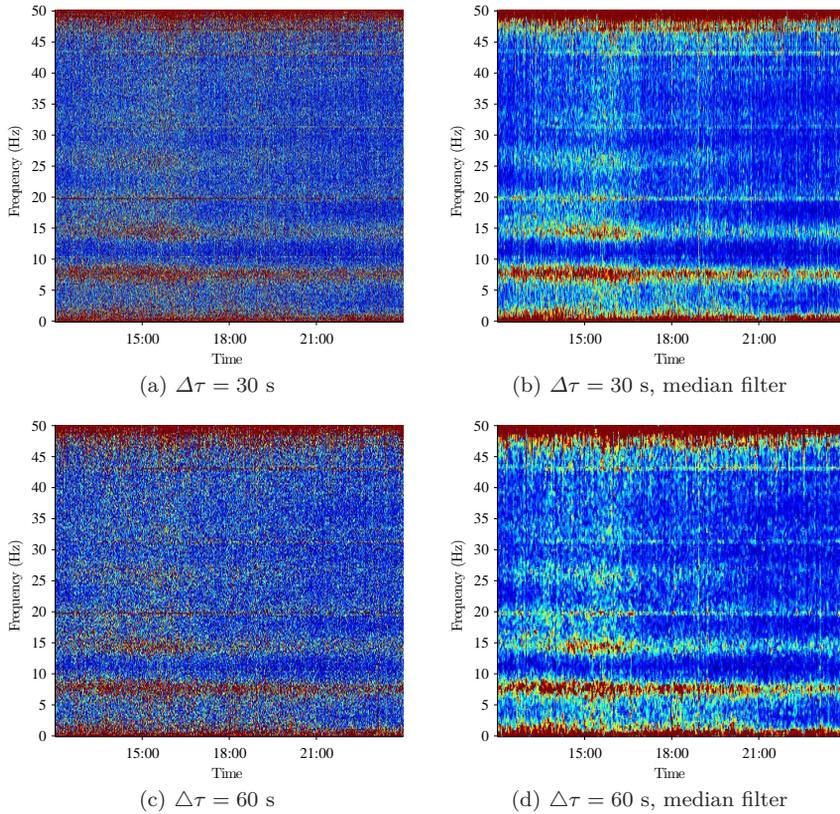


Fig. 1. Comparison of spectrograms with different values of $\Delta\tau$ and the use of median filter.

1.3 Computational results

Computational experiments were performed with the data of magnetic field intensity in E/W direction on the 1st of January, 2015 starting at 12:00 hours.

Fig. 1 shows the spectrograms for the data. Primary use of the spectrogram is to analyze the frequency distribution of the signal. One can clearly observe Schumann resonances and the maximum at around 50Hz. The latter one is present due to the frequency used in the power grid of Lithuania. Parts b) and d) in Fig. 1 are the same parts a) and c) correspondingly but with the median filter of dimensions 3×3 applied. The median filter makes the spectrogram easier to analyze.

The spectrogram can also be used for computing the power of the signal. Power would be computed as the sum of power spectral density with only one interval on the time domain. One can also compute spectrograms dividing the time domain to several parts and then add up the components of the power spectral density. The spectrogram comes in use if one needs to consider only a particular frequency range or clip the power spectral density. Although the classical approach to compute the power of a signal by taking the square of RMS value of the intensity is much more faster to obtain.

2 Algebraic analysis of the magnetometer data

2.1 Some elements from the algebraic analysis

As I_t is obtained from the magnetometer device it is a discrete sequence, for eg. a set of magnetic field intensities in E/W direction. So let the sequence be denoted as $\{I_t\}_{t=0}^{+\infty}$. If this sequence has an H -rank $Hr\{I_t\}_{t=0}^{+\infty} = m$ it can be algebraically reconstructed [3]. The concept of the H -rank is presented in [2].

$$\begin{vmatrix} I_0 & I_1 & \dots & I_m \\ I_1 & I_2 & \dots & I_{m+1} \\ \vdots & \vdots & \ddots & \vdots \\ I_{m-1} & I_m & \dots & I_{2m-1} \\ 1 & \rho & \dots & \rho^m \end{vmatrix} = 0. \tag{4}$$

Algebraic reconstruction of the sequence $\{I_j\}_{j=0}^{+\infty}$ is performed by firstly solving the characteristic Eq. (4). Then coefficients $\mu_k, k = \overline{1, m}$, according to the Eq. (5) are determined.

$$I_n = \sum_{k=1}^m \mu_k \rho_k^n, \quad n = 0, 1, \dots \tag{5}$$

Practical use of the algebraic reconstruction of the sequence could take role in recovering short time data losses. Having coefficients $\mu_k, k = \overline{1, m}$, and roots of the characteristic equation $\rho_k, k = \overline{1, m}$, enables one to compute I_n for $n > 2m - 1$.

2.2 Computational results

In this section the data from Section 1.3 is used. It must be noted first that the real world sequence does not have a rank thus the pseudorank was found first. The detailed procedure for algebraic extrapolation and notes on finding pseudoranks could be found in [3]. In this example the best value for pseudorank resulted in $m = 15$ which corresponded to machine epsilon of $\varepsilon = 10^{-15}$ and RMSE of the extrapolation of 0.0595.

The extrapolated sequence is shown in Fig. 2(b) and (c). Note that the period of 130 time steps corresponds to 1s (due to the sampling rate of the data). Also pay attention to the vertical axes in Fig. 2(b) and (c) as the data is normalized. If one does not normalize the data before the algebraic extrapolation the resulting sequence tends to diverge in the first iterations.

It is evident that short time losses of the signal could be replaced with the algebraic extrapolation of the sequence $\{I_t\}_{t=0}^{+\infty}$. The reason for this approach to be more useful than just simply copying the last available values or computing moving average is quite simple in fact. Reconstructed sequence contains algebraic relations preserved by the use of the concept of the H -rank. This makes the result to follow the underlying algebraic model of the series to a some degree.

3 Conclusions

1. Spectrograms comes in use for computing power of a signal if one needs to consider only a particular frequency range or clip the power spectral density to a certain value.

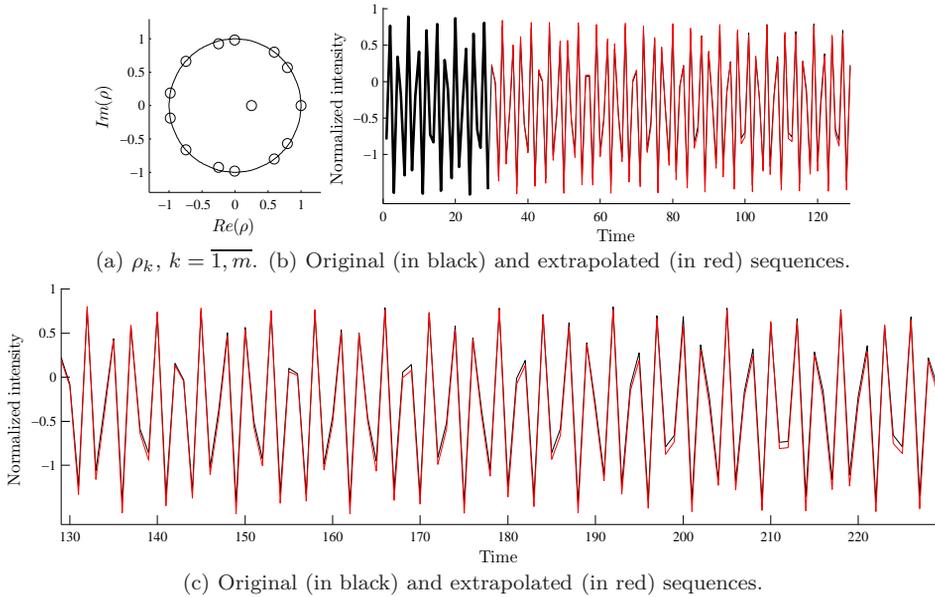


Fig. 2. Extrapolating the magnetic field intensity.

2. Optimal time step $\Delta\tau$ for the STFT (in visually representing the spectrogram) showed to be 10 to 30 seconds. This applies to the local magnetic field of Lithuania.
3. The algebraic reconstruction of the sequence could take role in recovering short time signal losses.

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REZIUMĖ

Lokalaus magnetinio lauko intensyvumo algebrinė ir spektro analizė

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Centrinėje Lietuvoje yra įrengtas Europoje analogų neturintis magnetometras, registruojantis lokalaus magnetinio lauko svyravimus. Prietaiso jautrumas yra pikotesių eilės. Jau daugiau kaip metus išnaudojama unikali galimybė gauti itin tikslūs duomenis. Magnetinio lauko intensyvumas, jeigu

žvelgsime į jį kaip į skaičių seką, rango neturi. Tai bendra realaus pasaulio sekų savybė. Nepaisant to, algebriniai metodai, pavyzdžiui paremti H -rangu, leidžia analizuoti tokius signalus. Darbe nagrinėjamas tokių svyravimų spektrinis pasiskirstymas, aptarta darbo su dideliais duomenų masyvais specifiika. Taip pat pristatoma sukurta programinė įranga, skirta darbui su magnetometro duomenimis.

Raktiniai žodžiai: magnetinio lauko intensyvumas, signalo spektrinis pasiskirstymas, sekos rangas.