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Spatial classification rule with distance in three dimensional space

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Abstract. Spatial Classification Rule with Distance (SCRD) method is used in two dimensional coordinate system, which limits the usage of existing spatial information in MRI, CT and other three dimensional (layered) images. The SCRD method is extended to be applied in three dimensional coordinate space. Artificial experiment is performed in order to show ability to use SCRD method with three dimensional medical or other similar images where training sample is available.

Keywords: spatial classification, supervised classification, image analysis, three dimensional space.

Introduction

The importance of spatial information in image analysis was described in many articles (see e.g., [3, 4, 12]). Spatial autocorrelation represents the degree to which that correlation changes with distance [5]. In the papers (see e.g., [6, 1, 2]) the incorporation of geostatistical information of features into plug-in versions of classifiers is based on the marginal distribution of the observation to be classified. The incorporation of geostatistical information of features into plug-in versions of classifiers based on the marginal distribution of the observation to be classified were shown in some papers (see e.g., [2]).

The statistical supervised classification method based on plug-in Bayes discriminant function (PBDF) was extended by incorporating more influence from the spatial dependency into classification problem in the paper [9]. This method is called spatial classification rule with distance (SCRD) [9]. SCRD method showed its better accuracy comparing with other common methods in [9] in artificial experiment and in the paper [10] in real life situation. In the paper [11] the importance of training sample selection was analyzed and the importance of having data from all classes while classifying an object was shown.

While analyzing SCRD method only two dimensional (2D) coordinate system was considered. 2D methods version can be used in many situations in spatial statistics and in image analysis. Such method can be used in remote sensing image classification, in medical image analysis where pixels of the image must be classified. In medical imagery we often deal with MRI or CT images. In many cases each layer of these images is analyzed independently. These images can be classified with SCRD method layer by layer using 2D coordinate system, but knowing the distance between the layers the SCRD method can be extended in order to use the three dimensional (3D) coordinate system. In this paper SCRD method is extended in to 3D coordinate space and the artificial experiment is performed to show it's performance.

1 Method extension

The method used in this letter is a spatial classification rule based on the Plug-in Bayes Discriminant function (PBDF) with posterior distribution of class label depending on distances among unclassified locations and training sample locations is called SCRD [9].

According to the method, features are modeled by stationary Gaussian Random Field (GRF) $\{Z(s) : s \in D \subset \mathbb{R}^3\}$, and class labels are modeled by discrete Markov Random Field (MRF). Such modeling is common in image analysis. Here s is a state of the pixel. Previously state s was described by two coordinates s_x and s_y. In three dimensional space additional space coordinate s_z is added.

$Z(s) = \mu_l + \varepsilon(s)$ is the marginal model of observation $Z(s)$ in class Ω_l, with the mean μ_l and with the error term $\varepsilon(s)$ which is generated by zero-mean stationary GRF $\{\varepsilon(s) : s \in D \subset \mathbb{R}^3\}$ with covariance function defined by model $cov\{\varepsilon(s), \varepsilon(u)\} = \sigma^2 r(d(s,u))$ for all $s, u \in D \subset \mathbb{R}^3$, where σ^2 is variance as a scale parameter. In this letter the exponential covariance function is used $C(h) = \sigma^2 \exp\{-|h|/\alpha\}$. $r(d(s,u)) = r(h) = \exp\{-|h|/\alpha\}$ is the spatial correlation function, where α is the correlation range parameter which shows how far the correlation remains and h is the Euclidean distance between s and u locations:

$$h = d(s,u) = \sqrt{(s_x - u_x)^2 + (s_y - u_y)^2 + (s_z - u_z)^2}. \tag{1}$$

The PBDF to the classification problem is

$$W_t(Z_0; \hat{\Psi}) = \left(Z_0 - \frac{(\hat{\mu}^0_{1t} + \hat{\mu}^0_{2t})}{2}\right) \frac{(\hat{\mu}^0_{1t} - \hat{\mu}^0_{2t})}{\hat{\sigma}^2_{0t}} + \gamma(y) \tag{2}$$

where $\hat{\mu}^0_{lt} = \hat{\mu}_l + \alpha'_0(z_n - X_y\hat{\mu})$, and $\hat{\sigma}^2_{0t} = \hat{\sigma}^2 R_{on}$, $\gamma(y) = \ln(\pi_1(y)/\pi_2(y))$.

The classification rule SCRD is based on the following posterior distribution of $Y(s_0)$ specified by

$$\pi_1(y) = \left(\sum_{i \in I_0} \frac{\delta(y_i = 1)}{d(s_i, s_0)}\right) \Big/ \left(\sum_{i \in I_0} \frac{1}{d(s_i, s_0)}\right), \tag{3}$$

where $\delta(\cdot)$ is the 0–1 indicator function and $d(\cdot,\cdot)$ denotes the Euclidean distance function between locations as described in Eq. (1). For the case of two classes $\pi_2 = 1 - \pi_1$.

The extension of the SCRD method to be used with three spatial coordinates influences the calculations of π_1, π_2, $\gamma(y)$, $C(h)$, $r(h)$ and R_{on}.

$I_0 = \{i : s_i \in N_0,\ i = 1, \ldots, n\}$ and n_1 is the number of locations from N_0 with label equal 1. Here N_0 is a set of s_0 states neighboring pixel selected by a neighborhood scheme.

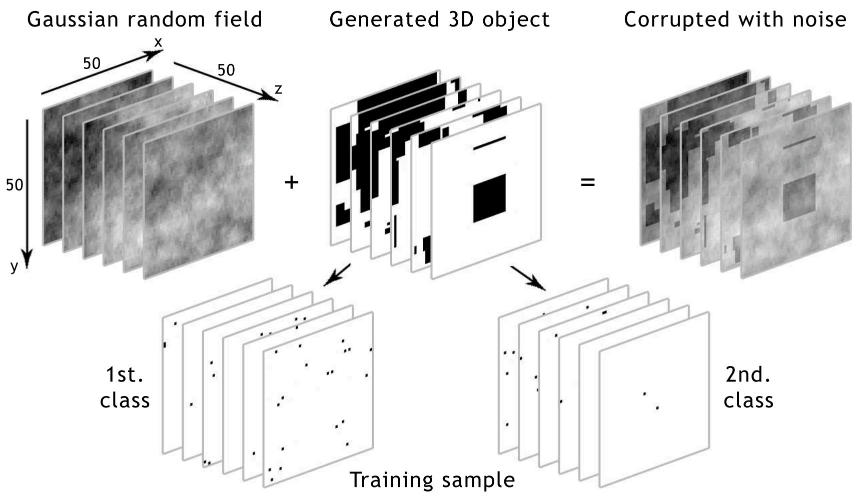

Fig. 1. Experiment scheme. 100 objects were prepared according to this scheme.

$NN_C(8,2)$ neighborhood scheme was used for neighbor selection from training sample. Neighborhood scheme importance was previously analyzed in [11] and $NN_C(n,m)$ scheme was shown as the scheme producing good classification results and solving some important issues. This method selects n nearest neighbors form the training sample and if n_i (selected neighboring points from the i-th class) is smaller then m, then additional $m - n_i$ neighbors are selected from the training sample from the i-th class and it is done for all classes. In this case there is always at least m elements from all classes and this means that the information from all classes is used for classification for every pixel to be classified.

2 Description of experiment

To show the extended SCRD methods performance the artificial experiment was performed. 100 different three dimensional images of three dimensional objects (cuboids) were generated. The size of images was $50 \times 50 \times 50$ px.

Three dimensional GRF fields were created using R program [7] and it's package Random Fields [8]. GRF were generated using exponential covariance function with correlation range parameter $\alpha = 20$. The generated GRF is added to the original image with proportion $3:1$. The obtained image is used in classification with extended SCRD method.

α and corruption level influence were analyzed in previous papers for two dimensional space (see e.g. [10, 11]) and parameters used in this experiment makes the classification problem hard enough. If the corruption level was lower we can expect very high accuracy and visual results will not be informative enough.

The training sample is generated for every image from the initially generated image. The training sample is smaller then 0.8% of all pixels. The preparation of the experiment scheme is presented in Fig. 1.

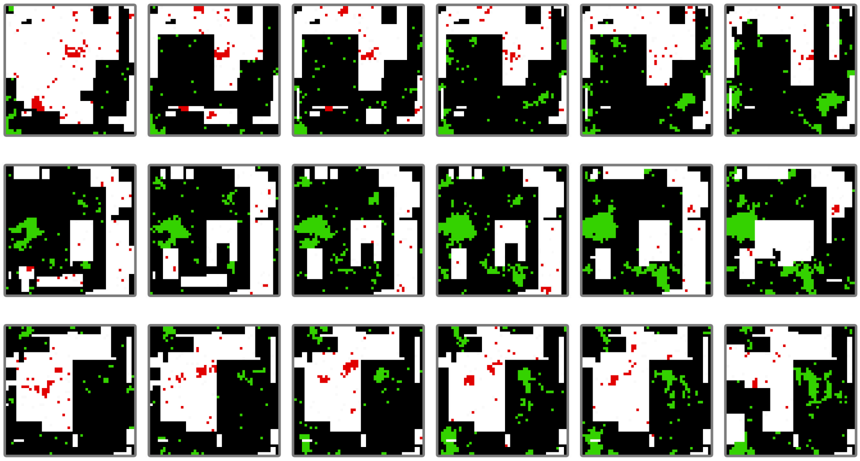

Fig. 2. Classification results showing layer by layer classified images. Red color – miss-classification of the first class. Green color – miss-classification of the second class.

3 Results

After the classification the results were received. The average classification accuracy of 91.5% was obtained. The minimum accuracy was 79.3% and the maximum accuracy was 96.6%. This accuracy is similar to the accuracy obtained earlier analysis the performance of SCRD in two dimensional space [9, 10]. The accuracy is calculated according to the equation:

$$\text{accuracy} = \big((n - n_{err})/n\big) \times 100, \tag{4}$$

here n_{err} is the number of miss-classified pixels.

Visual classification results are presented in Fig. 2. Here several different image layers of classification results are presented. From these results can be seen that miss classification is very small and even very thin parts are well classified. Also the images in Fig. 2 are presented layer by layer and from this the usage of the three dimensional classification ability can be seen.

Conclusions

The of SCRD methods extension described in this paper, shows the ability to use this method in MRI or CT image analysis when training sample in these images is available.

The classification accuracy in three dimensional images is very similar as in two dimensional images presented in previous works [9, 10, 11].

This method can be applied in per pixel classification of MRI images, to classify liver cancer in some cases.

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REZIUMĖ

Erdvinė klasifikavimo taisyklė su atstumais trimatėje erdvėje

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Erdvinė klasifikavimo taisyklės su atstumais (SCRD – Spatial Classification Rule with Distance) metodas yra naudojamas dvimatėje koordinačių sistemoje, o tai apriboja erdvinės informacijos panaudojimą MRI, CT ir kitų trimačių (sluoksninių) vaizdų analizėje. SCRD metodas yra praplečiamas tam, kad jį būtų galima taikyti trimatėje koordinačių erdvėje. Pateikiamas dirbtinis eksperimentas, kuris pademonstruoja SCRD metodo pritaikymo galimybę trimačiams medicininiams ar kitiems panašiems vaizdams, kai yra egzistuojanti mokymo imtis.

Raktiniai žodžiai: erdvinis klasifikavimas, klasifikavimas su mokymu, vaizdų analizė, trimatė erdvė.