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The mathematic model of grades transfer from one grading scale to other

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Santrauka. The article analyzes the problems of the use of ECTS grading scale, presents the constructed mathematic model of grades transfer from one grading scale to other. This model is made on the ground of the ECTS grading table and converts the marks on the principle of most probable grade equivalent evaluating the rating of a mark within the group of analyzable grades.

Raktiniai žodžiai: grading scale, ECTS grading table, empirical probabilistic distribution.

1 Introduction

The variety of grading scales used in different countries and their usage determines the necessity of the algorithming of grades conversion. The problem of grades conversion is analyzed in the works of different authors. In 1997 G. Haug examined the differences of grading scales used in different countries and emphasized that the interpretation of grades was not more objective than an assessment process itself [5]. According to the author an exact and objective conversion of grades from one grading scale to the other using mathematical formulas is impossible because not only grading scales, but also their usage in different countries considerably varies.

Big attention was given to the conversion of grades into ECTS grading scale. S. Nunes, L. Ribeir, G. David described the method of grades conversion into ECTS grading scale placing emphasis on separate conversion cases legitimized in Portugal [8]. P. Warfvinge presented the model of grades conversion into ECTS assessment scale.

The aim of the research: to review the problems of the usage of ECTS grading scale, to construct mathematic model of the grades transfer from one grading scale to other.

This research is important for Lithuanian higher education institutions. Created model will be integrated into e-learning system Moodle. Grade transfer computerization would guarantee the transparent and correct interpretation of marks in different higher education institutions.

2 ECTS grading scale and its application problems

On purpose to warrant the clarity, comprehensibility and comparability of grades obtained in different countries, institutions and subject areas ECTS grading scale was developed. This scale was recommended to follow in addition to national scale as a translation device into other grading systems. ECTS grading scale was based on the statistical distribution of passing grades in each programme [3]. ECTS grading scale is a norm-referenced [6]. That highlights how the national scale was actually being used in that context and allowed for comparison with the statistical distribution of grades in a parallel programme of another institution [3]. ECTS grading scale supplements the national grading scale, however it does not replace it. According to T. Karran the attraction of national assessment scales lies in their oneness [6]. Italy and Norway are the only two of countries under investigation whose national grading scales correspond to ECTS grading scale.

The first stage of the implementation of ECTS grading scales is the collection of statistical data. Statistical data illustrate how national grading scales are actually being used. At the second stage of the implementation of ECTS grading scale the curve of the statistical marks distribution of every control group is divided into five parts (top 10%, next 25%, next 30%, next 25%, the lowest 10%), also called A, B, C, D, E [3]. Grades A, B, C, D, E are the direct tool converting the grades of one scale into the grades of the other scale.

ECTS grading scale has been criticized a lot. P. Warfvinge emphasizes that a chosen norm-referenced nature of the ECTS grading scale may be very uncomfortable in many pedagogic cultures [11]. Applying ECTS grading scale students ranking appears which is not accepted by Swedish educators. According to L. Dahlgren, A. Fejes, M.A. Dahlgren and N. Trowald students ranking contradicts students' cooperation and interaction which encourage effective students' study [1].

During implementation of ECTS grading scale in higher schools the problems of the application of this grading scale were brought to light. According to T. Karran at several universities, the local absolute or criterion-referenced local scale is directly mapped on the ECTS scale without taking the required distribution of grades into account [7]. According to P. Warfvinge higher schools chose easier but incorrect ECTS application practice which does not require taking into consideration the data of previous assessments and ranking students [11]. The study covering 20 universities and conducted by T. Karran showed that the alignment of ECTS grades varies within nation states and show that, despite the fact that ECTS grading is a norm-referenced system, while national systems are usually criterion-referenced, the ECTS conversion tables provided by universities indicate straight line transference from institutional to ECTS grades [7].

Whereas the second stage of the implementation of ECTS grades scale turned out to be difficult to implement the procedure of grades conversion was simplified with a help of ECTS grading table which rests on the first stage of five-point system [3]. ECTS grading scale based on a predetermined percentage structure is to be replaced by a simple statistical table completed for each degree programme or group of homogeneous programmes [3].

1 lentelė. The relation of grading scales A and B.

Grading scale A	Grades distribution in grading scale A (in percentage terms)	Grading scale B	Grades distribution in grading scale B (in percentage terms)
a_1	$k_1\%$	b_1	$l_1\%$
a_2	$k_2\%$	b_2	$l_2\%$
...	...	...	...
a_n	$k_n\%$	b_m	$l_m\%$
Total	**100%**		**100%**

3 Mathematical model of grades conversion

The mathematical model of grades conversion is made on the ground of ECTS grading table. The relation of two grading scales is described in below given ECTS grading table.

In Table 1 $a_1, a_2, \ldots, a_n$ signify the grades of A grading scale, and $b_1, b_2, \ldots, b_m$ – the grades of B grading scale. Marks of both grading scales are presented from highest to the lowest one. The sizes of grading scales are described by indexes n and m. These indexes satisfy the relationships described by inequalities (1):

$$n \geqslant 1, \qquad m \geqslant 1. \tag{1}$$

The data of grading scales A and B are recorded as empirical probabilistic distributions. Whereas some countries use increasing grading scales, and the others decreasing ones the markings of these grading scales are also different, a new variable – assessments index i – is introduced. Assessments indexes number the grades of scales in decreasing order and correspond the characteristic values of distributions under consideration. Characteristic values of distributions are one after one going natural numbers. The probabilities of the acquisition of the values of the being made empirical probability distributions are calculated using the formulas (2).

$$p_{Ai} = \frac{k_i}{100}, \quad i = \overline{1,n}, \qquad p_{Bj} = \frac{l_j}{100}, \quad j = \overline{1,m}. \tag{2}$$

For the conversion of grades from grading scale A to grading scale B it is necessary to make a two-dimensional empirical probability distribution (Table 2). The values of this distribution (i,j), $i = \overline{1,n}$, $j = \overline{1,m}$. The probability p_{ij} of the acquisition of distribution value (i,j) is a probability that a student's knowledge and skills on the grading scale A are assessed by a mark with index i, and on B grading scale will be assessed by a mark with index j.

The probabilities of two-dimensional empirical probability distribution are calculated using the formula (3):

$$p_{ij} = \min\left(p_{Ai} - \sum_{k=0}^{j-1} p_{ik};\ p_{Bj} - \sum_{k=0}^{i-1} p_{kj}\right), \quad i = \overline{1,n},\ j = \overline{1,m},$$
$$p_{i0} = 0, \quad p_{0j} = 0. \tag{3}$$

2 lentelė. Two-dimensional empirical probability distribution of grade conversion.

A	B			
	1	2	...	m
1	p_{11}	p_{12}	...	p_{1m}
2	p_{21}	p_{22}	...	p_{2m}
...	...	...	...	...
n	p_{n1}	p_{n2}	...	p_{nm}

Grade's equivalent is attached on the ground of made two-dimensional empirical probability distribution. If grades are not rating the most probable equivalent is attached to a grade.

$$a_i = b_k, \quad \text{if } p_{ik} = \max(p_{i1}, p_{i2}, \ldots, p_{im}), \quad i = \overline{1,n}. \tag{4}$$

If the probabilities of the acquisition of some grades are equal the maximum grade equivalent is taken, i.e., the mark whose grading index is smaller. Mark a_i on the grading scale A corresponds to mark b_k on the grading scale B, if the probability of the appearance of distribution value (i,k) satisfies the relationships described by inequalities (3).

$$\begin{gathered} a_i = b_k, \quad \text{if } p_{ik} = \max(p_{i1}, p_{i2}, \ldots, p_{im}), \\ \text{if } p_{ik} = p_{il}, \quad \text{then } k < l, \ i = \overline{1,n}. \end{gathered} \tag{5}$$

If the students are rating the marks are sorted out in the order of decreasing of assessment value and every grade gets corresponding rating. The number sk_i of grades corresponding to a converted mark a_i is redistributed using (3) the formula. Grade b_1 corresponds s_{i1} the highest a_i grades, b_2 – s_{i2} the next a_i grades and so on.

$$\begin{gathered} s_{ij} = \left[\sum_{k=1}^{j} \frac{p_{ik}}{\sum_{l=1}^{m} p_{il}} sk_i + 0.5 \right] - \sum_{k=0}^{j-1} s_{ik}, \quad i = \overline{1,n}, \ j = \overline{1,m}, \\ s_{i0} = 0. \end{gathered} \tag{6}$$

If the rating of a converted mark a_i in the analyzed assessment set is r, grade rating a_i in marks group v, the number of grades corresponding to this rating rsk_r, is redistributed using the formula (3). Grade b_1 corresponds rating r corresponding grades, b_2 – rating r corresponding grades and so on.

$$\begin{gathered} c_{rj}^i = \min \left(s_{ij} - \sum_{k=0}^{v-1} c_{kj}^i; \ rsk_r - \sum_{k=0}^{j-1} c_{rk}^i \right), \quad i = \overline{1,n}, \ j = \overline{1,m}, \\ c_{0j}^i = 0, \quad c_{r0}^i = 0. \end{gathered} \tag{7}$$

Then the grade ai of rating r corresponds to the mark bk in grading scale B, if relationships describes by equality are satisfied (3).

$$\begin{gathered} a_i = b_k, \quad \text{if } c_{rk}^i = \max\left(c_{r1}^i, c_{r2}^i, \ldots, c_{rm}^i\right), \quad i = \overline{1,n}, \ k = \overline{1,m}, \\ \text{if } c_{rk}^i = c_{rl}^i, \quad \text{then } k < l. \end{gathered} \tag{8}$$

4 Conclusions

ECTS grading scale does not guarantee the comparability, transparency and correct interpretation of marks received in different countries, institutions and study programs. The conducted analysis showed that the most part of higher education institutions chooses easier but incorrect practice of ECTS grading scale.

The mathematical model of grades conversion is made on the ground of ECTS grading table and converts grades on the principle of the most probable grade equivalent evaluating the rating of a grade in the group of analyzed grades.

Students rating and united usage of the made model would guarantee the transparent and correct interpretation of marks in different higher education institutions.

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SUMMARY

Pažymių konvertavimo iš vienos vertinimo skalės į kitą matematinis modelis
J. Lieponienė, R. Kulvietienė

Straipsnyje analizuojamas ECTS vertinimo skalės taikymas, sukurtas vertinimų konvertavimo iš vienos vertinimo skalės į kitą matematinis modelis. Šis modelis yra sudarytas ECTS vertinimų lentelės pagrindu ir pažymius konvertuoja labiausiai tikėtino vertinimo atitikmens principu, įvertinant pažymio reitingą analizuojamų pažymių grupėje.

Keywords: vertinimo skalė, ECTS vertinimo lentelė, empirinis tikimybinis skirstinys.