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Analysis of Lithuanian aggregate consumption by means of dynamic programming*

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Abstract. In the paper, we model household consumption from the perspective of the modern representative agent-based approaches. Household chooses a stochastic consumption plan to maximise the expected value of their time-additive nonlinear utility function subject to asset budget constraint. We apply the dynamic programming for this multi-period problem using the Bellman equation. Beforehand we estimate the structural parameters of the given problem. We employ numerical methods to compute equilibrium. Finally we obtain that observed consumption was just below equilibrium in 2008.

Keywords: consumption, dynamic programming, stochastic optimization.

Introduction

Household consumption is the largest component of aggregate expenditure in most economies. In Lithuania it accounts for about 65% of spending. Therefore it is important for macroeconomists to be able to explain the determinants of consumer spending via a well-specified consumption function. Although literature on modelling consumption is large [2,4,5,8], in Lithuanian academic literature the analysis of household consumption, at macroeconomic level, is relatively scarce. Typically consumption is an integral part of a larger structural macroeconomic model [10,11] and there is one recent publication devoted only for the modelling of consumption [9]. In the latter the authors use consumption as the error-correction type of model. Their results are quite close to the general ideas of Friedman's permanent income hypothesis. Modern theories of consumption are based on analysis of optimal consumption behaviour over time under constraint [1]. In equilibrium, a rational consumer chooses optimum levels of consumption in each period so as to maximise utility. Different forms of utility functions are used by the economists. The most widely used utility functions are the ones with constant absolute risk aversion (CARA) or constant relative risk aversion (CRRA). Typically the function is chosen according to the ease of theoretical exposition or according to the need for model calibrations and simulations.

In this paper, we model household consumption from the perspective of the modern representative agent-based approaches. Household chooses a stochastic consumption

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plan to maximise the expected value of their time-additive nonlinear utility function subject to asset budget constraint. This multi-period problem can be solved by using the Bellman equation. We employ numerical methods to compute equilibrium.

1. Estimation of structural parameters

Households choose a stochastic consumption plan to maximise the expected value of their time-additive utility function subject to an assets budget constraint

$$\max_C \mathbb{E}_t \sum_{i=0}^{\infty} (1+\delta)^{-i} u(c_{t+i}) \tag{1}$$

subject to

$$R(A_{t+i} + y_{t+i} - c_{t+i}) = A_{t+i+1}, \tag{2}$$

where $C = \{c_t, c_{t+1}, c_{t+2}, \dots, c_{t+i}, \dots\}$ with $i = 0, 1, \dots, \infty$; $R = 1 + r$ and A_t is given. $\mathbb{E}_t$ denotes the mathematical expectations operator conditional on information available at time t; δ is the rate of subjective time preference and acts like an interest rate. r is constant rate of real interest; c_t is consumption; A_t are assets apart from human capital; $u(\cdot)$ is the one-period utility function that is assumed strictly concave and time separable, and y_t are earnings which are stochastic. In addition, a non-negativity constraint on consumption must be imposed. We assume that this constraint is always fulfilled.

To estimate the structural parameters in (1) one needs to specify the functional form of the utility function. As the most popular functions among the economists are CARA and CRRA type utility functions we test both of the functions. CARA type utility function is

$$u(c_t) = \frac{1}{\gamma}\bigl(1 - \exp(-\gamma c_t)\bigr), \tag{3}$$

where γ is the coefficient of absolute risk aversion.

CRRA preferences have the following form

$$u(c_t) = \begin{cases} \frac{c_t^{1-\sigma}}{1-\sigma}, & \text{if } \sigma \neq 1, \\ \ln c_t, & \text{if } \sigma = 1, \end{cases} \tag{4}$$

where σ is the coefficient of relative risk aversion.

There is no consensus about the size of discount factor $\beta = \frac{1}{1+\delta}$, absolute and relative risk aversion coefficients. Common assumption is that $0 < \beta < 1, \gamma \geqslant 0, \sigma \geqslant 0$. Very often authors assume that latter coefficients are constant over time. In [1] authors fix the coefficient of relative risk aversion at the value of 1.5 and experiment with assumptions on the discount factor (0.99, 0.98, 0.96, 0.91). In quarterly calibrated Lithuanian DSGE model [7], the coefficient of relative risk aversion is equal to 1.0 and discount factor is 0.99. However in [6] the discount factor was chosen 0.96 to meet 4 percent constant interest rate.

Empirical analysis is conducted using quarterly Lithuanian data covering period from year 1995 to the first quarter of 2009. Real consumption is a subcomponent of the gross domestic product deflated by private consumption deflator. We calculate real interest rates as deposit interest rates less inflation calculated from consumer price index. In the analysis we also use distribution of consumption expenditure from household budgets statistics which is available for year 2003–2007 on annual basis. Consumption per head is calculated dividing consumption by the number of Lithuanian inhabitants and normalised dividing observations by the mean value.

For estimation of absolute and relative risk aversion we use the first-order condition which is so-called Euler equation

$$u'(c_t) = R\beta\mathbb{E}_t u'(c_{t+1}). \tag{5}$$

This equation gives the dynamics of marginal utility in any two successive periods. Therefore we minimise squared difference between marginal utility at t and $t+1$ to estimate the parameters of the utility functions

$$\min \sum_{t=1}^{n} \left(u'(c_t) - R\beta\mathbb{E}_t u'(c_{t+1})\right)^2, \tag{6}$$

where n is a number of observations. When the utility function is CARA type we obtain estimate for the coefficient of the absolute risk aversion γ. Respectively, when we choose CRRA type utility function we estimate the coefficient for the relative risk aversion σ. As in [1] we experiment with different assumptions on the discount factor (0.99, 0.98, 0.97, 0.96). Although interest rates was somewhat different before and after year 2000 (see Table 1), but we will follow [7] and keep interest rates fixed at 4 percent for the comparison of the results.

We present the results for both utility functions in Table 2. From the table one can see that with smaller discount factor both coefficients of risk aversion decrease.

As discount factor might be treated as other interest rate, we compare the obtained results with actual interest rates. From Table 1one can see that the average real interest

Table 1. Comparison of nominal and real interest rates over different time periods

Period	1995–2008	1995–2000	2000–2008
Nominal interest rates	1.06	1.09	1.04
Real interest rate	0.97	0.91	1.02

Table 2. Evolution of absolute and relative risk aversion given different discount factors

β	0.99	0.98	0.97	0.96
$\gamma \times 10^{-3}$	1.87	1.31	0.67	0.23
σ	1.62	1.09	0.55	0.15

rate is negative ($R < 1$) over the period 2000–2008. However the average interest rate was negative until year 2000 and it turned to positive in consecutive period. Given that discount factor should be smaller than 1, we choose CRRA type of utility function with $\beta = 0.98$ and respectively CARA type utility function with $\gamma = 0.00131$ corresponding $\sigma = 1.09$ and $\delta = 0.02$ for further analysis. Our findings are somewhat different from the parameter selection in [7]. According the analysis if one chooses discount factor equal to 0.99 the coefficient of relative risk aversion should be equal to 1.62 rather 1.

Having estimated the structural variables we move to stochastic optimization of the given problem where we restrict our analysis only to CRRA type of utility function.

2. Dynamic programming

The intertemporal separability of the objective function and the accumulation constraints allow us to use dynamic programming methods to solve the above problem, which can be decomposed into sequence of two-period optimization problems. As in [2] and [5] for dynamic programming we use the Bellman equation, which general form is

$$V_t(A_t) = \max_{c_t} \big(u(c_t) + \beta \mathbb{E}_t\big[V_{t+1}(A_{t+1})\big]\big), \tag{7}$$

where $V_t(\cdot)$ is a value function. The value function is stochastic as future income are uncertain and enters (7) as an expected value. The first-order conditions, so-called Euler equation, are obtained taking derivative of (7) with respect to consumption c_t subject to constraint (2).

The state variable (A) at time t is the consumer's certain amount of resources at the end of period t, i.e. $R(A_t + y_t - c_t)$ and the value function respectively is $V_t(R(A_t + y_t - c_t))$, where subscript t means that the value of resources available depends on the information set at time t. Given the latter and applying (7) for the problem (1) subject to (2) we obtain

$$V_t(A_t) = \max_{c_t} \Big(u(c_t) + \beta \mathbb{E}_t\big[V_{t+1}\big(R(A_t + y_t - c_t)\big)\big]\Big). \tag{8}$$

This equation gives us a way of solving the original optimization problem. The idea is to start at the end and proceed to earlier times recursively. Suppose that the consumer's horizon ends at time T. The final period where a choice is made is in $T - 1$. In the period T, the individual consumes the remaining wealth and the labour income, i.e., $c_T = A_T + y_T$, if we assume that there is no bequest, i.e., $A_{T+1} = 0$. The optimization problem at time $T - 1$ is

$$V_{T-1}(A_{T-1}) = \max_{c_{T-1}} \Big(u(c_{T-1}) + \beta \mathbb{E}_{T-1}\big[V_T\big(R(A_{T-1} + y_{T-1} - c_{T-1})\big)\big]\Big) \tag{9}$$

subject to

$$A_T = R(A_{T-1} + y_{T-1} - c_{T-1}). \tag{10}$$

This procedure can now be used to derive optimal plans for consumption in each period until we finally arrive to period 0.

To obtain optimal consumption plans one has to estimate Euler equations. Very often authors linearise nonlinear Euler equations using first or second order Taylor polynomials [1,2,4,5]. Linearised Euler equations were estimated using the general moment method and in many studies the estimated coefficients had opposite signs or were not stable [3,8,9]. For described above framework, we do not calculate Euler equations analytically rather we find optimal consumption plans directly applying numerical methods. We obtain that equilibrium consumption should be close to 62.6 mill. in 2008. As to the latest data real private consumption stands at 58.6 mill., indicating that our estimation is slightly higher than the observed value.

3. Conclusions

In the paper, we model household consumption from the perspective of the modern representative agent-based approaches. Household chooses a stochastic consumption plan to maximise the expected value of their time-additive nonlinear utility function subject to asset budget constraint. We apply the dynamic programming for this multi-period problem using the Bellman equation. Beforehand we estimate the structural parameters of the given problem using two type utility functions: constant average risk aversion and constant relative risk aversion. Having estimated the parameters, we calculate the Bellman equations and employ numerical methods to compute equilibrium. Finally we obtain that observed consumption was just below equilibrium in 2008.

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REZIUMĖ

A. Jakaitienė, A. Žilinskas, J. Žilinskas. Lietuvos visuminio vartojimo analizė dinaminio programavimo metodu

Straipsnyje sprendžiamas vartojimo tarplaikinio optimizavimo modelis grindžiamas reprezentatyviojo vartotojo siekiu pasirenkant optimalų vartojimo planą maksimizuoti viso gyvenimo vartojimo naudingumą. Maksimizuojant naudingumą atsižvelgiama į biudžetinį apribojimą. Šį dinaminį optimizavimo uždavinį galima išspręsti taikant dinaminio programavimo metodą, kai daugiaperiodis uždavinys suskaidomas į dviejų periodų uždavinius. Dinaminio programavimo uždavinys sprendžiamas naudojantis Belmano funkcionalinėmis lygtimis. Darbe įvertinti sprendžiamo uždavinio parametrai. Pusiausvyros vartojimas apskaičiuojamas taikant skaitmeninius metodus. Gauta, kad visuminis vartojimas 2008 metais buvo šiek tiek mažesnis negu pusiausvyros vartojimas.

Raktiniai žodžiai: vartojimas, dinaminis programavimas, stochastinis optimizavimas.