

## Additional mathematical education for pupils

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**Abstract.** In this paper we make overview of the additional mathematical education for pupils in Lithuania. Mathematical contests and olympiads, clubs and schools, lectures of professional mathematicians and other activities are discussed including the history and contemporary condition of all these activities. We give special credit to the enthusiasm and work of the educators from schools and university and scientists in the field of additional education.

*Keywords:* additional education, competitions, curriculum, pupils, mathematical olympiads, schools.

During recent years the level of mathematical knowledge of pupils entering universities is deteriorating. This is due to small amount of weekly hours for teaching mathematics, imbalanced school curricula, non-compulsory state exam of mathematics, lack of teachers and the prestige of teaching profession in general, as well as other various reasons. Universities apply different means in order to facilitate the adaptation process of freshmen, including the introduction of equivalency courses. Since September 2008 the equivalency courses will be also introduced at the Faculty of Mathematics and Informatics of Vilnius University (FMI VU).

However, despite of all general problems mentioned above, some pupils still have the inclination towards the exact sciences. Some of them study in different schools with additional mathematical education, prepare themselves for mathematical olympiads and participate in them. Our task is to provide the opportunities for additional education for as many pupils as possible. The improving performance of the teams of Lithuanian pupils in Baltic, international and world olympiads show that we have talented pupils. Of course, this would not be possible without consistent work of excellent teachers of mathematics at schools and tutors at universities, who care about additional education for pupils.

Additional mathematical education for pupils is the important area of education for young people, therefore always urgent. The problem of mathematical education has been investigated many times (see, e.g., [1]).

In this article we try to present a wider view of additional mathematical education in Lithuania, including the review of the new means of education which have appeared in recent years.

How do we perceive the additional mathematical education? It is education for pupils when teaching or studying mathematics takes place outside the lessons at school. The aim of all means and events of additional education, whoever organises them, is the same: to encourage pupils' interest in mathematics, to develop their creativity, to find and cherish those pupils who like mathematics and are talented, to make

the work of teachers of mathematics more active. Therefore, it includes additional sessions, participating in the activity of the mathematical clubs, preparation for mathematical olympiads and competitions, participation in them, studying at various mathematical schools, attendance of lectures which are provided by the tutors of university or professional mathematicians, etc.

We will survey only those means and events which are in some ways related to universities, to the departments which facilitate mathematical education and to Institute of Mathematics and Informatics (IMI) as well as to the Lithuanian Society of Mathematicians and to the Lithuanian Association of Teachers of Mathematics. We will start with the survey of olympiad movement in Lithuania.

Lithuanian Mathematical Olympiads, which are being organised since 1952, are already historical. In fact, they were started in 1951 with Vilnius Olympiad. These annual olympiads, which were originally organized by prof. J. Kubilius, are now arranged by the Lithuanian Society of Mathematicians, the Ministry of Education and Science (MES), FMI VU, IMI. The tutors from various universities and mathematicians from IMI participate in the work of jury.

The collections of tasks and solutions from mathematical olympiads are later published as separate books (see, e.g., [3–5]), which are very useful for preparing young mathematicians for olympiads and in additional teaching of mathematics in general.

The competition for young mathematicians in Šiauliai University began since 1967 with prof. B. Balčytis in charge. Now the main organiser is the Department of Mathematics (head of the department ass. prof. J. Genys). Pupils of the 5th–12th grades from the town of Šiauliai used to participate in the Olympiad. Since 1972 pupils of 9th–11th grades were invited from Šiauliai town and region. In 1974 it already became the North Lithuanian Olympiad for young mathematicians, which later turned into elimination competition for Lithuanian Olympiad for young mathematicians. In the first ten places one could usually see the teams from town of Šiauliai and Klaipėda, and from regions of Kretinga, Mažeikiai, Plungė, Ukmergė, Pasvalys, Raseiniai, Tauragė, Kaišiadorys and Kėdainiai. Among the winners there were names of the pupils from such schools as Panevėžys J. Balčikonis Gymnasium, Mažeikiai “Gabijos” Gymnasium, Kretinga J. Pabrėža Gymnasium, Šiauliai J. Janonis Gymnasium, Plungė “Saulės” Gymnasium, Raseiniai Žemaitis Gymnasium, Klaipėda “Ažuolyno” Gymnasium, Mažeikiai M. Račkauskas Gymnasium. In the recent years even 4 prize places went to the pupils from Vilnius Lyceum. In these educational institutions the preparation of pupils for mathematical olympiads has deep traditions. It is a hard work of the mathematics teachers. Paulius Šarka from Kretinga J. Pabrėža Gymnasium, Ignas Budvytis and Regimantas Valentonis from Panevėžys J. Balčikonis Gymnasium and Mindaugas Nausėda from Raseiniai Žemaitis Gymnasium were the winners in this olympiad for several years in a row.

Due to the initiative of ass. prof. A. Zabulionis, in 1986 the prof. J. Kubilius Cup as mathematical team-contest was initiated. At that time mathematical team olympiads still were a novelty. Now this olympiad, organised annually by the tutors from Vilnius University, Vilnius Pedagogical University (VPU) and scientists from IMI, has become very popular. In terms of education, team olympiads are very useful because they teach how to work in a team. Also team members, seeking for the better results, have to share

their work; the pupils have to evaluate their own abilities and those of the other team members.

The first J. Matulionis competition for young mathematicians, which was organised for the very first time in 1990, will be organised for the 20th time next year (2009). The idea to organise this competition occurred after the discussions with the tutor from Vilnius University, A. Zabulionis, who that year was in charge of the National olympiad for young mathematicians. This competition is taking place every year at the Faculty of Fundamental Sciences of Kaunas Technology University (FFS KTU) and is organised by the tutors and student representatives from KTU, and is financed by organisers and supporters. Much work for organising this competition was contributed by prof. V. Pekarskas, ass. prof. L. Papreckienė and many other employees of FFS KTU. All the pupils of the 9th–12th grades can participate in this competition without prior selection or registration in advance. The pupils arrive by their own means and there is no participation charge. Evaluation commission decides on the winners of the first, second and third places for the pupils of the 9, 10, 11 and 12 grades. The winners and their teachers are awarded with certificates of gratitude from the KTU Rector, with prizes, and the best of them are invited to the National Olympiad. The number of participants has increased from 110 in 1990 to 788 in 1999. After that the number of participants slightly decreased: 588 in 2003, 502 in 2004, 336 in 2005, 592 in 2006, 600 in 2007 and 475 in 2008.

The mathematical competition of VPU, which is being organised since 1992, is the mathematical olympiad for the pupils, which is the selective competition for Lithuanian National Mathematical Olympiad for the pupils. It was initiated by the tutors from the Faculty of Mathematics and Informatics ass. prof. V. Bernotas, ass. prof. J. Šinkūnas, ass. prof. R. Skrabutėnas, ass. prof. S. Grigelionis, later the leadership was taken over by ass. prof. A. Kaučikas and ass. prof. E. Mazėtis. The olympiad is taking place every year in the beginning of March. All willing pupils of the 9th–12th grades can participate. 4 tasks are prepared for every grade and the pupils with the best performance are invited to the Lithuanian National Mathematical Olympiad. The number of participants usually reaches 130–180, although it was as high as 250 one year. Many of the winners of the VPU competition later win the first places in the Lithuanian National Mathematical Olympiads.

The olympiads for the pupils of younger age are less common. The participants of the olympiads mentioned above are usually pupils from senior school years, whereas there were no such events for the pupils of younger age. Since 1999 personal olympiads for the pupils of the 5th–6th and the 7th–8th grades were added by ass. prof. R. Kašuba to the Lithuanian Mathematical Team Olympiad. The winners and laureates of them, 'infected' by participation in the olympiads, usually become more active participants in the olympiads for the pupils of senior years.

The Mathematical Olympiads for the pupils of the 4th–5th grades and the 6th–8th grades are being organised in Šiauliai University by the Faculty of Educology (prof. A. Kiseliovas). In 2008 the tenth such olympiad took place. The number of participants in these olympiads is quite high. For example, in 2003 in the olympiad in Šiauliai for the 4th–5th grades there were over 500 participants from the whole country, and in the olympiad for the pupils of the 6th–8th grades there were 100 pupils from every grade.

The other famous competition is the international mathematical competition KANGAROO, which is being organised in some Lithuanian schools since 1995, and in the most of Lithuanian schools since 2000. Interesting tasks of this competition encourage its constantly increasing popularity. The competition is organised for five (now already six) groups of age. Every year more than 1000 schools and over 60 thousand pupils participate in this competition. This competition would not be that popular in Lithuania without a huge input of dr. J. Mačys (IMI) – the head of “Kangaroo” committee, ass. prof. R. Kašuba, expert teachers M. Stričkienė (MES), L. Nonėvič (Vilnius A. Mickevičius Gymnasium), K. Čiuprinska (Vilnius John Paul II Gymnasium), L. Narkevičius (KTU Gymnasium) and other mathematicians, involved in the committee of tasks preparation for “Kangaroo”, as well as in the international “Kangaroo” camps. The expert teacher S. Rudalevičienė (Vilnius “Ryto” Secondary School) is the head of “Kangaroo” camps.

The publications of “Kangaroo” competition tasks [6–8], which have been for a long while published by TEV publishing house, is a very good material for teaching pupils to solve the tasks which are not traditional.

We mentioned the national olympiads. Apart from them there are quite a few regional mathematical olympiads organised in the country.

Since 1999 in Pasvalys Region the prof. B. Grigelionis Cup is taking place every year (ass. prof. A. Apynis, prof. V. Bagdonavičius).

Every year, since 2000, ass. prof. R. Kašuba organises in Raseiniai region the Cup of prof. J. Kubilius, successfully involving the students in mathematics to help with task preparation and to participate in the evaluation of works.

Since 2000 Mathematical Team Olympiad for the pupils of 5–10 grades to win the Cup of prof. V. Statulevičius is taking place in Utena “Aukštakalnio” Basic School. The organisers of this event, ass. prof. V. Sičiūnienė and ass. prof. J. Šinkūnas are also participating in the educational project for children and youth “Into the Depth of Mathematics”, organised by Utena “Aukštakalnio” Basic School.

In 2007 the sixth team olympiad of young mathematicians to win the Prize of prof. V. Liutikas took place in Kretinga. The organisers of this event are: ass. prof. A. Bakštytys, Department of Mathematics of ŠU, as well as the educational department of municipality administration of Kretinga region.

Since 2002 the mathematical team olympiad of Rietavas to win the Cup of teacher K. Šikšnius is taking place. The organisers of this event are the municipality of Rietavas, Rietavas Laurynas Ivinskis Gymnasium and prof. E. Stankus.

Since 2005 ass. prof. A. Apynis and his colleagues prepare tasks for the mathematical olympiads for pupils of Alytus region to win the Cup of K. Klimavičius.

It is already two years since the Department of Didactics of Mathematics and Informatics of FMI VU prepares the tasks for Vilnius Mathematical Olympiad.

In 2007 the first team olympiad for young mathematicians took place in Kvėdarna (Šilutė region). The organiser of this event is prof. D. Jurgaitis.

We would like to mention at least a couple of teachers who give all their strength and time to work additionally with pupils, organise activities of mathematical clubs, prepare pupils for olympiads of different levels, etc. Those names are: the expert teachers I. Bagdonienė, B. Budvytis, A. Skūpas (Vilnius Lyceum), L. Narkevičius

(KTU Gymnasium), V. Narmontas (Kretinga J. Pabrėža Gymnasium), A. Choliavkin (Vilnius “Juventos” Gymnasium), E. Tumėnaitė (Panevėžys J. Balčikonis Gymnasium), P. Grebeničenkaitė (Šiauliai Didždvaris Gymnasium), J. Mackevičienė (Visaginas “Gerosios vilties” secondary school), Z. Šiaulienė (Vilnius Tuskulėnai secondary school) and many others.

In short we will survey the schools of additional mathematical education and other activities of university tutors, which encourage pupils’ interest in mathematics.

A significant influence for mathematical education of the youth was made by the two-year Lithuanian distance learning school for young mathematicians, which was established in 1969 by the Lithuanian Society of Mathematicians. During twenty years of this school’s existence 7469 pupils graduated from it. This school was fostered by the chairman of the Lithuanian Society of Mathematicians prof. J. Kubilius; prof. K. Grincevičius was the first chairman of the school council, much work has been contributed by prof. P. Survila, ass. prof. H. Jasiūnas, ass. prof. A. Jonušauskas, ass. prof. A. Matuliauskas, ass. prof. A. Nagelė, ass. prof. E. Neniškytė, ass. prof. P. Rumšas, ass. prof. P. Vaškas and many other tutors from the Faculty of Mathematics and Mechanics of VU, which was later renamed to the Faculty of Mathematics.

The Lithuanian School for Young Mathematicians (LSYM) was re-established in 1998 due to the initiative of ass. prof. A. Apynis (VU) and continues the work of the Lithuanian Long Distance School for Young Mathematicians, which was operating during years 1969–1989. The founders of this school are: Vilnius University, Institute of Mathematics and Informatics, Kaunas Technological University, Vilnius Pedagogical University, Šiauliai University. The council consists of 13 representatives from these institutions, MES and secondary schools. Ass. prof. A. Apynis has been the head of the council since the very beginning of re-establishment of the Lithuanian School for Young Mathematicians. Since 1998 nine generations of pupils have graduated from Lithuanian School for Young Mathematicians – 3112 pupils altogether. This year in September the Lithuanian School for Young Mathematicians will celebrate its tenth anniversary. The studies at the Lithuanian School for Young Mathematicians are long distance and it takes two years to graduate from it. The intake to LSYM takes place in September. The pupils from the 11th grade (or the pupils of the corresponding level from gymnasiums), who submit the solutions of entrance tasks and receive positive evaluation are accepted.

What do the pupils of LSYM study? While compiling the programme the first priority is to provide pupils with the opportunity to deepen their knowledge of school mathematics. There are also subjects included in order to increase the scope of their knowledge, which reveal the possibilities to apply mathematics, although they are not within the limits of school curriculum. During the learning programme of LSYM, which takes two years, the pupils have to solve eight tasks after receiving methodical instructions and examples of solutions. The papers of the pupils of the Lithuanian School for Young Mathematicians are evaluated by the students – future teachers from VU, VPU, ŠU, for whom it is a very good practice. The pupils of LSYM, who have solved at least six tasks at acceptable level, are invited to the Faculty of Mathematics and Informatics of VU for their final task. The teachers of mathematics from schools and gymnasiums of Vilnius and Širvintos are willingly helping in the organisation of

the final task and evaluation of the final works - big thanks goes to them. The pupils who fulfil the whole learning programme are awarded with certificates, which are recommendation for studying the exact sciences. The website of LSYM is very popular. There pupils can find out the scores for their solutions of the tasks and various news, related to studies.

After every school year the book "For young mathematician" is published, which consists of all eight tasks, their solutions and methodical instructions. Eight such books have been published so far (see [2]), which are helpful both for pupils and teachers.

The National Academy for Pupils (NAP), which was established in 2004 is a fulfilled old dream of ass. prof. B. Narkevičienė from KTU. In the National Academy for Pupils there are eight sections of subjects: mathematics, informatics, chemistry, Lithuanian language, economics, physics, biochemistry, and music. Altogether, there were over 750 children from whole Lithuania who participated in development and prospecting programs of NAP. The teaching is conducted via internet (during the whole year) and via sessions (three times a year they take place in Kaunas, Vilnius, Palanga and Nida). During three years of the project funded by ESP there were 180 children who graduated from NAP. Moreover, qualification courses for the teachers, during which they have a possibility to receive supervisions by scientists and expert teachers, are being organised. The leaders of the mathematics section are expert teacher L. Narkevičius (KTU Gymnasium) and dr. A. Elijio (VU). There is a close cooperation with professor A. Andžans (University of Latvia), NAP retains connections with its graduates, who study in Lithuania and abroad. There was also issued the first methodical publication of NAP [9].

In 2005 Mathematics School "Rokunda" of Pasvalys region was established, which successfully operates with ass. prof. A. Apynis in the leading role. Ass. prof. A. Apynis regularly (once a month) delivers lectures for the pupils of Pasvalys and Pakruojis region on various topics of mathematics. He also organises various competitions and tournaments in this region.

Since 2005 the school of young mathematicians "Matematikos Olimpų" operates and the leader is ass. prof. R. Kudžma (VU). The lectures in the sessions of this school are delivered by tutors, doctorate students, and the students of senior year from VU.

There are popular series of articles by ass. prof. R. Kašuba (since 1998) about mathematics and mathematical tasks in the magazine "Kompiuterija". These are the articles where the tasks are formulated, which are willingly solved by the pupils. Ass. prof. R. Kašuba is also the organiser of the olympiad camps for pupils, the leader of olympiad teams participating in the olympiads of various levels, the facilitator of preparation of teams of pupils for international, world and Baltic countries olympiads, and the participant of various regional olympiads and events.

Ass. prof. A. Kaučikas (VPU) was working for a long time with the talented pupils from the secondary schools of Visaginas and Utena and was preparing them for olympiads. There are quite a few winners of National Olympiads among the pupils who were prepared by him.

For nine years the Saturday school for participants of mathematical olympiads for the 5th–6th grades has been operating in Kaunas (the leader and organiser of sessions is the expert teacher from KTU Gymnasium L. Narkevičius). Since the current school

year this school was expanded and it now encompasses of the pupils of the 2nd–8th grades. From the children who were attending this school we have already had more than one prize winner of the National Olympiads, and two of them have also participated in the IMO.

It is not possible to name here all Lithuanian teachers who are preparing their pupils for various olympiads – schools, towns, regions, districts – which are also being organised by the same teachers, who care about the mathematical education and the development of Lithuanian youth. It is their huge input towards Lithuanian future.

While preparing this survey, the information provided by ass. prof. A. Apynis, ass. prof. V. Bernotas, prof. D. Jurgaitis, ass. prof. R. Kašuba, ass. prof. E. Mazėtis, ass. prof. B. Narkevičienė, expert teacher L. Narkevičius, ass. prof. L. Papreckienė was used. The author sincerely thanks all of them.

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### REZIUMĖ

#### *E. Stankus. Mokinių papildomas matematinis ugdymas*

Straipsnyje analizuojamos papildomo mokinių matematinio lavinimo problemos. Autoriaus supratimu – tai įvairių lygių matematikos konkursai ir olimpiados, papildomo ugdymo būreliai bei mokyklos, profesionalių matematikų paskaitos ir kitokie renginiai, lavinantys mokinių matematinį gebėjimą. Apžvelgiama mokinių papildomo matematinio ugdymo, olimpiadinio judėjimo Lietuvoje raida ir dabartinė padėtis. Atkreipiamas dėmesys į reikšmingą entuziastų – vidurinių mokyklų ir gimnazijų mokytojų, universitetų dėstytojų ir mokslininkų darbą šioje srityje.

*Raktiniai žodžiai:* matematikos olimpiados, mokiniai, mokyklos, papildomas ugdymas, programos, varžybos.