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R/S financial market analysis

Svetlana DANILENKO (VGTU)
e-mail: svetlana.danilenko@fm.vgtu.lt

Abstract. This work discusses the application of R/S analysis to financial markets. The most of the work is devoted to the calculation of Hurst parameter. The Hurst parameters of the Baltic state shares indices are presented.

Keywords: R/S analysis, Hurst parameter, financial markets, share index.

1. Introduction

R/S method had been developed by a hydrologist H.E. Hurst in his work [1] in which he investigates the level of the river Nile. Later B.B. Mandelbrot [2, 3] developed this idea in his works and called the parameter described by H.E. Hurst by the name of the author. Hurst parameter has been used in many works. Due to its appearance in E.E. Peters [4, 5] works, it is now popular for analysis of financial markets [6, 7, 8].

Hurst method discloses the following properties of the statistical data: clusterisation, persistence, short range dependence, anti-persistence, presence of periodical or non-periodical cycles, etc. [9]. With the help of Hurst parameter it is possible to distinguish a random sequence from a non-random sequence, even if the random sequence is not Gauss sequence [4].

R/S analysis method is one of the most commonly used methods for the evaluation of the Hurst parameter.

2. Application of R/S analysis to financial markets

The application of R/S analysis to financial markets will be discussed.

Let $S = (S_n)_{n\geqslant 0}$ – share index for the Baltic states, then the logarithm of the share index $h_n = \ln \frac{S_n}{S_{n-1}}$, $n \geqslant 1$ – the proportional change in the share index or the logarithmic change.

We will describe the essence of R/S analysis application for the investigation of $h = (h_n)_{n\geqslant 1}$ sequence properties.

Let us comprise $H_n = h_1 + \cdots + h_n$, $n \geqslant 1$.

Let us define

$$R_n = \max_{k\leqslant n}\Big(H_k - \frac{k}{n}H_n\Big) - \min_{k\leqslant n}\Big(H_k - \frac{k}{n}H_n\Big). \tag{1}$$

Let $\overline{h}_n \equiv \frac{H_n}{n}$, where $(h_1, h_2, \ldots, h_n)$ be an empirical mean, then $H_k - \frac{k}{n}H_n = \sum_{i=1}^{k}(h_i - \overline{h}_n)$ is H_k deviation from its empirical mean $\frac{k}{n}H_n$. The measure R_n characterises the degree of dispersion of $H_k - \frac{k}{n}H_n$, $k \leqslant n$ deviations.

Let the empirical dispersion

$$S_n^2 = \frac{1}{n}\sum_{k=1}^{n} h_k^2 - \left(\frac{1}{n}\sum_{k=1}^{n} h_k\right)^2 = \frac{1}{n}\sum_{k=1}^{n}\left(h_k - \overline{h}_n\right)^2 \tag{2}$$

and the adjusted range of the cumulative sums H_k, $k \leqslant n$

$$Q_n \equiv \frac{R_n}{S_n}. \tag{3}$$

Based on the R/S we are testing the null hypothesis (H_0) the share index is a random walk. If the null hypothesis is correct then if the value of n is large the values of R_n/S_n should be proportional to the square root of n

$$R_n/S_n \sim cn^{0.5}. \tag{4}$$

However, in financial data analysis it usually is

$$R_n/S_n \sim cn^{H}, \tag{5}$$

where H – Hurst parameter that is significantly different from 0.5.

Hurst parameter can be evaluated by graphically representing (plotting) $\log(R_n/S_n)$ dependence on $\log(n)$ and then calculating the slope by using the ordinary least squares regression method, i.e., the Hurst parameter H is calculated using an equation

$$\log(R_n/S_n) = \log(c) + H \cdot \log(n). \tag{6}$$

The essence of Hurst parameter [5]:

- $H = 0.5$ – means white (Gauss) noise. The correlation is zero.
- $0.5 < H \leqslant 1$ – means black noise. Such a time series is characterised by the long range dependence effect and is said to be persistent. In theory, the future is affected by the present. The correlation is positive, i.e., we should expect positive (negative) values of h_n following the positive (negative) values of h_n.
- $0 \leqslant H < 0.5$ – means pink noise. Such a time series is anti-persistent. The correlation is positive, i.e., we should expect positive (negative) values of h_n following the positive (negative) values of h_n.

3. Overview of the Baltic states' share indices

The Vilnius, Riga and Tallinn stock exchanges are the only stock exchanges in their countries [10]. The OMX Baltic states stock market index group is comprised of the Baltic states stock market comparability, trade, all shares and sector indices.

The indicators of the Baltic states stock markets are the OMXB, OMXV, OMXR and OMXT share indices.

When analysing the share markets one base has been chosen: the values of the stock market indices have been normalised at the beginning of the investigation period (1st January 2000). Their development between 2000–2007 is demonstrated in Fig. 1.

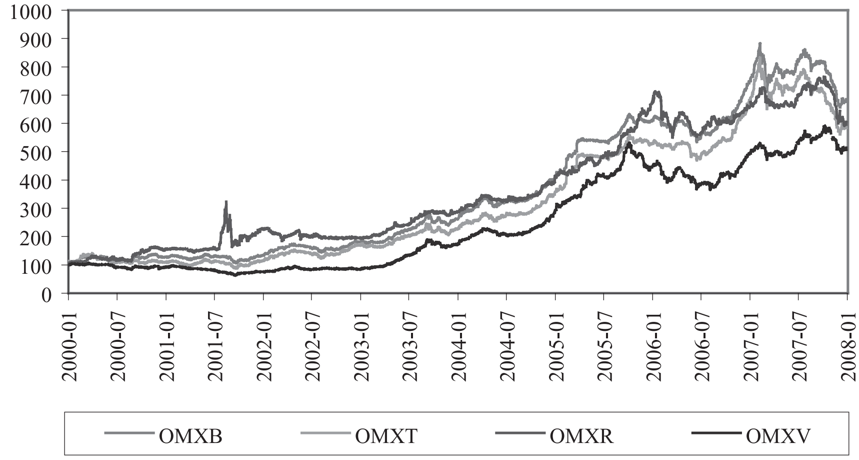
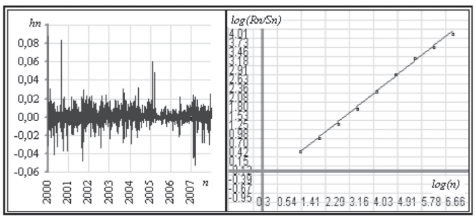


Fig. 1. Development of the share indices of the Baltic countries in 2000–2007.

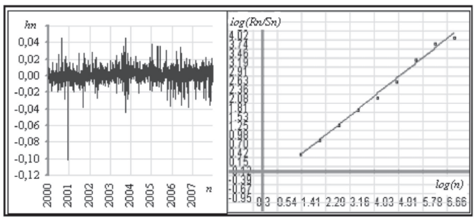
4. Application of the R/S analysis to the Baltic stock market indices

The Hurst parameters of the Baltic states share indices are calculated with the help of R/S analysis. They are obtained using SELFIS [11] – a programme that is freely distributed for common use.

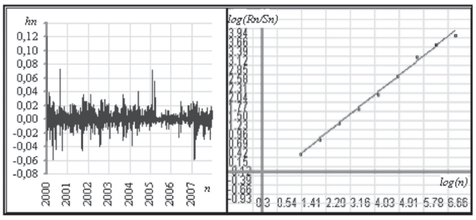
Fig. 2 presents the logarithmic changes of the considered share indices as well as the graphical evaluation of the Hurst parameter (according to the (6) formula) using the R/S analysis.



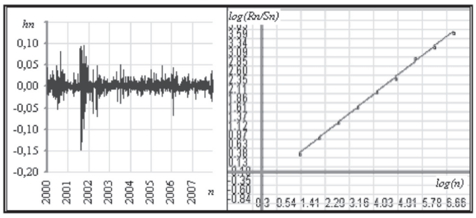
(a) OMXB



(b) OMXV



(c) OMXT



(d) OMXR

Fig. 2. Change in the share indices and the R/S analysis graphical evaluation of the Hurst parameter.

Table 1. The results of R/S analysis

Country	Share index	Number of observations	Hurst parameter
Baltic states	OMXB	2065	0.661
Lithuania	OMXV	2535	0.674
Latvia	OMXR	2553	0.591
Estonia	OMXT	2560	0.656

The values of share indices' Hurst parameters using the R/S analysis method are presented in the Table 1.

The obtained values of the Hurst parameters indicate that the Hurst parameter for all countries' share indices time series is within the following interval $0.5 < H \leqslant 1$ which shows that there is a week correlation in them.

5. Hurst parametr evaluation methods

The R/S analysis method is one of the methods of the Hurst parameter evaluation. However there are many methods of evaluating the Hurst parameter with the most commonly used being the following [12]:

- Ratio variance of residuals – a log-log plot of the aggregation level versus the average of the variance of the residuals of the series should be a straight line with slope of $H/2$.
- The periodogram method – this method plots the logarithm of the spectral density of a time series versus the logarithm of the frequencies. The slope provides an estimate of H.
- The Whittle method – the method is based on the minimization of a likelihood function, which is applied to the periodogram of the time-series. It gives an estimation of H and produces the confidence interval.
- The Abri-Veitch method – Wavelets are used for the Hurst exponent to be estimated. The energy of the series in various scales is studied to calculate the Hurst exponent.

The share indices' Hurst parameters evaluated using the above methods are presented in the Table 2.

Table 2. Evaluation of Hurst parameter using different methods

Country	Share index	Ratio variance of residuals	Periodogram method	Whittle method	Abri-Veitch method
Baltic states	OMXB	0.633	0.576	0.615	0.687
Lithuania	OMXV	0.778	0.617	0.602	0.617
Latvia	OMXR	0.627	0.498	0.537	0.593
Estonia	OMXT	0.648	0.596	0.570	0.640

6. Conclusion

Hurst parameter is widely applied for the time series analysis. When little information is held of the investigated system, the Hurst parameter helps to classify the time series. When the Hurst parameter is $0.5 < H \leqslant 1$ it is considered that it is a log memory process; however, when $0 < H \leqslant 0.5$ it is held that the process is changing quickly. The value of Hurst parameter equal to 0.5 indicates that the change in such a process grows with the speed of the square root of n (where n is time).

R/S analysis method is one of the little methods that evaluate the Hurst parameter. This method uses the rescaled range statistic (R/S statistic). The R/S statistic is the range of partial sums of deviations of a time series from its mean, rescaled by its standard deviation. A log-log plot of the R/S statistic versus the number of points of the aggregated series should be a straight line with the slope being an estimation of the Hurst exponent.

The Hurst parameter of financial data is usually bigger than 0.5. This is supported by the share indices' Hurst parameters findings for the Baltic states.

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REZIUMĖ

S. Danilenko. Finansų rinkų R/S analizė

Darbe nagrinėjamas R/S analizės metodo taikymas finansų rinkoms. Pagrindinis dėmesys skiriamas Hursto parametro skaičiavimui. Pateikiami įvertinti pagal R/S analizės metodą Baltijos šalių akcijų indeksų Hursto parametrai.

Raktiniai žodžiai: R/S analizė, Hursto parametras, finansų rinka, akcijų indeksas.