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On some cardinal invariants of space of finite subsets

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Abstract. In article is investigated relationships between some cardinal invariants of topological space and it's space of finite subsets with Vietoris topology. In general we note coincidence of them.

Keywords: topological space, Vietoris topology, hyperspace, the space of finite subsets.

All spaces are assumed to be Hausdorff. Points of space $\exp_{<\aleph_0} X$ are finite subsets of topological space. For finite subset $P \subset X$ coresponds point $(P) \in \exp_{<\aleph_0} X$.

DEFINITION [2]. Vietoris topology on the space of closed subsets of space X $\exp X$ we call topology wich base is formed of sets $\langle V, V_1, V_2, \dots, V_m \rangle$ there V and V_i are open in X and $\langle V, V_1, V_2, \dots, V_m \rangle = \{(F) \exp X\colon\ F \subset V, F \cap V_i \neq \oslash, i = 1, \dots, m\}$.

Remark. The space $\exp_{<\aleph_0} X$ is ocupated with Vietoris topology inducated from $\exp X$. The spaces $\exp_{\leqslant n} X = \{A\colon\ A \subset X, (A) \leqslant n\}$ are also investigated. Obviesly $\bigcup_{n=1}^{\infty} \exp_{\leqslant n} X = \exp_{<\aleph_0} X$.

THEOREM 1.*Weight $\omega(x)$ of topological space X coincide with weight $\omega(\exp_{<\aleph_0} X)$ of space* $\exp_{<\aleph_0} X$.

Proof. Space X is closed embedable into space $\exp_{<\aleph_0} X$, so $\omega(x) \leqslant \omega(\exp_{<\aleph_0} X)$.

On the other hand $\langle \bigcup_{i=1}^{m} V_i, V_1, V_2, \dots, V_m \rangle^* = \langle \bigcup_{i=1}^{m} V_i, V_1, \dots, V_m \rangle \cap \exp_{<\aleph_0} X$ where $V_i; i = 1, \dots, m$ are sets belonging to some open base of x forms base of space $\exp_{<\aleph_0} X$. Conseguently $\omega(\exp_{<\aleph_0} X) \leqslant \omega(x)$.

THEOREM 2. *Net weight $n\omega(x)$ of topological space X coincide with net weight $n\omega(\exp_{<\aleph_0} X)$ of space* $\exp_{<\aleph_0} X$.

Proof. Thus X is closed embedable into $\exp_{<\aleph_0} X$, so $n\omega(x) \leqslant n\omega(\exp_{<\aleph_0} X)$. Let $\widetilde{N}$ is net of space X closed respectivelly to finite unions, than family N^* where $\widetilde{N}^* = \{N^*\}$, on space $\exp_{<\aleph_0} X$ (there $N^* = N \cap \exp_{<\aleph_0} X$) form net on $\exp_{<\aleph_0} X$. Really, let $(P) \in \exp_{<\aleph_0} X$, $P = \{p_1, \dots, p_k\}$ and $p_i \in V_i$. $V^* = \langle \bigcup_{i=1}^{m} V_i, V_1, \dots, V_m \rangle^*$ it's neiberhood, than $N^* = (\bigcup_{j=1}^{k} N_j)^*$, $N_j \in N$ and $N = \bigcup_{j=1}^{k} N_j$ there $p_i \in N_j \subset V_i$ for some $j = 1, \dots, k$ has property $(P) \in N^* \subset V^*$, concignently $n\omega(\exp x) \leqslant n\omega(x)$.

THEOREM 3. *Density of topological space X, $d(x)$ coincide with density of space* $\exp_{<\aleph_0} X$, $d(\exp_{<\aleph_0} X)$.

Proof. Let J is dens in space X and $|J| = d(x)$. Then all finite sets of space X $J^* = \{(P)\colon\ P \subset J;\ |P| < \aleph_0\}$ is dense in $\exp_{<\aleph_0} X$ and has capasity $d(x)$. Really for each basic set $\widetilde{U} = \langle U, U_1, \ldots, U_k\rangle$ we have that exists $p_i \in J$, that $p_i \in U_i$, $i = 1, 2, \ldots, k$, but than $P = \{p_1, \ldots, p_k\}$ is such that $(P) \in \widetilde{U} \quad ((P) \in J^*)$.

On the other hand let J^* is family of finite subset and J^* is dense in $\exp_{<\aleph_0} X$ and $|J^*| = d(\exp_{<\aleph_0} X)$, than $J = \{p\colon\ p \in P,\ (P) \in J^*\}$ is dence in X. Sufficent discust neiberhoods of $(X) \quad \langle X, V\rangle$. Thus J^* is dence in $\exp_{<\aleph_0} X$, we can finde $(P) \in J^*$ that $(P) \in \langle X, V\rangle$ consiquently exists $p \in P$, that $p \in V$, so $d(\exp_{<\aleph_0} X) \geqslant d(x)$.

THEOREM 4. *Character of topological space $X \quad \chi(X)$ coincide with character of space* $\exp_{<\aleph_0} X \quad \chi(\exp_{<\aleph_0} X)$.

Proof. Thus X is embedable into $\exp_{<\aleph_0} X$ then $\chi(X) \leqslant \chi(\exp_{<\aleph_0} X)$. Let $(P) \in \exp_{<\aleph_0} X \quad P = \{p_1, \ldots, p_k\} \quad p_i \neq p_j;\ i, j = 1, 2, \ldots, k$ and let $U^* = \langle U, U_1, \ldots, U_k\rangle^*$ it's neiberhood such $U_i \cap U_j = \oslash \quad i \neq j$. Let observe collection $\langle \bigcup_{i=1}^{k} V_i, V_1, \ldots, V_k\rangle^*$ where $V_1, \ldots, V_k$ are sets from local base's of points p_i $i = 1, 2, \ldots, k$ in space X such $p_i \in V_i \subset U_i$ for all $i = 1, \ldots, k$.

Clare that capasity of family $\langle \bigcup_{i=1}^{k} V_i, V_1, \ldots, V_k\rangle^*$ do not exside $\max\{\chi(p_i), i = 1, \ldots, k\}$. Conciquently $\chi(\exp_{<\aleph_0} X) \leqslant \chi(X)$ and $\chi(X) = \chi(\exp_{<\aleph_0} X)$.

THEOREM 5. *Let X is topological space than pseudocharacter of space X – $p\chi(X)$ is eguil to pseudocaracter of* $\exp_{<\aleph_0} X \quad p(\chi(\exp_{<\aleph_0} X))$.

Proof. Thus X is embedable into $\exp_{<\aleph_0} X$ then $p\chi(X) \leqslant p(\exp_{<\aleph_0} X)$ Invers inequality can be proof like in theorem 4.

References

1. A.V. Archangelsky, On clasification and structure topological spaces and it cardinal invariants, *Uspechi matematicheskich nauk*, **35**, 29–84 (1978).
2. E. Michael, Topologics on spaces of subsets, *Trans Amer. Math. Soc.*, **71**, 152–182 (1951).
3. R. Engelking, *General topology*, Warszava (1978).

REZIUMĖ

G. Praninskas. Apie kai kuriuos baigtinių poaibių erdvės kardinalinius invariantus

Čia įrodomi topologinės erdvės ir jos baigtinių poaibių erdvės su Vietorio topologija svorio, tinklinio svorio, tankio, charakterio ir pseudocharakterio sutapimas.