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On the identification of Hammerstein–Wiener systems

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1. Introduction

Special classes of nonlinear systems applied in engineering are nonlinear systems with both block-oriented Hammerstein and Wiener structures, respectively [1, 3, 4, 7, 8, 14]. There are a lot of papers devoted to the different aspects of the parametric identification of Hammerstein and Wiener systems and much less on that of the Hammerstein–Wiener (H-W) systems with so-called hard nonlinearities [2, 6, 12, 13]. On the other hand, the abovementioned systems are common in nonlinear control applications where hard nonlinearities such as the saturation, preload, dead-zone, etc., are present [5]. Especially frequently saturation nonlinearities as an input or an output nonlinearity are observed here, too. In such a case, respective observations of a nonlinear system to be identified could be partitioned into distinct data sets according to different descriptions. However the boundaries of sets of observations depend on the value of unknown thresholds – observations are divided into regimes dependent on whether some observed threshold variable is smaller or larger than the threshold. Therefore, the problem of identification of unknown parameters of linear blocks of the H-W systems could be solved, if a simple way of partitioning the available data sets were found in the case of unknown thresholds of both saturations. Afterwards the estimates of parameters of regression functions could be calculated by processing particles of non-clipped observations to be determined. Comparing with [9, 10, 11] we extend here our research on the parametric identification of linear parts of block-oriented H-W systems with saturation nonlinearities by processing input-output observations.

2. Statement of the problem

The H-W system given in Fig. 1 consists of a static nonlinearity $f(\cdot,\eta)$ followed by a linear part $G(q,\mathbf{\Theta})$ and by the other static nonlinearity $f(\cdot,\beta)$. The linear part of the H-W system is dynamic, time invariant, causal, and stable. It can be represented by a time invariant dynamic system (LTI) with the transfer function $G(q,\mathbf{\Theta})$ as a rational function of the form

$$G(q,\mathbf{\Theta}) = \frac{b_0 + b_1 q^{-1} + \ldots + b_m q^{-m}}{1 + a_1 q^{-1} + \ldots + a_m q^{-m}} = \frac{B(q,\mathbf{b})}{1 + A(q,\mathbf{a})} \tag{1}$$

with a finite number of parameters

$$\mathbf{\Theta}^T = (b_0, b_1, \ldots, b_m, a_1, \ldots, a_m),$$

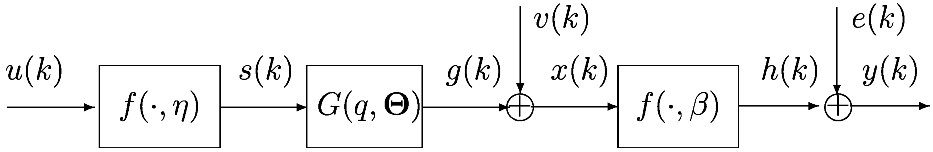

Fig. 1. The block-oriented H-W system with the process noise $v(k)$ and that of the measurement $e(k)$. The linear dynamic part $G(q,\mathbf{\Theta})$ of the H-W system is parameterised by $\mathbf{\Theta}$, while the static nonlinear parts $f(\cdot,\eta)$ – by η and $f(\cdot,\beta)$ – by β. Signals: $u(k)$ is input, $y(k)$ is output, $s(k)$, $g(k)$, $x(k)$, $h(k)$ are unmeasurable intermediate signals.

$$\begin{aligned}\mathbf{b}^T &= (b_0, b_1, \ldots, b_m),\\ \mathbf{a}^T &= (a_1, \ldots, a_m),\end{aligned}\tag{2}$$

that are determined from the set $\mathbf{\Omega}$ of permissible parameter values $\mathbf{\Theta}$. Here q^{-1} is a backward time-shift operator and the set $\mathbf{\Omega}$ is restricted by conditions on the stability of the respective difference equation.

The nonlinear parts $f(\cdot,\eta)$ and $f(\cdot,\beta)$ are saturation nonlinearities of the forms

$$f(z,\eta) = \begin{cases} -\eta & \text{if } z \leqslant -\eta,\\ z & \text{if } -\eta < z \leqslant \eta,\\ \eta & \text{if } z > \eta,\end{cases}\tag{3}$$

and

$$f(z,\beta) = \begin{cases} -\beta & \text{if } z \leqslant -\beta,\\ z & \text{if } -\beta < z \leqslant \beta,\\ \beta & \text{if } z > \beta,\end{cases}\tag{4}$$

respectively, both of which should be partitioned into three functions.

The output signal

$$y(k) = h(k) + e(k) = f(x(k),\beta) + e(k),\tag{5}$$

is generated by the output nonlinearity of the H-W system (4) as a response to the unknown intermediate signal

$$x(k) = g(k) + v(k),\tag{6}$$

with

$$g(k) = G(q,\mathbf{\Theta})s(k) = G(q,\mathbf{\Theta})f(u(k),\eta).\tag{7}$$

and $u(k)$ as a true input of the H-W system. The process noise $v(k)$ and the measurement noise $e(k)$ are added to an intermediate signal $x(k)$ and to the output $y(k)$, respectively. Noises are mutually noncorrelated sequences of independent Gaussian variables with zero means and variances σ_v^2, σ_e^2.

The aim of the given paper is to estimate parameters (2) of the linear part (1) by processing N pairs of data $u(k)$ and $y(k)$ $\forall\, k \in \overline{1,\mathrm{N}}$, of the H-W system (Fig. 1) under the assumption that no less than 50% of their observations, respectively, have passed through both saturation nonlinearities (3), (4) without clipping.

3. The parameter estimation procedure

Assuming that the process noise $v(k) \equiv 0$, one could approximate the H-W system (1)–(7) by the infinite impulse response (IIR) system described by

$$\begin{aligned} y(k) = b_0u(k) + b_1u(k-1) + \ldots \\ + b_mu(k-m) + a_1y(k-1) + \ldots + a_my(k-m) + e(k) \end{aligned} \tag{8}$$

$\forall\, k \in \overline{\nu, \mathrm{N}}$, or the expression in the matrix form

$$\mathbf{Y} = \mathbf{\Lambda\Theta}. \tag{9}$$

Here

$$\mathbf{Y} = \big(y(\nu), y(\nu+1), \ldots, y(N-1), y(N)\big)^T \tag{10}$$

is the $(\mathrm{N}-\nu) \times 1$ vector consisting of $y(k)$ values, $\nu = m+1$, and

$$\mathbf{\Lambda} = \begin{bmatrix} u(\nu) & \ldots & u(1) & -y(\nu-1) & \ldots & -y(1) \\ u(\nu+1) & \ldots & u(2) & -y(\nu) & \ldots & -y(2) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u(\mathrm{N}) & \ldots & u(\mathrm{N}-\nu) & -y(\mathrm{N}-1) & \ldots & -y(\mathrm{N}-1-\nu) \end{bmatrix} \tag{11}$$

is the full rank $(\mathrm{N}-\nu) \times (2m+1)$ regression matrix, consisting of observations of the non-noisy input $u(k)$ and noisy output $y(k)$ $\forall\, k \in \overline{\nu, \mathrm{N}}$.

Let us now rearrange the data in the vector $\mathbf{Y}$ in an ascending order of their values reordering the associated rows of matrix $\mathbf{\Lambda}$, too. One could carry out it by interchanging equations in the initial system (9). Then the vector $\mathbf{Y}$ and matrix $\mathbf{\Lambda}$ should be partitioned into three data sets:

$\mathbf{Y_1} = \mathbf{\Lambda_1\Theta}$ – the left-hand data set,

$\mathbf{Y_2} = \mathbf{\Lambda_2\Theta}$ – the middle data set, and

$\mathbf{Y_3} = \mathbf{\Lambda_3\Theta}$ – the right-hand data set according to three regimes of nonlinearities (3), (4).

Here $\mathbf{Y_1}, \mathbf{Y_2}, \mathbf{Y_3}$ are $(\mathrm{N}_1-\nu) \times 1, \mathrm{N}_2 \times 1$ and $\mathrm{N}_3 \times 1$ vectors, respectively, $\mathbf{\Lambda}_1, \mathbf{\Lambda}_2, \mathbf{\Lambda}_3$ are $(\mathrm{N}_1-\nu) \times (2m+1), \mathrm{N}_2 \times (2m+1)$ and $\mathrm{N}_3 \times (2m+1)$ matrices, respectively, $\mathrm{N} = \mathrm{N}_1 + \mathrm{N}_2 + \mathrm{N}_3 - \nu$.

Thus the initial system (9) is reordered into the system

$$\tilde{\mathbf{Y}} = \tilde{\mathbf{\Lambda}}\mathbf{\Theta}; \tag{12}$$

with

$$\tilde{\mathbf{Y}} = \begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \\ \mathbf{Y}_3 \end{bmatrix}, \quad \tilde{\mathbf{\Lambda}} = \begin{bmatrix} \mathbf{\Lambda_1} \\ \mathbf{\Lambda_2} \\ \mathbf{\Lambda_3} \end{bmatrix} \tag{13}$$

by interchanging the equations in (9).

The left-hand side data set $\mathbf{Y}_1$ ($\mathrm{N}_1 - \nu$ samples) consists of the values equal to negative β, the middle data set $\mathbf{Y}_2$ (N_2 samples) of the values higher than negative β, but lower than β, and the right-hand side data set $\mathbf{Y}_3$ (N_3 samples) of the values equal to β, if the measurement noise $e(k)$ is also absent. The observations of the middle data set $\tilde{y}(k)\ \forall\, k \in \overline{\mathrm{N}_1 + 1, \mathrm{N}_2}$ are equivalent to those output observations $y(k)$ that passed nonlinearity (4) without clipping.

In spite of the unknown threshold β, one could get most equations corresponding to the middle data set $\mathbf{Y_2}$ simply by choosing the upper interval bound slightly lower than the 75 % and the lower interval bound slightly higher than the 25 % of the reordered equations in (12). It could be mentioned that the equations corresponding to the middle data set still contain the input observations $u(k)\ \forall\, k \in \overline{\nu, \mathrm{N}}$ that have been clipped by the nonlinearity (3), when their values are less or equal to $-\eta$ or more than η in (3). These equations of system (12) should be rejected, too. It could be done, first, using the same rearrangement of the input data $u(k)\ \forall\, k \in \overline{\nu, \mathrm{N}}$, second, marking the observations that are present in both side-sets of the rearranged input data, third, reordering the rows in $\mathbf{Y_2}$ and $\mathbf{\Lambda_2}$ in the right order again, and, fourth, deleting equations containing the input data to be marked. Afterwards, by compressing the vector $\hat{\mathbf{Y}}_\mathbf{2}$ and matrix $\hat{\mathbf{\Lambda}}_\mathbf{2}$, respectively, one will have $\hat{\mathbf{Y}}_\mathbf{2}$ and $\hat{\mathbf{\Lambda}}_\mathbf{2}$ with some portions of missing data within them, belonging to the unknown left-hand and right-hand side sets of rearranged $\tilde{u}(k)$ and $\tilde{y}(k)\ \forall\, k \in \overline{\nu, \mathrm{N}}$.

The estimates of parameters (2) of the transfer function $G(q, \mathbf{\Theta})$ are calculated according to

$$\hat{\mathbf{\Theta}} = \left(\hat{\mathbf{\Lambda}}_\mathbf{2}^T \hat{\mathbf{\Lambda}}_2\right)^{-1} \hat{\mathbf{\Lambda}}_2^T \hat{\mathbf{Y}}_2. \tag{14}$$

Here

$$\hat{\mathbf{\Theta}}^T = \left(\hat{\mathbf{b}}, \hat{\mathbf{a}}\right)^T, \quad \hat{\mathbf{b}}^T = \left(\hat{b}_0, \hat{b}_1, \ldots, \hat{b}_m\right), \quad \hat{\mathbf{a}}^T = \left(\hat{a}_1, \ldots, \hat{a}_m\right) \tag{15}$$

are $(2m+1) \times 1, (m+1) \times 1, m \times 1$ vectors of the estimates of parameters (2), respectively.

Really, the measurement noise $e(k)$ is present, therefore it could be difficult to decide whether some observed threshold value that is close to the boundaries of respective data-sets, belongs to the middle data-set. In order to determine how the same process noise realization and different realizations of measurement noise affect the accuracy of estimation of unknown parameters, we have used the Monte Carlo simulation with 10 data samples, each containing 100, 500, 1000 input-output observation pairs, respectively. 10 experiments with the same realization of the process noise $v(k)$ and different realizations of the measurement noise $e(k)$ of different levels of its intensity were carried out. The intensity of noises was assured by choosing respective signal-to-noise ratios (SNR) (the square root of the ratio of signal and noise variances). For the process noise, the $\mathrm{SNR^v}$ was equal to 100 and for the measurement noise $\mathrm{SNR^e}$: 1, 10, 100. As inputs for all given nonlinearities, the periodical signal and white Gaussian noise with variance 1 were chosen. In each ith experiment the estimates of parameters

Table 1. Averaged estimates of the parameters b_1, a_1 with their confidence intervals (the first line for each estimate corresponds to the periodical signal, while the second line to the Gaussian white noise as inputs)

Estimates	$SNR^e = 1$	$SNR^e = 10$	$SNR^e = 100$	N	N_2
$\hat{b}_1$	-1.22 ± 0.46 -1.38 ± 0.57	-1.14 ± 0.39 -0.98 ± 0.31	-0.97 ± 0.33 -0.98 ± 0.35	100 100	49 51
$\hat{a}_1$	0.01 ± 0.45 0.03 ± 0.39	0.02 ± 0.27 0.05 ± 0.18	0.09 ± 0.25 0.07 ± 0.23	100 100	49 51
$\hat{b}_1$	0.14 ± 0.58 0.83 ± 0.43	0.34 ± 0.37 0.87 ± 0.36	0.81 ± 0.19 0.92 ± 0.24	500 500	247 251
$\hat{a}_1$	0.47 ± 0.45 0.26 ± 0.38	0.33 ± 0.21 0.28 ± 0.07	0.15 ± 0.07 0.09 ± 0.02	500 500	247 251
$\hat{b}_1$	0.38 ± 0.15 0.37 ± 0.17	0.34 ± 0.08 0.36 ± 0.09	0.3 ± 0.04 0.3 ± 0.03	1000 1000	500 498
$\hat{a}_1$	-0.06 ± 0.19 -0.07 ± 0.18	-0.35 ± 0.14 -0.48 ± 0.16	-0.49 ± 0.06 -0.47 ± 0.08	1000 1000	500 498

were calculated. During the Monte Carlo simulation, averaged values of the estimates of parameters and their confidence intervals were calculated. In Table 1 for each input the averaged estimates of parameters of the simulated H-W system (Fig. 1) with the linear part (1) ($b_1 = 0.3$; $a_1 = -0.5$) and saturation nonlinearities (3), (4) with $\eta = 1.0$, $\beta = 1.5$ and with their confidence intervals (the significance level $\alpha = 0.05$) are presented. It should be noted that, in each experiment here, the value of SNR^v was fixed and the same, while the values of SNR^e were varying due to different realizations of $e(k)$. The Monte Carlo simulation (Table 1) implies that the accuracy of parametric identification of the H-W system depends on the intensity of process and measurement noises as well as on the type of the input signal and the number of non-clipped input-output observation pairs to be processed according to (14).

The problem of identification of block-oriented H-W systems could be essentially reduced by a simple input data rearrangement in an ascending order of their values. Thus, the available data are partitioned into three data sets. Later on the estimates of unknown parameters of linear regression model could be calculated by processing the respective middle data sets of the rearranged input and associated output with missing observations that are clipped by the saturation input-output nonlinearities.

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REZIUMĖ

R. Pupeikis. Apie Hameršteino–Vinerio sistemų identifikavimą

Straipsnyje nagrinėjamas Hameršteino–Vinerio sistemų tiesinės dalies, aprašomos skirtumine lygtimi su nežinomais koeficientais ir soties tipo netiesiškumais junginys. Parodyta, kad pertvarkius įėjimo-išėjimo signalų stebėjimus pagal didėjančias jų reikšmes, galima išskirti vidurinę stebėjimų dalį. Nežinomų tiesinės Hameršteino–Vinerio sistemos dalies koeficientų įverčiai gaunami mažiausiųjų kvadratų metodo algoritmu, apdorojant stebimų, bet pertvarkytų įėjimo-išėjimo signalų duomenis. Pateikti modeliavimo rezultatai.