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The numerical simulation of military skills

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1. Introduction and main system of equations

We examine the problems of holding under control the trajectory of mine. We suppose that initial speed of a mine is known and the meteorological conditions are taken into consideration. The movement of mine may be described by the following system of the equations [1, 2, 3]:

$$\begin{cases} m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -k\frac{\mathrm{d}x}{\mathrm{d}t} + F_1\cos(\phi), \\ m\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = -k\frac{\mathrm{d}y}{\mathrm{d}t} + F_2\sin(\phi), \\ m\frac{\mathrm{d}^2z}{\mathrm{d}t^2} = -k\frac{\mathrm{d}z}{\mathrm{d}t} - mg, \end{cases} \tag{1}$$

with the following initial conditions:

$$x(0) = 0, \quad y(0) = 0, \quad z(0) = 0, \tag{2}$$

$$m\frac{\mathrm{d}x}{\mathrm{d}t} = v_0\cos(\alpha_1)\cos(\alpha_2), \quad m\frac{\mathrm{d}y}{\mathrm{d}t} = v_0\cos(\alpha_1)\sin(\alpha_2), \quad m\frac{\mathrm{d}z}{\mathrm{d}t} = v_0\sin(\alpha_1),$$

where t denotes time of flying of mine, k is the coefficient of aerodynamic resistance, m is the mass of a missile, g – the acceleration due to the force of gravity, $x(t)$ and $y(t)$ (x_t, y_t) denote the horizontal co-ordinates of the moving body at time moment t (x-axis is turn to the target), $z(t)$ (z_t) – vertical co-ordinate, F_1 – the front force of wind, F_2 – the side force of wind, φ is the angle between the directions of a wind and x-axis, v_0 – initial velocity. We suppose that the direction of the initial velocity vector coincides with the direction of the trench-mortar tube. At the aiming, the position of the trench-mortar tube is determined by two angles: α_1 – angle between the initial velocity vector and xy-plane and α_2 – angle of correction, that is the angle of compensation of influence of side winter.

It is rather difficult to choose the aerodynamic resistance coefficient k, which may be established by experiment [4]. It depends on the speed of mine and of meteorological conditions: pressure of atmosphere, air temperature and air humidity in every point of trajectory. These parameters change during the movement because the mine achieves greatest attitude. We propose to choose it by co-ordinating the calculus and the results of

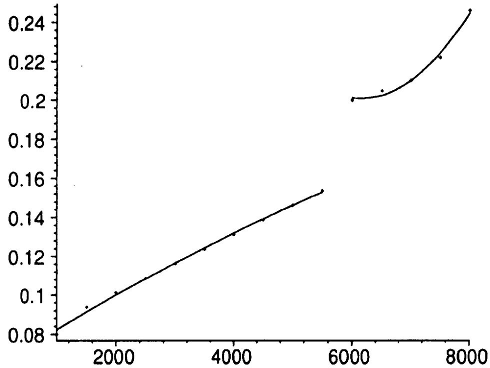

Fig. 1. Dependence of the coefficient of resistance k from the distance to the target.

the experience, and it is generalized in the following table [4]. We choose the coefficient of air resistance k correspondent to different distances from the target in accordance with the data of table [4] and calculus and we construct the approximation polynomial. Comparing data we state the dependence of resistance coefficient upon the distance to the target (see Fig. 1). If k is known, it is possible to calculate the trajectory of mine's flying. Inserting in the numerical system (1) values of coefficients and parameters of initials conditions in the expression of solutions, we transform this solutions in the functions of angles α_1, α_2, and of time t.

The system of non-linear equations can be solved using the numerical methods:

$$\begin{cases} x(\alpha_1,\ \alpha_2, t) = L, \\ y(\alpha_1,\ \alpha_2,\ t) = 0, \\ z(\alpha_1,\ \alpha_2,\ t) = \Delta h, \end{cases} \tag{3}$$

where Δh – the difference between levels (if the trench mortar is below the target, we insert the negative number).

Having solved the system (3), using the method of simple iteration, can be found the angle of aiming α_1, the angle of correction α_2 and the time of flying t_n ($40^0 \leqslant \alpha_1 \leqslant 85^0$, $-8^0 \leqslant \alpha_2 \leqslant 8^0$, $10 \leqslant t_n \leqslant 80\ s$).

The example of such calculus is presented in the Fig. 2.

In this case distance to the target is $L = 7808\ m$, the angle of throwing $50^0\ 14'$, the initial velocity $v_0 = 352\ m$, the altitude of trajectory (the maximum altitude of the

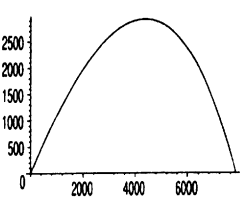

Fig. 2. The trajectories of mine flying.

trajectory) $h_{max} = 2934\ m$, the length of flying $t_n = 49\ s$. In addition we can use the analytical solution and find duration of all necessary parameters for practical application: the instantaneous speed, the altitude of trajectory, the angle of a fall and etc. It is possible to examine the different interesting cases. For example, in the case when the target is moving, by inserting in the right side of the system (3) the corresponding parameters, we get the following solutions.

2. Trajectory parameters selection

The distance of the target and the direction to the target (the angle of the north of clockwise) we can calculate if we have the information of two scout posts (see Fig. 3).

In this figure α_{a1}, α_{a2} – the angles of azimuth, α_{s1}, α_{s2} – the angles of site (reading of plane xy), r_1, r_2 – distances to the target, x, y, z – coordinates of target, x_i, y_i, z_i – coordinates of the scout posts.

We solve the task of optimization. We calculate the probabilities coordinates of the target applying the method of minimization of error polynomial. The absolute errors of the angles of azimuth, the angles of site and distances to the target may be written in formula:

$$\begin{cases} \varepsilon_a = \sum\limits_{i=1}^{i=2} \left[\arctan \left(\dfrac{x - x_i}{y - y_i} - \alpha_{ai} \right) \right], \\ \varepsilon_s = \sum\limits_{i=1}^{i=2} \left[\arcsin \left(\dfrac{z - z_i}{sqrt(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} - \alpha_{si} \right) \right], \\ \varepsilon_r = \sum\limits_{i=1}^{i=2} \left[\sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} - r_i \right]. \end{cases} \tag{4}$$

Now we write polynomial of the relatives errors of early refer quantity:

$$S = \left(\frac{\varepsilon_a}{\Delta\varepsilon_a} \right)^2 + \left(\frac{\varepsilon_s}{\Delta\varepsilon_s} \right)^2 + \left(\frac{\varepsilon_r}{\Delta\varepsilon_r} \right)^2. \tag{5}$$

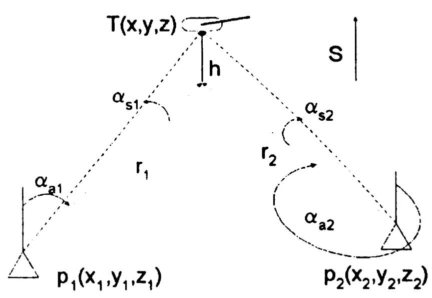

Fig. 3. The scheme of measuring distances to the target, the angles of azimuth and the angles of site.

Table 1

α_{ai}	α_{si}	r_i	$\Delta\varepsilon_a$	$\Delta\varepsilon_s$	Δ_r
51	4.5	6400	0.5	0.5	30
34	8	3600	0.5	0.5	20

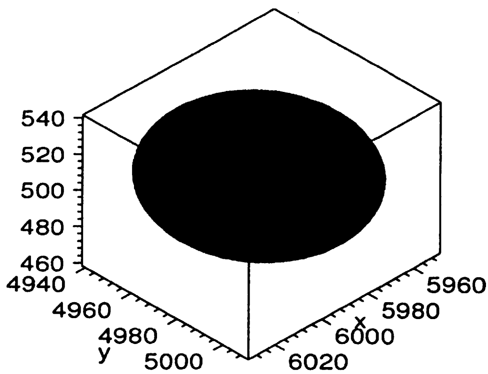

Fig. 4. The ellipsoid of errors.

$\Delta\varepsilon_a$, $\Delta\varepsilon_s$, $\Delta\varepsilon_r$ are absolute errors of corresponding measure instruments. The coordinates of target x, y, z we find by solving the system of non-linear algebraic equations (7):

$$\begin{cases} \dfrac{\partial S}{\partial x} = 0, \\ \dfrac{\partial S}{\partial y} = 0, \\ \dfrac{\partial S}{\partial z} = 0. \end{cases} \tag{6}$$

The find extreme is minimum, because Hessian of system (7) is positive by determination. For example, solving the task with parameters indicated in the Table 1, we obtain such values:

$$L = 7800\,m, \quad \Delta h = 500\,m, \quad \alpha_a = 50^0\,15'.$$

Use the chi-square distribution (with 6 degrees of freedom and 95% confidence interval), we can draw the ellipsoid of errors (see Fig. 4).

3. Choice of optimal numbers of mines

Actually it is impossible to avoid the abundance random errors. For example, gusts of wind, variations in air humidity, temperature and density are possible. Some errors may

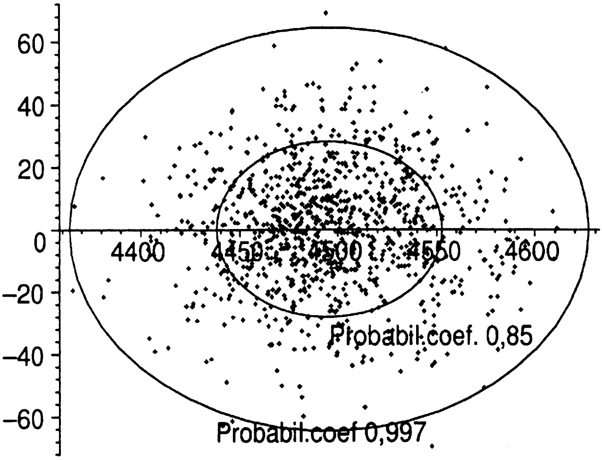

Fig. 5. The ellipse of dispersion of 1000 points hitting.

occur due to the perfection of technology of mines and arms production, for example, the variation of initial speed.

We can get random errors of interesting variables by using the generating program. For example, we can indicate the initial speed of mine v_0, the coefficient of resistance k and the influence of wind in the following manner

$$\begin{aligned} &v_0 + random[normal]\Delta v_0, \qquad &, k + random[normal]\Delta k, \\ &F_1 + random[normal]\Delta F_1, \qquad &F_2 + random[normal]\Delta F_2, \end{aligned} \tag{7}$$

here $random$ $[normal]$ – MAPLE command generating standard normal distribution $N(0;1)$ and Δv_0, Δk, ΔF_1, ΔF_2 – the maximum random errors of indicated variables discussed in specific situation.

By generating the random initial conditions (4) and solving system (3) we can find the angles α_1 and α_2. For these angles we find scatter points of the fall of mines caused by random errors. In this way we get the dispersion Ellipse of points hitting the target, corresponding to the chosen dispersion of our parameters. We present the example of such calculus in Fig. 4.

The co-ordinates of the hitting points are calculated with 0.5% maximum error of initial velocity and 1% maximum error of air resistance coefficient. The distance to the target $L = 4500\ m$. The ellipse of dispersion of the hitting points with the confidence level equals to 0.85 and 0.997. The standard quadratic variance $\sigma_x = 39.8$, $\sigma_y = 19.6$ and means $m_x = 4495\ m$, $m_y = 0.11\ m$. If the distance to the target increases, the initial speed and the length of trajectory increase too. By repeatedly generating the random variables it is possible to repeat the numerical experience and to examine the influence of same parameters by changing their values.

The numerical experience can help to estimate the possibilities of certain fire-arm and to estimate properly the meteorological factors to the efficiency of shooting. By examining the influence of initial conditions to the final results, so we analyze the change of shooting result depending on initial conditions.

We shall discus random errors affecting the moving of mines and estimate its maximum values. First of all the "wearing of gun tube" is of grand importance to the results of shooting. It determines the random error of initial speed Δv_0. Which is measured before shooting. The increment of mine mass from standard (it is indicated in the passport) and the temperature of charge also influence or differ from the initial speed. Air resistance, atmosphere pressure and air humidity at different attitudes, determine the random error of coefficient of resistance k. The rush (of a wind) determines the random errors of values F_1, F_2.

Let us analyze two problems. The first – how many mines must be shot to the armed target of size $8 \times 4\ m^2$ (for example the tank) situated at a definite distance for hitting with the confidence probability 0.9.

The second – how many volleys must be shot to a group target $200 \times 300\ m^2$ situated at a definite distance for hitting more 95% of the target by the shell and splitters.

In the first case we shall use the two-dimensional normal distribution. We know the standard quadratic variance σ_x, σ_y and means m_x, m_y. Then the probability of hitting the target equals:

$$P\{x \subset (-4; 4) \text{ and } y \subset (-2; 2)\} = \frac{1}{\pi\sigma_x\sigma_y} \int_{-2}^{2} \int_{-4}^{4} e^{-\frac{(x-(L_x-m_x))^2}{2\sigma_x^2}} e^{-\frac{(y-m_y)^2}{2\sigma_y^2}} \, dx dy. \quad (8)$$

If $L = 1000\ m$, $\sigma_x = 9.9$, $\sigma_y = 4.5$ and means $m_x = 998.8\ m$, $m_y = 0.05\ m$, then probability hitting the target is $P = 0.1$. Because this probability is stable for determined distance, we may apply the binomial distribution and find necessary number of shooting for hitting the target. For example

$$\text{if } N = 10 \text{ then } P_{total} = 0.68, \quad \text{if } N = 15 \text{ then } P_{total} = 0.9.$$

We shall use the method Monte-Carlo to solve the second problem and we shall estimate the probability of hit group target. So we must find the necessary number of mines that the target would be hit with the chosen confidence (certainty). Let us assume that the battery of trench-mortar receives an order-mission to annihilate the group target of 300 m in width and 200 m in depth. If one shoots utilizing the sight three and one angular sight, possible schemes of targets are represented in Fig. 6.

We form the random variables with the mean values coinciding with the co-ordinates of points indicated in Fig. 6 (18 points) and mean square variances σ_{xi} (for x-axis) and σ_{yi} (for y-axis) ($1 \leqslant i \leqslant 6$, $1 \leqslant j \leqslant 3$). They may be different for distinct mortar.

$$x_i = x_i + \sigma_{xi} \times random[normald], \quad y_i = y_i + \sigma_{yi} \times random[normald]. \quad (9)$$

Having generated 18 random hits corresponding to the scheme represented by Fig. 6 we can calculate the results of one realization. For that purpose the target is divided in

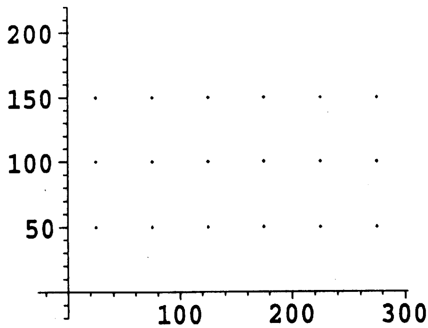

Fig. 6. Possible schemes of targets.

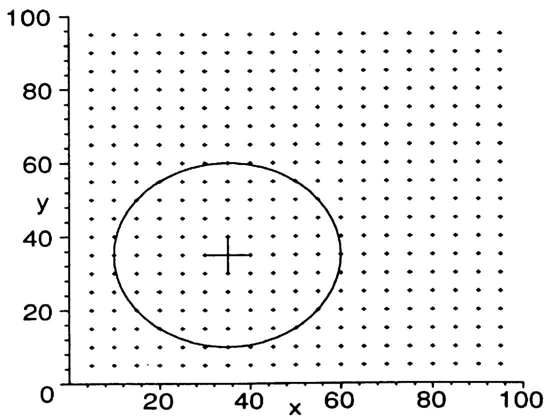

Fig. 7. The fragment of group target.

the zones according to the type of target. If soldiers under fire are without cover they may be hurt by shell-splinters if they are not further than a 25 m from explosion place. In the Fig. 7 the cross note the point hit. Points in the hit zone are hurt.

The quotient of squares fallen in the zone of hit n_i with all squares n corresponding to the hurt part of group target (the ratio between hurt area S_i and whole square S):

$$U_i = \frac{n_i\, s}{n\, s} = \frac{S_i}{S}. \tag{10}$$

We repeat the numerical experience N times and every time we evaluate the hurt part of target (10). We continue the experience until the mean value of target hurt part U would be evaluated with precision desirable (its value no change more within the limits of given precision when N increase):

$$MU = \frac{1}{N}\sum_{i=1}^{N} U_i. \tag{11}$$

Further we determine the variance of this value and find the probability to injure some part U_0 of group target

$$P(U > U_0),$$

and the mean value of mines N_n which missed the target.

After each volleys of eighteen hits one determines the degree of target destruction with given accuracy and calculate the number of mines that have missed the target. If the calculations are repeated a given number of times, one finds mean value and evaluates the percentage $N\%$ injured part of target and average number of missing of the target using the formula (11).

4. Final remark

We present the method for numerical simulation of trench-mortar tasks. These programs, helps to calculate parameters of firing for destroing the target. One can calculate this parameters in 20–30 seconds and realize task in the real time. One can control the arm with computer. Data that can be calculated: the standard quadratic variance, number of mine for realizing successful firing, destroing separate or group targets with selected confidence probability and etc. These programs can be used for officers training.

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Matematinis modeliavimas karyboje

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Nagrinėjama netiesinių lygčių sistema, aprašanti minosvaidžio iššautos minos judėjimą. Pasinaudojama eksperimento duomenimis ir surandama oro pasipriešinimo koeficiento priklausomybė nuo taikinio nuotolio. Tai leidžia „ištiesinti“ uždavinį atskiruose intervaluose. Tada galima per 20–30 sekundžių suskaičiuoti šaudymo parametrus ir vykdyti užduotį realiame laike. Jei ginklo valdymo mechanika automatizuota, jo valdymą galima atlikti kompiuteriu. Galima paruošti duomenis būsimam apšaudymui, t.y., susiskaičiuoti vidutinius kvadratinius nuokrypius, surasti šūvių ar salvių skaičių, reikalinga sunaikinti atskirą ar grupinį taikinį su norima pasikliovimo tikimybe. Šios programos gali būti naudojamos kaip „treniruoklis“ apmokant būsimuosius karininkus.