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# The mathematical modeling of counter-attack

Albertas PINCEVIČIUS, Rimantas-Jonas RAKAUSKAS,
Gintautas MISEVIČIUS, Svajonė VAŠTERIENĖ (LKA)
*e-mail: pincev@takas.lt, rimantas.rakauskas@tmk.lka.lt*

## 1. Formulation of the problem

In this paper authors proceed the examination of problems of modeling military operations which take origin in [1, 2]. We examine the situation when well prepared advanced detachment unexpectedly attacks the more abundant column of enemy from an ambush. In difference to proceeding model [1] one supposes the utilization of more powerful arms during the battle: machine-guns, grenade-guns and anti-tank guns (see Fig. 1).

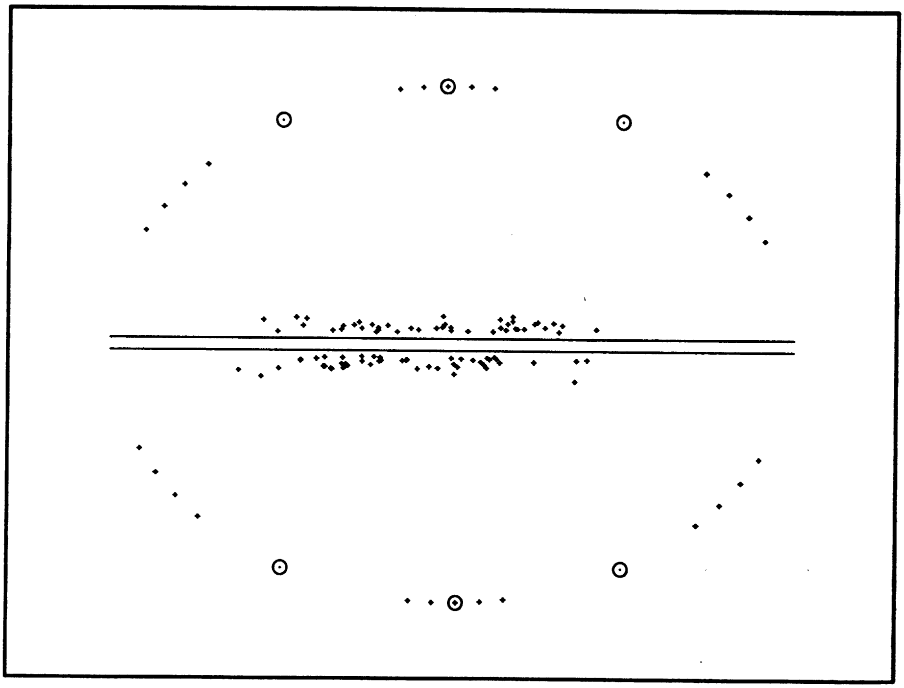

Fig. 1. One detachment (about 30 soldiers) unexpectedly attacks and forces debut from military machines one company (about 100 soldiers) of enemies and fires on machine-guns – ⊙ and grenade-guns – ⊕.

This battle may be described by the system of equations:

$$\begin{cases} \dfrac{dm_x}{dt} = -r_y p_x m_y, \\ \dfrac{dm_y}{dt} = -r_x p_y m_x + r_k p_{yk} m_k + r_g f(m_y) m_g. \end{cases} \tag{1}$$

Here $m_x, m_y$ – the mean values of numbers of fighters, $r_x, r_y$ – the fire-powers of fighters (that is the mean number of shots by one fighter during the unit of time – minute, hour, day, etc.), $p_x, p_y$ – mean probabilities of hitting the target, $r_k$ – the fire-power of machine-guns (the mean value of hits number in the time unit), $m_k$ – the number of machine-guns (in our case—four), $r_g$ – the mean number of hits of grenade-guns, $m_g$ – the number grenade-gun, $f(m_y)$ – the mean value of probabilities of hitting the target by the grenade-guns. We shall evaluate this probability separately because the explosion of grenade may put out of action same various. In the work [2] we get the differential equations under condition that one shoot hits only one soldier.

Let us discuss the method of choosing the coefficients system of equations. The fire-powers $r_x, r_y, r_k, r_g$ are determined by the weapons and both the cartridges and the grenades. In the battle intensive one soldier uses up about 12 cartridges a minute in the average, machine-gun about 100–200 in minute, grenade-gun antinfantryman use up 4–6 grenades in minute and anti-tank gun – 1–2 grenades a minute. Having in mind that the duration of the intensive battle is less than half an hour we shall reckon the time in minutes.

### 1.1. *Evaluation of probabilities of hitting the target in cases of shooting with automatic rifle and machine-gun*

The probabilities of hitting the target get in the cases of shooting with automatic rifles and machine-guns are different when enemy defends oneself in the unprepared positions and one fires by machine-gun not separated targets but enemy dislocation. We suppose that fire-power of submachine gunners decreases because the part of soldiers becomes wounded mean while the wounded machine-gunner may be replaced by other and fire-power does not decrease during the battle. We propose following formula of hitting the target:

$$p_{yk} = \frac{s_y}{S_y} m_y,$$

where $S_y$ is the area of the defense territory, $\frac{S_y}{m_y}$ – the real mean value of area for one soldier of the unit, $s_y$ – the area taken up by one soldier.

We shall evaluate these quantities. The defending oneself unity due to debut the battle machines and distribution in the both sides of the rout in the area

$$S_y = 600 \times 25 \times 2 \approx 30000\text{m}^2.$$

Since they defend unprepared position the "the area of shadow" of laying soldier

$$s_y \approx 0.7\text{m}^2 \text{ and } \frac{s_y}{S_y} \approx 0.000023.$$

The probability $p_y$ of hitting the target by the automatic rifle is expedient to calculate differently since one shoots after partial taking aim, $s_y$ mean value of area of one soldier, that is of that of target $s_y \approx 0.2 \div 1$ m$^2$ and the place of possible appearance of target

$$S_y \approx 1 \times 10\text{m}^2 \text{ and } \frac{s_y}{S_y} \approx 0.06.$$

If the attacking soldiers are located in prepared and unknown for enemy position and to arrange the misleading targets, in this case the soldiers of the defending side shoot in all directions. The probability of hitting the target $p_x$, that is the probability of hitting attacking soldiers dislocated arrows is the following: $s_x$ – the mean area of one soldier, that is the area of "target" $s_x \approx 0.2\,\text{m}^2$, the area of dislocations of unit (since the positions of attacking soldiers in the best case will be known with the exactness 20–30 m and one shoots all this space)

$$S_x = 2\pi r l = 2 \times 3.14 \times 650 \times 30 \approx 120000\text{m}^2 \text{ and } \frac{s_x}{S_x} \approx 0.0000016.$$

1.2. *The estimation of probability of hitting the target by the grenade-gun or by other heavy weapon*

The probability of hitting the target will be estimated using the law of large numbers, that is the such property of random variables where for sufficiently big number of experiments the mean value of the observated random variable with the significant probability of confidence coincide with arithmetical mean of observed values. Having distributed arbitrarily the defending soldiers in both sides of the rout in the area of $S_y = 600 \times 25 \times 2$ m$^2$ and having generated random hitting in this area and putting $r$ the distance in which the grenade-splinters wound the soldiers we can find the average number of soldiers put out of action. Repeating the numerical experiment sufficiently large number of times we get the average number of soldiers wounded by one grenade. The probability of hitting depends on the number of combatants and on the parameters of grenade. Having calculated the probabilities of hitting different numbers of combatants and approximated the obtained statistical relation by the least square method we get the function $f(m_y)$ for corresponding sort of weapons.

In the case of radius of demolition being $r = 5$ m, by approximation of found points by least square method we get such function:

$$f(m_y) = 0.000964 m_y - 0.00692. \tag{2}$$

See this cases in Fig. 2.

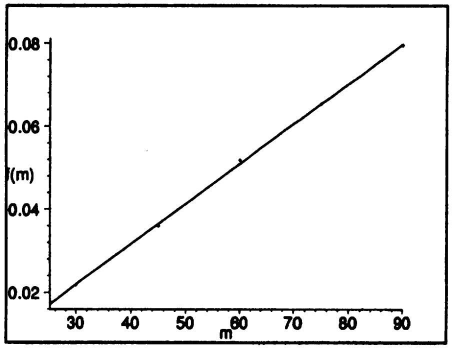


Fig. 2. The dependence of probabilities of hitting the target $f(m_y)$ on number of soldiers being in the zone under the fire when the grenade wound inside of circle of radius 5 m. The coordinates of points are get in the numerical experiment and we get line of approximation by the method of least squares.

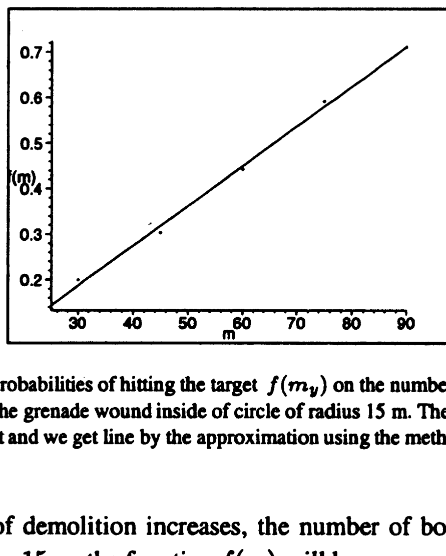


Fig. 3. The dependence of probabilities of hitting the target $f(m_y)$ on the number of soldiers being in the zone under the area firing when the grenade wound inside of circle of radius 15 m. The coordinates of points are get in the numerical experiment and we get line by the approximation using the method of least squares.

When the radius of demolition increases, the number of bounded soldiers also increases. In the case $r = 15$ m, the function $f(m)$ will be:

$$f(m_y) = 0.008734m_y - 0.00745. \tag{3}$$

Thus we may estimate the influence of heavy weapons grenade-guns, trench-mortar in course of a battle.

We substitute the corresponding values of coefficient found in Paragraph 1.1 and that of function $f(m_y)$ (3) in the system of equations (1), and we get the following form:

$$\begin{cases} \dfrac{\mathrm{d}m_x}{\mathrm{d}t} = -0.0000196m_x m_y, \\ \dfrac{\mathrm{d}m_y}{\mathrm{d}t} = -0.72m_x - 0.03787m_y + 0.2262. \end{cases} \tag{1a}$$

This system is written in the case when two anti tank grenade-guns shoot in average 3 shots in minute and four machine-guns – 500 shots a minute. One solves this system with initial conditions:

$$m_{x0} = 24. \quad m_{y0} = 90.$$

## 2. The results of calculus and its consideration

One solve the system of equations numerically by using the package of computer algebra MAPLE [3]. In Fig. 4 and Fig. 5 the phase diagrams of systems solutions are indicated in the cases when one shoot by anti tank trench-mortar (system of equations (1a)) and the phase diagrams of analogous system when one shoots by anti foot-soldiers grenades.

The dependence of mean number of soldiers on time is represented in Table 1.

We see that utilization of heavy weapons causes attacking side to suffer very small losses. The utilization of heavy grenade-guns in the case of counter-attack is more effective but heavy weapons cause the transport problems.

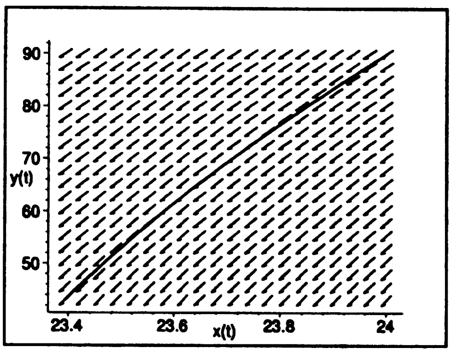


Fig. 4. The phase diagram of solution of system (1a). One shoots by grenade-gun $r = 5\ m$.

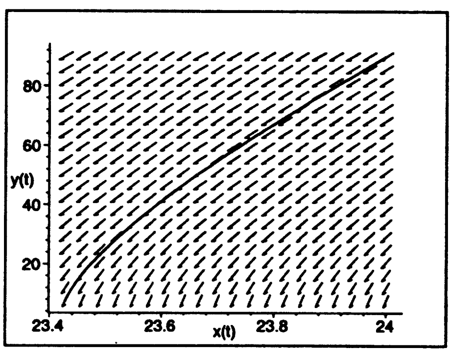


Fig. 5. The phase diagram of solution of system (1a). One shoots by anti tank grenade-gun $r = 15$ m.

Table 1

| Time [min] | $r = 15$ m | | $r = 5$ m | |
|---|---|---|---|---|
| | $m_y$ | $m_x$ | $m_y$ | $m_x$ |
| 0 | 90 | 24 | 90 | 24 |
| 0.5 | 87 | 23 | 88 | 23 |
| 1 | 85 | 23 | 87 | 23 |
| 1.5 | 83 | 23 | 85 | 23 |
| 3 | 76 | 23 | 82 | 23 |
| 5 | 68 | 23 | 76 | 23 |
| 7 | 61 | 23 | 71 | 23 |
| 10 | 51 | 23 | 64 | 23 |
| 12 | 45 | 23 | 60 | 23 |
| 15 | 37 | 23 | 53 | 23 |
| 20 | 25 | 23 | 43 | 23 |
| 25 | 15 | 23 | 33 | 23 |
| 30 | 7 | 23 | 24 | 23 |

Application of created programs gives the possibility to analyze the variants of dislocation for both the attacking and the defending sides (distances between the soldiers and their dislocation) and to find optimal conditions in the cases of attack and defense.

Analogously one can analyze the influence to the course of battle of different sorts of arms. It is possible using the MAPLE and discontinuing by intervals function (see [3]) to analyze without difficulties the moment of debut by battle machines when the possibility of hitting the target is significantly large and many other process too.

## References

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## Kontratakų matematinis modeliavimas

A. Pincevičius, R.-J. Rakauskas, G. Misevičius, S. Vašterienė

Darbe modeliuojama kontrataka panaudojant galingesnius ginklus: kulkosvaidžius, granatsvaidžius. Atsisakyta ankstesniame darbe [1] daromos prielaidos, kad vienu šūviu gali būti sužeidžiamas tik vienas karys. Pasiūlytas modelis padės būsimiesiems karininkams išmokti išsirinkti palankiausią tiek puolimo, tiek gynybos variantą.