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# Investigation of the stability of a metal cutting process

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## Introduction

In the practice of metal treatment by cutting it is frequently necessary to deal with oscillations of the cutting tool, a treated detail and units of the machine tool. These oscillations in many cases are an obstacle on the way of increasing the productivity and quality of detail treatment on metal-cutting machines. It is most difficult to eliminate and, at the same time, investigate self-excited oscillations. The frequency of self-excited oscillations can reach up to 5000 Hz and higher. The stability of the process of chip formation is one of the main conditions, to be satisfied by a metal-cutting machine.

## Dynamics of cutting process

For the development of a theory of self excited oscillations at cutting, it is necessary to use the conformity with the law of the accompanying deformations of a treated metal [2, 3].

The specialty of the process of cutting is related to plastic properties of metal. Therefore delay of cutting force acts on the lathe tool, in relation to coordinates of lathe tool. The self-excited oscillations at metal cutting are a result of delay of forces, which shake the system.

The reason of the delay of forces at cutting a metal is the features of deformation process [3]. The edge a of lathe tool $A$ (Fig. 1) does not permanently participate in deformation of the main chip, but only incises the layer of material and thus causes the deformation process. At small oscillations of the system in the direction of $x$, the oscillation of thickness of the chip and of the force $P$ is periodically detained. The cutting force is variable [7].

The equation of the delay of force $P$ from the coordinate is as follows:

$$\Delta P(t) = B\Delta x(t - \tau_p). \tag{1}$$

The equation of the delay of friction force is

$$\Delta Q(t) = f\Delta P(t - \tau_Q). \tag{2}$$

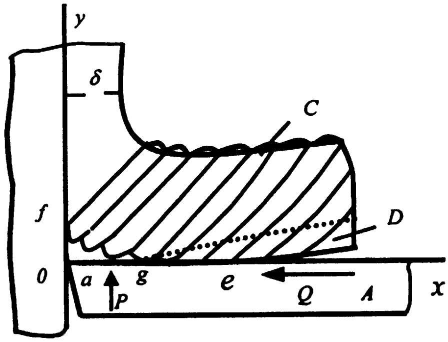


Fig. 1. The process of the chip formation.

Equations of the small oscillation of a dynamic system are:

$$\ddot{x}(t) + \frac{b_x}{m_x}\dot{x}(t) + \omega_x^2 x(t) = -\frac{fB}{m_x}x(t - \tau_p - \tau_Q), \tag{3}$$

$$\ddot{y}(t) + \frac{b_y}{m_y}\dot{y}(t) + \omega_y^2 y(t) = -\frac{B}{m_y}x(t - \tau_p), \tag{4}$$

where $\omega_x^2 = \frac{c_x}{m_x}$; $m_x$ and $m_y$ are masses, $c_x$ and $c_y$ are the coefficients of elasticity.

In the system of equations (3)–(4), the time of delay depends on the functions $\dot{x}$ and $\dot{y}$

$$\tau_p = \frac{l_p}{v_s + \dot{y}}, \quad \tau_Q = \frac{l_Q}{v_s + \dot{y} + \xi\dot{x}}. \tag{5}$$

Here $l_p$ and $l_Q$ are the path of delay, $v_s$ is the cutting speed.

$B = kb_c\mu\delta^{\varepsilon-1}$ is a relative cutting force, $\delta$ is the thickness of a chip, $\mu$ is the power estimating the characteristics of metals and the form of the lathe tool, $b_c$ is the width of the chip, and $k$ is relative pressure.

*Linear analysis.* After linearising equation (3), we get a linear differential equation

$$\ddot{x}(t) + \frac{b_x}{m_x}\dot{x}(t) + \omega_x^2 x(t) + \frac{fB}{m_x}x\left(t - \frac{l_p + l_Q}{v_s}\right) = 0. \tag{6}$$

A characteristic quasi-polynomial of equation (6) is

$$P(\lambda) = \lambda^2 + \alpha_1\lambda + \alpha_2 + k_1 e^{-h_Q}, \tag{7}$$

where

$$\alpha_1 = \frac{b_x}{m_x}, \quad \alpha_2 = \omega_x^2, \quad k_1 = \frac{fB}{m_x}, \quad h_Q = \frac{l_p + l_Q}{v_s}.$$

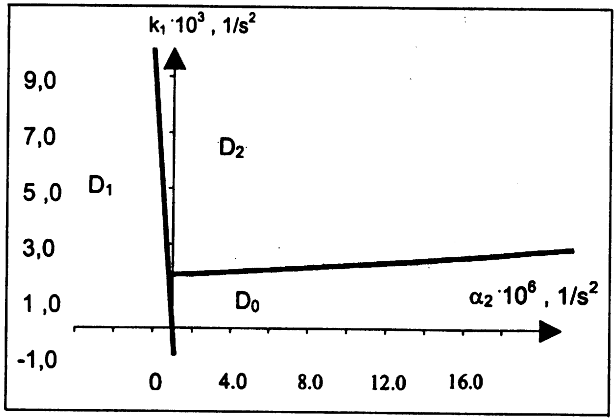


Fig. 2. $D$-expansion in the plane of parameters $k_1$ and $\alpha_2$.

We will look for the distribution of roots of equation (6) in the plane of parameters $k_1$ and $\alpha_2$ using the method of $D$-expansion. We get equations of remaining curves of $D$-expansion in the following parametrical forms:

$$\begin{cases} k_1 = \dfrac{\alpha_1 \sigma}{\sin(\sigma h_Q)}, & (8) \\ \alpha_2 = \sigma - \alpha_1 \sigma \operatorname{ctg}(\sigma h_Q). & (9) \end{cases}$$

As $\sigma \to 0$, from equations (8) and (9) we define the return point with the coordinates

$$\lim_{\sigma \to 0} \alpha_2 = -\frac{\alpha_1}{h_Q}, \quad \lim_{\sigma \to 0} k_1 = \frac{\alpha_1}{h_Q}.$$

According to the experimental results [2,3,4 ] we can calculate the values of the coefficient $\alpha_1$ and time of delay $h_Q$, $v_s = 140$ m/min, $l_p = 0,35$ mm, $l_Q = 0,32$ mm, $c_x = 40000$ N/mm, $m_x = 4,64 \cdot 10^{-3}$ Ns$^2$/mm, $b_x = 0,0118$ Ns/mm. If we substitute those values in equations (8) and (9) then we get $D$-expansion in the plane of the parameters $k_1$ and $\alpha_2$ (Fig. 2).

We have to emphasize that in the real cutting process only positive values of the parameters $\alpha_2$ and $k_1$ are important. We are interested in the area $D_0$ of asymptotical stability and the area $D_2$ which describe self-excited oscillations arising in the process of cutting.

**Lemma 1.** *The inequality* $\operatorname{Re} \lambda < 0$ *is valid for all roofs of the quasi-polynomial in the area* $D_0$ (Fig. 2), *i.e., the area* $D_0$ *is the area of asymptotical stability.*

**Theorem 1.** *Let* $k_1, \alpha_1 \in D_0$, $h_Q > 0$ *and* $\alpha_2 > 0$. *Then the state of zero equilibrium of equation* (5) *is asymptotically stable.*

**Lemma 2.** *When, $\alpha_1 > 0$, $\alpha_2 = 0$ and $k_1 > k_{10}$ the quasi-polynomial (7) has a couple of complex joint roots with the positive real part while real parts of the other roots are negative.*

The proof of Lemmas 1 and 2 it is presented in work [5].

*Nonlinear analysis.* We investigate differential equations by the method of bifurcation the theory. The system of differential equations with delay, depending on the searching function is

$$\ddot{x}(t) + \alpha_1 \dot{x}(t) + \alpha_2 x(t) + k_1(\varepsilon) x(t - \tau_p - \tau_Q) = 0, \tag{8}$$

$$\ddot{y}(t) + \beta_1 \dot{y}(t) + \beta_2 x(t) + k_2 x(t - \tau_p) = 0. \tag{9}$$

Let us take in to account linear part of the small parameter $\varepsilon$ then

$$k_1(\varepsilon) = k_{10} + \varepsilon.$$

We change time

$$t = (1 + c)\tau \tag{12}$$

and get

$$\begin{cases} \ddot{x}(\tau) + \alpha_1 x(\tau)(1+c) + \alpha_2 x(\tau)(1+c)^2 = -\left[(1+c)^2(k_1+\varepsilon)x(\tau - h_Q + W_1)\right], & (13)\\ \ddot{y}(\tau) + \beta_1 \dot{y}(\tau)(1+c) + \beta_2 y(\tau)(1+c)^2 = -\left[(1+c)^2(k_2+\varepsilon)y(\tau - h_p + W_2)\right]. & (14) \end{cases}$$

We expand the functions in a power series of $\xi$

$$x(\tau) = \xi \cos \sigma_0 \tau + \xi^2 x_2(\tau) + \xi^3 x_3(\tau) + \dots, \tag{15}$$

$$y(\tau) = \xi y_1(\tau) + \xi^2 y_2(\tau) + \xi^3 y_3(\tau) + \dots, \tag{16}$$

$$c = \xi^2 c_2 + \xi^4 c_4 \dots, \tag{17}$$

$$\varepsilon = \xi^2 b_2 + \xi^4 b_4 + \dots. \tag{18}$$

We also expand the functions $x(\tau - h_Q + W)$ and $y(\tau - h_p + W_2)$ in the Taylor series:

$$x(\tau - h_Q + W) = x(\tau - h_q) + x'(\tau - h_q)W + \frac{1}{2}x''(\tau - h_q)W^2 + \dots, \tag{19}$$

$$y(\tau - h_p + W_2) = y(\tau - h_p) + y'(\tau - h_p)W_2 + \frac{1}{2}y''(\tau - h_p)W_2^2 + \dots. \tag{20}$$

After expanding the right and left parts in a series with respect to $\xi$ and after sorting the coefficients of the same powers of $\xi$, we get a sequence of linear nonhomogeneous differential equations with the period $2\pi/\sigma_0$. From these equations we can find the coefficients $c_2$ and $b_2$. Their calculation is presented in [6] more in detail.

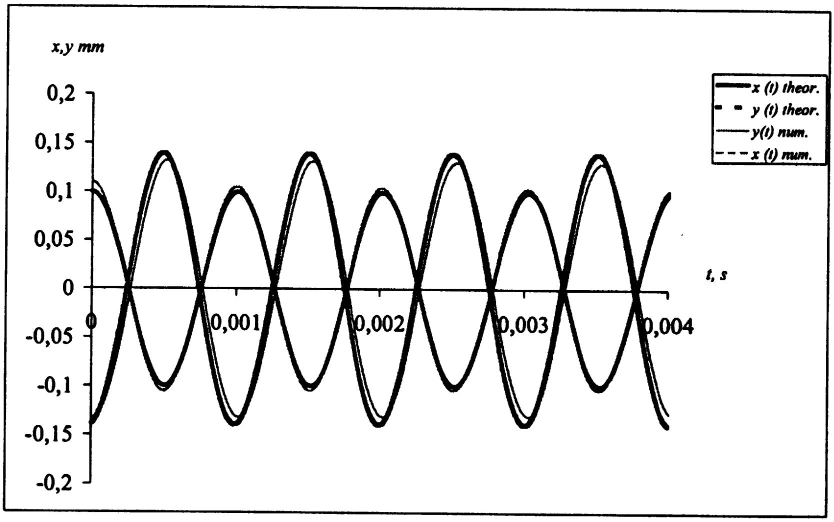


Fig. 3. Comparison of functions $y(t) \approx 0.011(0.12\sin 6237t - 1.336\cos 6237t)$ and $x(t) \approx 0.011\cos 6237t$ with the results of numerical experiment for $v_s = 140$ m/min, $s = 0.8$ mm, $b_c = 1.4$ mm.

From (18) we can obtain, that

$$\xi_* = \sqrt{\frac{\varepsilon}{b_2}} + 0(\varepsilon), \quad \text{then} \quad \tau \approx \frac{t}{1 + \frac{c_2}{b_2}\varepsilon}.$$

Finally, we get the periodical solution of system differential equations (10)–(11):

$$\begin{cases} x(t) = \sqrt{\frac{\varepsilon}{b_2}} \cos \frac{\sigma_0 t}{1 + \frac{c_2}{b_2}\varepsilon} + 0(\varepsilon), \\ y(t) = \sqrt{\frac{\varepsilon}{b_2}} \left( A_s \sin \frac{\sigma_0 t}{1 + \frac{c_2}{b_2}\varepsilon} + A_c \cos \frac{\sigma_0 t}{1 + \frac{c_2}{b_2}\varepsilon} \right) + 0(\varepsilon). \end{cases}$$

We solve systems (3) and (4) by the Runge-Kutto method of the results of computational experiment are presented in Fig. 3.

## Conclusions

The results of computational experiment correspond to the theoretical solution of system (3)–(4). We can model the metal cutting process, when we change the parameter $k_1$. Thus, when coefficients $\alpha_1$, $\alpha_2$, $\beta_1$ and $\beta_2$ have different values, we can find conditions, for the system of differential equation to have a stable periodical or asymptotically stable solution and not arive self-excited oscillations not arise in the metal cutting process.

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# Metalų pjovimo proceso stabilumo tyrimas

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Straipsnyje tiriamos metalų pjovimo proceso metu susižadinančių autosvyravimų lygtys. Atlikta šių lygčių tiesinė analizė taikant $D$-suskaidymų metodą. Išskirta asimptotinio stabilumo sritis, kurioje autosvyravimai neturėtų susižadinti. Netiesinė šių lygčių analizė atliekama taikant bifurkacijų teoriją ir gaunama apytikrio sprendimo išraiška. Gautas sprendinys palygintas su skaitinio eksperimento rezultatais. Turint gautus rezultatus, keičiant parametrų reikšmes, galima matematiškai modeliuoti pjovimo procesą, kad jis vyktų asimptotinio stabilumo srityje.