LMD konf. darbai, 255–260
1997, Vilnius

Problems of laser beam width measuring with CCD cameras

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1. INTRODUCTION

Digital image processing is a fascinating subject in several aspects. Human beings perceive most of the information about their environment through their visual sence. While for a long time images could only be captured by photography, we are now at the edge of another technological revolution which allows image data to be captured, manipulated, and evaluated electronically with computers.

With breath taking piece, computers are becoming more poweful and at the same time less expensive, so that widespread applications for digital image processing emerge. In this way, image processing is becoming a tremendous tool to analize image data in all areas of natural science. For more and more scientists digital image processing will be key to study complex scientific problems they could not have dreamed to tackle only a few years ago. A door is opening for new interdisciplinary cooperations, merging computer science with the corresponding research areas [1].

Charge-coupled-device (CCD) cameras are commonly used for acquiring images of incoherently radiating objects in a diverse range of applications spanning the microscopic in biology to the macrocosmic in astronomy. Their wide spectral response makes them useful for acquiring image data throughout the UV, visible, and IR wavelengths [2].

CCD cameras are very useful devices for the acquisition of laser intensity profiles [3]. They – are relatively inexpensive, – possess a wide spectral response, – feature a high spatial resolution, – are easy to handle and readout, – have a linear signal output vs. input, – are applicable for CW and pulsed beams.

CCD cameras are imperfect. Optical elements that produce the CCD sensor itself, such as field stops and lenses, limit resolution and introduce aberrations. The CCD sensor exibits photoconversion noise, readout noise, nonuniform flat-field response, and extraneous charge carriers resulting from bias, dark current, and both internal and external background radiation. The significance of these various imperfections in a given application depends on how CCD camera is implemented and use, such as whether it is coaled and whether stray radiation is present in the field of view containing the object of interest, but all of them are present to some degree in all applications.

Recently several national and international efforts to standardize the definion and measurement of beam radius and beam quality have been made [4,5].

Choclab (Belgium, Canada, Denmark, Finland, Germany, Italy, Lithuania, Russia, Spain, U.K.) exists to design, test and optimize procedures to characterize laser beams and the optical components such as mirrors and lenses that quide them [6].

The project co-operats with the international standards commitees CEN and ISO to make sure that results will be embodied immediately in industry standards. The project works in four related areas: – Beam width, divergence and propagation, – beam pointing stability, – beam shape, – optics characterisation.

Choclab is also studying the power, energe and temporal characteristics of beams, spectral profile and stability, and the complete characterization of diode lasers.

The present paper is a part of the reseach carried out by the project. It is devoted to problems of statistical estimation of laser beam width on a base of image data obtained by use of CCD cameras. In Section 2 a simple mathematical model of laser beam measured with CCD cameras is presented. Estimation problems are discussed in Section 3. The last section contains a description of an iterative estimating procedure. Performance of the procedure when applying to real and simulated data will be discussed elsewhere.

2. MATHEMATICAL MODEL OF LASER BEAM MEASURED WITH CCD CAMERAS

For measurements of laser beam intensity using CCD cameras we apply the following mathematical model:

$$s(x, y) = f(x, y) + \mu + \varepsilon(x, y), \quad (x, y) \in \mathbf{K}, \tag{1}$$

where $f(x, y)$ is true intensity level of the laser beam at the mesh point $(x, y) \in \mathbf{K}$ of CCD camera, $\mathbf{K} = \mathbf{K}_{n,h} \stackrel{\text{def}}{=} \{(ih,\ jh),\ i, j = 0, 1, \ldots, n\}$ (h being a step size of the mesh), $s(x, y)$ is its measurement with the offset value μ and random errors (noise) $\varepsilon(x, y)$. We suppose that $\varepsilon(x, y)$, $(x, y) \in \mathbf{K}$ are independent and identically distributed (iid) random variables (rv's) with zero mean and variance δ^2. For a scalar (vector-, matrix-valued) function ψ denote by

$$F_\psi \stackrel{\text{def}}{=} h^2 \sum_{(x,y)\in\mathbf{K}} \psi(x, y) f(x, y),$$

$$S_\psi \stackrel{\text{def}}{=} h^2 \sum_{(x,y)\in\mathbf{K}} \psi(x, y) s(x, y)$$

the corresponding scalars (vectors, matrices). It is supposed f is nonnegative and $F_{x^2+y^2}$ converges to a finite limit as $nh \to \infty$. Hence, it is natural to assume that

$$f(x, y)(x^2 + y^2) \approx 0, \quad (x, y) \in \mathbf{K}_{\infty,h} \setminus \mathbf{K}_{n,h}, \tag{2}$$

for sufficiently large nh.

The laser beam quality is characterized by quantities

$$\sigma_{x^2} \stackrel{\text{def}}{=} \frac{F_{x^2} - F_x^2}{F_1}, \quad \sigma_{xy} \stackrel{\text{def}}{=} \frac{F_{xy} - F_x F_y}{F_1}, \quad \sigma_{y^2} \stackrel{\text{def}}{=} \frac{F_{y^2} - F_y^2}{F_1}.$$

Thus, our aim is to evaluate the aforementioned characteristics on a base of the measurements $s(x, y)$, $(x, y) \in \mathbf{K}$. Since no assumptions are impossed on a parametric shape of f the statistical estimation prolem we deal with is nonparametric one. The quantities F_ψ, $\psi \in \{x^2, xy, y^2\}$ and their estimators have the greatest impact on overall perfomance of estimators of σ_ψ, $\psi \in \{x^2, xy, y^2\}$. Therefore in this paper, we restrict ourselves to the problem of estimating F_ψ, $\psi \in \{x^2, xy, y^2\}$.

The offset value μ and the 'noise' variance δ^2 are unknown. Usually they are estimated from data of CCD camera obtained witout the laser beam. An alternative way is to exploit the condition (2). We do not consider this question further and suppose that δ^2 is given. The influence of the offset value is briefly discussed in the next section. For simplicity, in the sequel we also assume $F_x = F_y = 0$.

Let

$$F_\Psi \stackrel{\text{def}}{=} h^2 \sum_{(x,y)\in\mathbf{K}} \Psi(x, y) f(x, y),$$

$$S_\Psi \stackrel{\text{def}}{=} h^2 \sum_{(x,y)\in\mathbf{K}} \Psi(x, y) s(x, y)$$

where

$$\Psi \stackrel{\text{def}}{=} \begin{pmatrix} x^2 & xy \\ xy & y^2 \end{pmatrix}.$$

For a symmetric 2×2 matrix A define norm $\|A\|_2^2 \stackrel{\text{def}}{=} a_{11}^2 + 2a_{12}^2 + a_{22}^2$ and quadratic form

$$Av^2 \stackrel{\text{def}}{=} a_{11}x^2 + 2a_{12}xy + a_{22}y^2, \quad v = (x, y) \in \mathbf{K}. \tag{3}$$

Now our estimation problem can be stated as follows: to find an estimator $\hat{F}_\Psi$ of F_Ψ for which the risk

$$R \stackrel{\text{def}}{=} \mathbf{E}\|F_\Psi - \hat{F}_\Psi\|_2^2,$$

i.e. the average square losses, would be as small as possible.

3. PROBLEMS OF LASER BEAM WIDTH ESTIMATION

In view of (2) it is reasonable to consider estimators of the following type

$$\hat{F}_\Psi = S_{\Psi|\mathbf{B}} \stackrel{\text{def}}{=} h^2 \sum_{(x,y)\in\mathbf{B}} \Psi(x, y) s(x, y), \tag{4}$$

where $\mathbf{B}$ is a subset of the set $\mathbf{K}$. When $\mathbf{B} = \mathbf{K}$ we obtain a 'naive' estimate

$$\hat{F}_\Psi = S_\Psi. \tag{5}$$

Basically, we restrict ourselves to the case of ellipses

$$\mathbf{B} = \mathbf{B}_A(\rho) \stackrel{\text{def}}{=} \{v \in \mathbf{K}\colon A^{-1}v^2 \leqslant \rho\}, \tag{6}$$

where A is a positive definite matrix.

The risk of the estimator. It is well known that

$$R(\mathbf{B}) \stackrel{\text{def}}{=} \mathbf{E}\|F_\Psi - S_{\Psi|\mathbf{B}}\|_2^2 = \text{Bias}(\mathbf{B}) + \text{Var}(\mathbf{B}), \tag{7}$$

where

$$\text{Bias}(\mathbf{B}) \stackrel{\text{def}}{=} \|\mathbf{E}S_{\Psi|\mathbf{B}} - F_\Psi\|_2^2, \tag{8}$$

$$\text{Var}(\mathbf{B}) \stackrel{\text{def}}{=} \mathbf{E}\|S_{\Psi|\mathbf{B}} - \mu I_{\Psi|\mathbf{B}} - F_{\Psi|\mathbf{B}}\|_2^2, \tag{9}$$

and $I_{\Psi|\mathbf{B}}$ denotes $F_{\Psi|\mathbf{B}}$ for $f \equiv 1$. It is straightforward to verify that

$$\text{Var}(\mathbf{B}) = h^4\delta^2 \sum_{(x,y)\in\mathbf{B}} \|\Psi(x,y)\|_2^2 = h^4\delta^2 \sum_{(x,y)\in\mathbf{B}} (x^4 + 2x^2y^2 + y^4). \tag{10}$$

The bias Bias($\mathbf{B}$) depends on the values of the unknown function f outside the set $\mathbf{B}$ and the offset value μ and is given by

$$\begin{aligned} \text{Bias}(\mathbf{B}) &= \mu^2\|I_{\Psi|\mathbf{B}}\|_2^2 + \|F_\Psi - F_{\Psi|\mathbf{B}}\|_2^2 \\ &= \mu^2 h^4 \|\sum_{v\in\mathbf{B}} \Psi(v)\|_2^2 + h^4 \|\sum_{v\in\mathbf{K}\setminus\mathbf{B}} \Psi(v) f(v)\|_2^2. \end{aligned} \tag{11}$$

The best choice of $\mathbf{B}$ is a solution of the following minimization problem

$$R(\mathbf{B}) \longrightarrow \min_{\mathbf{B}}. \tag{12}$$

To apply criterion (12) it is necessary to estimate the bias Bias($\mathbf{B}$) because it depends on the unknown function f. Since a natural estimate of Bias($\mathbf{B}$) given by $\|S_{\Psi|\mathbf{K}} - S_{\Psi|\mathbf{B}}\|_2^2 - \text{Var}(\mathbf{K}) + \text{Var}(\mathbf{B})$ has large variance the staightforward implementation of criterion (12) runs into problems. Therefore a criterion based on principle of balancing the bias and the variance is prefered to (12). Below we describe this criterion as well as a corresponding procedure for selection of the set $\mathbf{B}$.

It follows from (10) that Var($\mathbf{B}$) is nondecreasing and additive set function:

(a) $\mathbf{B}_1 \subset \mathbf{B}_2$ implies $\text{Var}(\mathbf{B}_1) \leqslant \text{Var}(\mathbf{B}_2)$;

(b) $\text{Var}(\mathbf{B}_1 \cup \mathbf{B}_2) = \text{Var}(\mathbf{B}_1) + \text{Var}(\mathbf{B}_2)$ provided $\mathbf{B}_1 \cap \mathbf{B}_2 = \emptyset$.

Since the function f is nonnegative the bias (11) is a nonincreasing set function provided both f and $\mathbf{B}$ are symmetric and $\mu = 0$.

Let $\mathbf{B}_k$, $k = 1, 2, \ldots$, be a given sequence of sets (contendes) such that $\mathbf{B}_k \subset \mathbf{B}_{k+1}$ and the sets $\mathbf{B}_{k+1}\setminus\mathbf{B}_k$, $k = 1, 2, \ldots$, are sufficiently large. Define

$$b_k \stackrel{\text{def}}{=} \|F_{\Psi|\mathbf{B}_{k+1}} - F_{\Psi|\mathbf{B}_k}\|_2^2 \tag{13}$$

and let $c > 0$ be a prespecified constant. Let k_0 denote the least integer k for which the inequality

$$b_k < c\,\text{Var}(\mathbf{B}_{k+1}\setminus\mathbf{B}_k) \tag{14}$$

holds. According to the principle of balancing the bias and the variance the set $\mathbf{B}_{k_0}$ is judged to be optimal. Note that in criterion (14) the function f is implicitly supposed to be such that $b_k/\operatorname{Var}(\mathbf{B}_{k+1}\backslash\mathbf{B}_k)$ forms a nonincreasing sequence.

Assume that $\mu = 0$. The bias term b_k defined in (13) is estimated by

$$\hat{b}_k \stackrel{\text{def}}{=} \|S_{\Psi|\mathbf{B}_{k+1}} - S_{\Psi|\mathbf{B}_k}\|_2^2 - \operatorname{Var}(\mathbf{B}_{k+1}\backslash\mathbf{B}_k) \tag{15}$$

Now, in view of (14) an estimate k^* of the optimal k_0 is defined as the least integer k such that

$$\hat{b}_k < c\operatorname{Var}(\mathbf{B}_{k+1}\backslash\mathbf{B}_k) \tag{16}$$

The offset influence. Suppose that a true value of the offset μ is zero. It is easy to check that the bias of $S_{\Psi|\mathbf{B}}$, $\mathbf{B} = \mathbf{B}_A(\rho)$ (see (6) and (11)), due to the erraneous offset μ is given by

$$\operatorname{Bias}_\mu(S_{\Psi|\mathbf{B}}) \stackrel{\text{def}}{=} \|\mathbf{E}S_{\Psi|\mathbf{B}} - F_{\Psi|\mathbf{B}}\|_2^2 = \mu^2\|I_{\Psi|\mathbf{B}}\|_2^2 = \frac{\pi^2}{16}\mu^2\rho^4\lambda_1^2\lambda_2^2(\lambda_1^4 + \lambda_2^4)$$

where λ_1 and λ_2 are the eigenvalues of A. (Here and everywhere below sums are approximated by the corresponding integrals without indication of the approximation error.) Compare the last expression with

$$\operatorname{Bias}_\mu(S_\Psi) \stackrel{\text{def}}{=} \|\mathbf{E}S_\Psi - F_\Psi\|_2^2 = \mu^2\|I_\Psi\|_2^2 = \frac{32}{9}\mu^2(nh)^8.$$

Note that in the last case $\operatorname{Bias}_\mu(S_\Psi) \to \infty$ as $nh \to \infty$ provided $\mu \neq 0$.

An example of Gaussian beam. Suppose that f is of Gaussian shape. Namely,

$$f(x, y) = f(v) = 2\pi(\det(A))^{-1/2}\exp\{-(1/2)A^{-1}v^2\}$$

(see (3)). Further, let $\mu = 0$ and $\mathbf{B} = \mathbf{B}_A(\rho)$ (see (6)). Recall that λ_1 and λ_2 are the eigenvalues of A. Then

$$\operatorname{Bias}(\mathbf{B}) = (\lambda_1^4 + \lambda_2^4)(\rho/2 + 1)^2e^{-\rho} \tag{17}$$

and

$$\operatorname{Var}(\mathbf{B}) = \frac{\pi}{8}h^2\delta^2\rho^3\lambda_1\lambda_2\big(\lambda_1^4 + (2/3)\lambda_1^2\lambda_2^2 + \lambda_2^4\big). \tag{18}$$

(note that $\operatorname{Var}(\mathbf{B})$ does not depend on f !). We keep the matrix A fixed and seek for the best ρ. According to (12), the optimal ρ^* is found from the equation

$$\hat{R}'_\rho = 0 \tag{19}$$

obtained by equating to zero the derivative of

$$\hat{R}_\rho \stackrel{\text{def}}{=} R(\mathbf{B}_A(\rho)) = \operatorname{Bias}(\mathbf{B}_A(\rho)) + \operatorname{Var}(\mathbf{B}_A(\rho)) \tag{20}$$

with respect to ρ. Formulas (17)-(20) yield

$$(1 + 2/\rho)e^{-\rho} = \kappa h^2\delta^2 \operatorname{area}(\mathbf{B}_A(1)),$$

where

$$\kappa \stackrel{\text{def}}{=} \frac{3}{2}\frac{\lambda_1^4 + (2/3)\lambda_1^2\lambda_2^2 + \lambda_2^4}{\lambda_1^4 + \lambda_2^4} \in [3/2, 2].$$

Thus,

$$\rho^* \approx -\ln\big[\kappa h^2\delta^2 \operatorname{area}(\mathbf{B}_A(1))\big].$$

Application of the criterion (14) leads to approximately the same result.

4. ESTIMATING PROCEDURE

Suppose $\mu = 0$. Let $c_0 \in [3, 8]$ and $c_1 \in (0, 4]$ be given constants, let $\gamma_k, k = 0, 1, \ldots, \gamma_0 \geq 1$, be an increasing sequence of numbers and $\mathbf{D}_0 \stackrel{\text{def}}{=} \{(x, y) \in \mathbf{K}: s(x, y) < c_0\delta\}$.

The estimation algorithm.

Step 0. Set $k = 0$. Calculate $A_0 \stackrel{\text{def}}{=} S_{\Psi|\mathbf{D}_0}$ and

$$\rho_0 = \gamma_0 \frac{h^2 \text{card}(\mathbf{D}_0)}{\pi \det(A_0)^{1/2}} \approx \gamma_0 \frac{\text{area}(\mathbf{D}_0)}{\text{area}(\mathbf{B}_{A_0}(1))}$$

(card($\mathbf{D}_0$) denotes the number of elements of the set $\mathbf{D}_0$).

Step 1. Set $k = k + 1$ and calculate $A_k \stackrel{\text{def}}{=} S_{\Psi|\mathbf{D}_k}$ where $\mathbf{D}_k \stackrel{\text{def}}{=} \mathbf{B}_{A_{k-1}}(\rho_k)$ and $\rho_k = \gamma_k \rho_{k-1}$.

Step 2. If $k \leqslant 1$ go to step 1, otherwise check the stopping condition (see (10), (15) and (16))

$$\|A_k - A_{k-1}\|_2^2 < c_1 h^4 \delta^2 \sum_{(x,y)\in \mathbf{B}_k \setminus \mathbf{B}_{k-1}} (x^4 + 2x^2y^2 + y^4).$$

If this condition is not satisfied execute step 1, otherwise set $k^* \stackrel{\text{def}}{=} k - 1$ and stop.

The estimator $A^* = S_{\Psi|\mathbf{B}_{A_{k^*}}(\rho_{k^*})}$ obtained is 'optimal' one.

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Lazerio pluošto sklaidos matavimo su KSĮ kameromis problemos

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Lazerio kokybės nustatymui reikia kuo tiksliau įvertinti jo pluošto sklaidą įvairiomis kryptimis pagal pluošto intensyvumo matavimų su KSĮ (krūvio sąsajos įrenginio) kameromis rezultatus. Darbe aprašytas paprasčiausias lazerio pluošto intensyvumo matavimų su KSI kameromis matematinis modelis ir aptartas sklaidos vertinimo uždavinys. Pateiktas iteratyvus algoritmas, kurio sustojimo taisyklė remiasi poslinkio ir dispersijos subalansavimo principu.