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Completeness classes for intuitionistic first-order temporal logic with time gaps

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1. Introduction

V. Glivenko proved in [4] that a propositional formula beginning with '$\neg$' is derivable in a classical propositional calculus iff it is derivable in an intuitionistic propositional calculus. In [6], classes of sequents (called purely Glivenko and Glivenko σ-classes) are presented. Every sequent which belongs to some of these classes is derivable in a classical predicate sequent calculus iff it is derivable in an intuitionistic predicate sequent calculus.

LB and LBJ are sequent calculi of first-order classical and intuitionistic temporal logics with time gaps, respectively. Similar classes for LB and LBJ are given in [2]. In the present paper, we show that the Glivenko classes (i.e., purely Glivenko and Glivenko σ-classes) are LBJ completeness with respect to TBJ classes.

The paper is organized as follows. First, we present shortly the semantics of the first-order classical temporal logic TB and then the semantics of its intuitionistic counterpart TBJ. Then we introduce a sequent calculus LB for TB, sequent calculi LBJ and LBJ* for TBJ, and give some properties of the calculi. In the end, Glivenko σ-classes are defined and it is shown that these classes are LBJ completeness with respect to TBJ classes.

2. Semantics of TB

The class of admitted time structures for the traditional logic of discrete linear time TL is the class of structures order isomorphic to ω. Let us call such a structure an ω-segment. In such a segment, there is always an earliest point, for every point there is a unique next point and every point can be reached from the earliest point by passing finitely often to the next point. An admitted time structure $\mathcal{J}$ for TB is a structure order isomorphic to a well-founded tree of ω-segments (see [3]). For every point in $\mathcal{J}$, there is a unique next point, but also points 'after the gap' (which cannot be reached by successively passing on to the next point) which are initial points in the next ω-segments.

A TB structure $\mathbf{K}$ is a tuple $\langle \mathcal{J}, \{D_i\}_{i\in\mathcal{J}}, \{s_i\}_{i\in\mathcal{J}}\rangle$, where $D_i \neq \emptyset$ is a set called the domain at the state i, $D_i \subseteq D_j$ if $i \leqslant j$; it is also assumed that D_0 contains all the constants of the language; s_i is a function mapping constants and free variables to elements of D_i, n-ary function symbols to functions from D_i^n to D_i, predicate symbols of arity 0 to $\{\bot, \top\}$ and predicate symbols of arity $n > 0$ to functions from D_i^n to $\{\bot, \top\}$. Also: 1) $s_i(d) = d$ for every $d \in D_i$, 2) if $s_i(t) = d$, then $s_j(t) = d$ for any term t and $j \geqslant i$, and 3) $s_i(\mathcal{F}) = \bot$.

The valuation functions K_i are defined in the traditional way. See also [3].

The valuation functions K_i for sequents are defined as follows: $K_i(\Gamma \to \Delta) = \top$ if there is a formula $B \in \Gamma$ such that $K_i(B) = \bot$ or there is a formula $C \in \Delta$ such that $K_i(C) = \top$, and $K_i(\Gamma \to \Delta) = \bot$, otherwise.

A formula A (a sequent Σ) is satisfied in a TB structure $\mathbf{K} = \langle \mathcal{J}, \{D_i\}_{i\in\mathcal{J}}, \{s_i\}_{i\in\mathcal{J}}\rangle$, we write $\mathbf{K} \models A$ ($\mathbf{K} \models \Sigma$), iff $K_0(A) = \top$ ($K_0(\Sigma) = \top$). A (Σ) is valid in TB, $TB \models A$ ($TB \models \Sigma$), iff A (Σ) is valid in every TB structure.

3. Semantics of TBJ

A framework of an admitted time structure for TBJ is an admitted time structure for TB; as earlier, we denote it by $\mathcal{J}$. (An admitted time structure for TBJ can be thought of as a well-founded tree of ω-segments, the points themselves in the ω-segments being partially ordered sets satisfying certain conditions; admitted time structures for TBJ are formally defined as follows.) Suppose that E and E_1 are partially ordered sets. If $(l \in E) \Rightarrow (l \in E_1)$ and $(l \leqslant m \text{ in } E) \Rightarrow (l \leqslant m \text{ in } E_1)$, then we write $E \preceq E_1$. Let us denote by $\mathcal{E}$ the class of partially ordered sets and by f a function: $\mathcal{J} \ni i \mapsto f(i) \in \mathcal{E}$, where $f(i) \preceq f(j)$ if $i \leqslant j$. The class of admitted time structures for TBJ is the class of pairs $(\mathcal{J}, f)$. Elements of the sets $f(i)$ are considered to be time points in TBJ. The time point $l \in f(i+1)$ is considered to be next to the time point $l \in f(i)$.

A TBJ Kripke frame $\bar{\mathrm{K}}$ is a tuple $\langle(\mathcal{J}, f), \{D_{i,l}\}_{i\in\mathcal{J}, l\in f(i)}, \{s_{i,l}\}_{i\in\mathcal{J}, l\in f(i)}\rangle$, where $D_{i,l} \neq \emptyset$ is a set called the domain at the time point l of the state i, $D_{i,l} \subseteq D_{i,n}$ if $l \leqslant n$, and $D_{i,l} \subseteq D_{j,l}$ if $i \leqslant j$; it is also assumed that $D_{0,0}$ contains all the constants of the language; $s_{i,l}$ is a function mapping constants and free variables to elements of $D_{i,l}$, n-ary function symbols to functions from $D_{i,l}^n$ to $D_{i,l}$, predicate symbols of arity 0 to $\{\bot, \top\}$ and predicate symbols of arity $n > 0$ to functions from $D_{i,l}^n$ to $\{\bot, \top\}$; also: 1) $s_{i,l}(d) = d$ for every $d \in D_{i,l}$, 2) if $s_{i,l}(t) = d$, then 2.1) $s_{i,k}(t) = d$ for any term t and $k \geqslant l$, and 2.2) $s_{j,l}(t) = d$ for any term t and $j \geqslant i$, and 3) $s_{i,l}(\mathcal{F}) = \bot$. It is also assumed that 1) if $s_{i,l}(P) = \top$, then $s_{i,m}(P) = \top$ for every $m \geqslant l$, where P is a predicate symbol of arity 0 and 2) if $s_{i,l}(R)(s_{i,l}(t_1), \ldots, s_{i,l}(t_n)) = \top$, $n > 0$, then $s_{i,m}(R)(s_{i,m}(t_1), \ldots, s_{i,m}(t_n)) = \top$ for every $m \geqslant l$, where t_i are terms and R is an n-ary predicate symbol.

The valuation functions $K_{i,l}$ ($l \in f(i)$) for TBJ formulas are defined as follows.

1) $A = P(t_1, \ldots, t_n)$: $K_{i,l}(A) = s_{i,l}(P)(s_{i,l}(t_1), \ldots, s_{i,l}(t_n))$, where P is an n-ary predicate symbol ($n \geqslant 0$).
2) $A = \mathcal{F}$: $K_{i,l}(\mathcal{F}) = s_{i,l}(\mathcal{F}) = \bot$.
3) $A = B \wedge C$: $K_{i,l}(A) = \top$ if $K_{i,l}(B) = K_{i,l}(C) = \top$, and $= \bot$, otherwise.
4) $A = B \vee C$: $K_{i,l}(A) = \top$ if $K_{i,l}(B) = \top$ or $K_{i,l}(C) = \top$, and $= \bot$, otherwise.
5) $A = B \supset C$: $K_{i,l}(A) = \top$ if $K_{i,m}(B) = \bot$ or $K_{i,m}(C) = \top$ for every $m \geqslant l$, and $= \bot$, otherwise.
6) $A = \forall x B(x)$: $K_{i,l}(A) = \top$ if $K_{i,m}(B(d)) = \top$ for every $m \geqslant l$ and every $d \in D_{i,m}$, and $= \bot$, otherwise. Here $B(d)$ is obtained from $B(x)$ by substituting d for every occurrence of x in $B(x)$.

7) $A = \exists x B(x)$: $K_{i,l}(A) = \top$ if $K_{i,l}(B(d)) = \top$ for some $d \in D_{i,l}$, and $= \bot$, otherwise. Here $B(d)$ is obtained from $B(x)$ by substituting d for every occurrence of x in $B(x)$.
8) $A = \circ B$: $K_{i,l}(A) = \top$ if $K_{i+1,l}(B) = \top$, and $= \bot$, otherwise.
9) $A = \Box B$: $K_{i,l}(A) = \top$ if $K_{j,l}(B) = \top$ for every $j \geqslant i$, and $= \bot$, otherwise.

The valuation functions $K_{i,l}$ for sequents are defined as follows: $K_{i,l}(\Gamma \to \Delta) = \top$ if there is a formula $B \in \Gamma$ such that $K_{i,m}(B) = \bot$ or there is a formula $C \in \Delta$ such that $K_{i,m}(C) = \top$ for every $m \geqslant l$, and $K_{i,l}(\Gamma \to \Delta) = \bot$, otherwise.

A formula A (a sequent Σ) is valid in a TBJ Kripke frame $\bar{K} = \langle(\mathcal{J}, f), \{D_{i,l}\}_{i \in \mathcal{J}, l \in f(i)}, \{s_{i,l}\}_{i \in \mathcal{J}, l \in f(i)}\rangle$, we write $\bar{K} \models A$ ($\bar{K} \models \Sigma$), iff $K_{i,l}(A) = \top$ ($K_{i,l}(\Sigma) = \top$) for every $l \in f(i)$ and every $i \in \mathcal{J}$. A (Σ) is valid in TBJ, $TBJ \models A$ ($TBJ \models \Sigma$), iff A (Σ) is valid in every TBJ Kripke frame.

LEMMA 3.1. *Let A be a formula and $\mathbf{K}$ be a TB structure such that $\mathbf{K} \not\models A$. Then there is a TBJ Kripke frame $\bar{K}$ such that $\bar{K} \not\models A$.*

Proof. Let $\mathbf{K} = \langle \mathcal{J}, \{D_i\}_{i \in \mathcal{J}}, \{s_i\}_{i \in \mathcal{J}}\rangle$. Let $f(i) = \{1\}$, $D_{i,1} = D_i$, and $s_{i,1} = s_i$ for every $i \in \mathcal{J}$. Let $\bar{K} = \langle(\mathcal{J}, f), \{D_{i,1}\}_{i \in \mathcal{J}}, \{s_{i,1}\}_{i \in \mathcal{J}}\rangle$. Clearly, $\bar{\mathrm{K}}$ is a TBJ Kripke frame. Let n denote the number of occurrences of propositional connectives, quantifieres, and temporal operators in A. By induction on n, we prove that if $K_i(A) = \top$, then $K_{i,1}(A) = \top$, and if $K_i(A) = \bot$, then $K_{i,1}(A) = \bot$.

Base case: $n = 0$. The proof follows from the fact that $s_{i,1} = s_i$.

Inductive case: $n > 0$. We consider only some cases.

Let $A = B \vee C$. 1) Let $K_i(A) = \top$, then $K_i(B) = \top$ or $K_i(C) = \top$. By the inductive hypothesis, $K_{i,1}(B) = \top$ or $K_{i,1}(C) = \top$, therefore $K_{i,1}(A) = \top$. 2) Let $K_i(A) = \bot$. Then $K_i(B) = \bot$ and $K_i(C) = \bot$. By the inductive hypothesis, $K_{i,1}(B) = \bot$ and $K_{i,1}(C) = \bot$, therefore $K_{i,1}(A) = \bot$.

$A = B \supset C$. 1) Let $K_i(A) = \top$, then $K_i(B) = \bot$ or $K_i(C) = \top$. By the inductive hypothesis, $K_{i,1}(B) = \bot$ or $K_{i,1}(C) = \top$, therefore $K_{i,1}(A) = \top$. 2) Let $K_i(A) = \bot$, then $K_i(B) = \top$ and $K_i(C) = \bot$. By the inductive hypothesis, $K_{i,1}(B) = \top$ and $K_{i,1}(C) = \bot$, therefore $K_{i,1}(A) = \bot$.

$A = \Box B$. 1) Let $K_i(A) = \top$, then, for every $j \geqslant i$, $K_j(B) = \top$; by the inductive hypothesis, $K_{j,1}(B) = \top$, therefore $K_{i,1}(A) = \top$. 2) Let $K_i(A) = \bot$, then there is $j \geqslant i$ such that $K_j(B) = \bot$; by the inductive hypothesis, $K_{j,1}(B) = \bot$, therefore $K_{i,1}(A) = \bot$.

Now suppose that A is a formula, $\mathbf{K}$ is a TB structure, and $\mathbf{K} \not\models A$. Then $K_0(A) = \bot$, $K_{0,1} = \bot$ and $\bar{K} \not\models A$.

4. Deduction systems

Sequent Calculus LB for classical temporal logic with time gaps TB is obtained from a variant of Gentzen's sequent calculus LK (without structural rules) by adding some rules for temporal operators which are taken from [3] and slightly changed by us. The sequent calculus LBJ for intuitionistic temporal logic with time gaps TBJ is obtained from LB by introducing the restriction that sequents can have at most one formula in the succedent. The sequent calculus LBJ* is a multisuccedent version of LBJ. The calculi LB, LBJ, and LBJ* can be found in [2].

*4.1. Some properties of LB, LBJ, and LBJ**

The structural rules of weakening and contraction and the rule of cut is admissible in LB, LBJ, and LBJ*. For proofs, we refer to [2].

A sequent or a formula with no free variables is called a sentence.

THEOREM 4.1. *LB is sound for TB: if a sentence is derivable in LB, then the sentence is valid in TB.*

Proof. The theorem is proved in the same way as Theorem 3.7 in [3].

This theorem does not hold if 'sentence' is replaced by 'sequent'. E.g., the sequent $\Sigma = \forall x \Diamond P(x,a) \to \Diamond P(a,a)$, where $\Diamond = \neg\Box\neg$, is derivable in LB. Taking P to be '$<$', we get $\forall x \Diamond(x < a) \to \Diamond(a < a)$. Let $T = \omega$, $D_i = N$, $s_i(a) = i$. Then, $\forall n \in N(n < s_{n+1}(a) = n+1)$. We have a countermodel for Σ. If a is a constant, e.g. 2, then $\forall n \in N(s_n(2) = 2)$, and the above structure is not a countermodel in this case.

Let us take another example. $LB \vdash \Sigma = \Diamond P(b,a) \to \exists x \Diamond P(x,a)$. Assuming that P is '$<$', we get $\Diamond(b > a) \to \exists x \Diamond(x > a)$. Let $T = \omega$, $D_0 = \{1\}$, $D_i = \{1,2\}$ for $i > 0$, $s_i(a) = 1$, and $s_i(b) = 2$ for all i. It is easy to see that $K_0(\Sigma) = \bot$ even if a and b are constants, e.g., $a = 1$ and $b = 2$. However, this is not a TB structure, because D_0 has to have all the constants, thus $2 \in D_0$, and we have no countermodel in this case.

THEOREM 4.2. *LB is complete for TB: if a sequent is valid in TB, then it is derivable in LB.*

Proof. The theorem is proved as Theorem 4.1 in [3].

Below we prove that LBJ is sound but incomplete with respect to TBJ.

LEMMA 4.3 (monotonicity property). *Let $K_{i,l}$ be a valuation function defined via a TBJ Kripke frame $\bar{K}$. If A is a formula and $K_{i,l}(A) = \top$, then $K_{i,m}(A) = \top$ for every $m \geqslant l$.*

Proof. The lemma is proved by induction on the number of occurrences of propositional connectives, quantifiers, and temporal operators in A.

Base case. The proof follows from the definition of $s_{i,l}$.

Inductive case. We consider only some cases. Let $m \in f(i)$ and $m \geqslant l$.

Let $A = \exists x B(x)$. There is $d \in D_{i,l}$ such that $K_{i,l}(B(d)) = \top$. $d \in D_{i,m}$ (since $D_{i,l} \subseteq D_{i,m}$). By the inductive hypothesis, $K_{i,m}(B(d)) = \top$. Hence $K_{i,m}(A) = \top$.

$A = \circ B$. $K_{i+1,l}(B) = \top$. By definition of the function f, we have that $m \geqslant l$ in $f(i+1)$. Hence $K_{i+1,m}(B) = \top$ by the inductive hypothesis. This yields $K_{i,m}(A) = \top$.

$A = \Box B$. $K_{j,l}(B) = \top$ for every $j \geqslant i$. We have that $m \geqslant l$ in $f(j)$. Hence $K_{j,m}(B) = \top$ by the inductive hypothesis. This gives $K_{i,m}(A) = \top$.

Let $\Gamma \to \Delta$ be a sequent. We write $K_{i,l}(\Gamma) = \top$ if $\Gamma \neq \emptyset$ and $K_{i,l}(B) = \top$ for every $B \in \Gamma$. We also write $K_{i,l}(\Delta) = \top$ if there is $B \in \Delta$ such that $K_{i,l}(B) = \top$.

THEOREM 4.4 (soundness). *Let* $Calc \in \{LBJ, LBJ^*\}$. *If* $Calc \vdash^V \Gamma \to \Delta$, *then* $TBJ \models \Gamma \to \Delta$, *where* $\Gamma \to \Delta$ *is a sentence.*

Proof. The lemma is proved by induction on $h(V)$. We prove the lemma for $Calc = LBJ$. The proof for $Calc = LBJ^*$ is almost the same.

Base case: $h(V) = 0$. This case is obvious.

Inductive case: $h(V) > 0$.

$$\frac{}{\Gamma \to \Delta}(i).$$

We consider only some cases.

$(i) = (\to\supset)$:

$$\frac{\Gamma, A \to B}{\Gamma \to A \supset B}(\to\supset).$$

Let $K_{i,l}(\Gamma) = \top$, $m \in f(i)$, and $m \geqslant l$. By Monotonicity property (see above), we have that $K_{i,m}(\Gamma) = \top$. If $K_{i,m}(A) = \bot$, then $K_{i,m}(A \supset B) = \top$; if $K_{i,m}(A) = \top$, then $K_{i,m}(B) = \top$ by the inductive hypothesis and therefore $K_{i,m}(A \supset B) = \top$. Finally, $K_{i,l}(A \supset B) = \top$.

$(i) = (\circ_1)$:

$$\frac{\Gamma \to A}{\Pi, \circ\Gamma \to \circ A}(\circ_1).$$

Let $K_{i,l}(\circ\Gamma) = \top$. Then $K_{i+1,l}(\Gamma) = \top$ and, by the inductive hypothesis, $K_{i+1,l}(A) = \top$. Hence $K_{i,l}(\circ A) = \top$.

$(i) = (\Box)$:

$$\frac{\Box\Gamma \to A}{\Pi, \Box\Gamma \to \Box A}(\Box).$$

Let $K_{i,l}(\Box\Gamma) = \top$ and $y \geqslant i$. $K_{j,l}(\Gamma) = \top$ for every $j \geqslant y \geqslant i$. We have that $K_{y,l}(\Box\Gamma) = \top$. By the inductive hypothesis, $K_{y,l}(A) = \top$. Hence $K_{i,l}(\Box A) = \top$.

$(i) = (\to \forall)$:

$$\frac{S = \Gamma \to A(b)}{\Gamma \to \forall x A(x)}(\to \forall).$$

Here S is not a sentence, and we cannot apply the inductive hypothesis. However, it is easy to show by induction on $h(V)$ that $LBJ \vdash^V \Sigma(a) \Rightarrow LBJ \vdash^{V'} \Sigma(c)$ and $h(V') \leqslant h(V)$, where $\Sigma(a)$ is a sequent in which a free variable a occurs, and $\Sigma(c)$ is obtained from $\Sigma(a)$ by substituting a constant c for every occurrence of a in $\Sigma(a)$. Thus, we get $S' = \Gamma \to A(c)$ instead of S, where c does not occur in S. Now we can apply the inductive hypothesis to S' and argue further in the traditional way. The case when $(i) = (\forall \to)$ is similar, because the conclusion is a sentence and therefore no free variable occurs in it; if free variables are introduced in the premise, then we can substitute constants for the occurrences of free variables just as above.

THEOREM 4.5. *LBJ is incomplete w.r.t. TBJ.*

Proof. The sequent $\Sigma = \circ(A \vee B) \to \circ A \vee \circ B$ is derivable in LBJ*. This and Theorem 4.4 imply that $TBJ \models \Sigma$. However, Σ is not derivable in LBJ.

By considering the sequent $\circ(A \vee B) \to \circ A \vee (E \supset \circ B)$, one can see that LBJ* is not complete for TBJ, either.

5. Glivenko σ-classes

THEOREM 5.1. *Let $(*)$ be a property of sequents such that if a sequent satisfies $(*)$, then the sequent is derivable in LBJ iff it is derivable in LB. If Σ satisfies $(*)$, then $LBJ \vdash \Sigma$ iff $TBJ \models \Sigma$.*

Proof. $(\Rightarrow)$: The proof follows from Theorem 4.4.
$(\Leftarrow)$: Assume that a sequent Σ satisfies $(*)$ and $LBJ \nvdash \Sigma$. Then $LB \nvdash \Sigma$. As LB is complete for TB (Theorem 4.2), there is a TB structure $\mathbf{K}$ such that $\mathbf{K} \not\models \Sigma$. It follows from Lemma 3.1 that then there is a TBJ Kripke frame $\bar{\mathrm{K}}$ such that $\bar{K} \not\models \Sigma$. Hence $TBJ \not\models \Sigma$.

Now, following [6], we define σ-classes and purely Glivenko σ-classes. Let $\odot \in \mathbb{A} = \{\vee, \wedge, \supset, \exists, \forall, \circ, \Box\}$. An occurrence of $\odot$ in a formula or a sequent is called an occurrence of the type $\odot^+$ ($\odot^-$) if $\odot$ is a positive (negative) occurrence in this formula or sequent. A set $\{U_1^{\alpha_1}, \dots, U_n^{\alpha_n}\}$, where $U_i \in \mathbb{A}$ (see above) and $\alpha_i \in \{-, +\}$, is called a σ-class. A sequent S belongs to a σ-class $\{U_1^{\alpha_1}, \dots, U_n^{\alpha_n}\}$ iff there are no occurrences of $U_1, \dots, U_n$ of the type $U_1^{\alpha_1}, \dots, U_n^{\alpha_n}$, respectively, in S. A σ-class $\mathfrak{A}$ is contained in a σ-class $\mathfrak{B}$ if every sequent which belongs to $\mathfrak{A}$ also belongs to $\mathfrak{B}$ (note that then $\mathfrak{B} \subseteq \mathfrak{A}$).

A sequent with one formula in succedent is called a singular sequent. A σ-class is called a purely Glivenko σ-class if every singular sequent which belongs to it is derivable in LB iff it is derivable in LBJ.

THEOREM 5.2. *A σ-class is a purely Glivenko σ-class iff it is contained in at least one of the following 10 σ-classes:*
$\{\vee^-, \supset^+\}$; $\{\supset^-, \vee^+, \exists^+\}$; $\{\circ^+, \circ^-, \supset^+, \forall^+\}$; $\{\circ^+, \circ^-, \supset^+, \forall^-\}$; $\{\circ^+, \Box^+, \supset^+, \forall^+\}$; $\{\circ^+, \Box^+, \supset^+, \forall^-\}$; $\{\circ^-, \Box^-, \supset^+, \forall^+\}$; $\{\circ^-, \Box^-, \supset^+, \forall^-\}$; $\{\circ^+, \Box^-, \supset^+, \forall^+\}$; $\{\circ^+, \Box^-, \supset^+, \forall^-\}$.

A σ-class $\mathfrak{A}$ is called a Glivenko σ-class if every sequent with empty succedent which belongs to $\mathfrak{A}$ is derivable in LB iff it is derivable in LBJ.

THEOREM 5.3. *A σ-class is a Glivenko σ-class iff it is contained in at least one of the following 10 σ-classes:*
$\{\supset^-\}$; $\{\supset^+, \vee^-\}$; $\{\supset^+, \forall^-, \circ^+, \circ^-\}$; $\{\supset^+, \forall^-, \circ^+, \Box^+\}$; $\{\supset^+, \forall^-, \circ^+, \Box^-\}$; $\{\supset^+, \forall^+, \circ^+, \circ^-\}$; $\{\supset^+, \forall^+, \circ^+ \Box^-\}$; $\{\forall^+, \circ^+, \Box^+\}$; $\{\supset^+, \forall^+, \circ^-, \Box^-\}$; $\{\supset^+, \forall^-, \circ^-, \Box^-\}$.

Proofs of Theorems 5.2 and 5.3 can be found in [2].

THEOREM 5.4. *Purely Glivenko and Glivenko σ-classes are LBJ completeness with respect to TBJ classes.*

Proof. The proof follows from Lemma 5.1.

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REZIUMĖ

R. Alonderis. Pilnumo klasės intuicionistinei pirmos eilės laiko logikai su laiko tarpsniais

Darbe yra pateikiama pirmos eilės intuicionistinė laiko logika TBJ su laiko tarpsniais. Parodoma, kad šios logikos sekvencinis skaičiavimas LBJ yra korektiškas, tačiau nepilnas logikos TBJ atžvilgiu. Apibrėžiamos Glivenko sekvencijų klasės skaičiavimui LBJ bei jo klasikiniam atitikmeniui LB. Sekvencija priklausanti kuriai nors Glivenko klasei yra įrodoma skaičiavime LB tada ir tik tada, kai ji yra įrodoma skaičiavime LBJ. Glivenko klasės apibrėžiamos sekvencijoms, kurių sukcedentas susideda iš vienos formulės (visiškai Glivenko σ-klasės), ir sekvencijoms turinčioms tuščią sukcedentą (Glivenko σ-klasės). Parodoma, kad Glivenko klasės yra LBJ pilnumo logikos TBJ atžvilgiu klasės.