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GREEN SUPPLY CHAIN DECISION MAKING AND CONTRACT COORDINAION: THE INFLUENCE OF BRAND REPUTATION AND CONSUMER REFERENCE EFFECTS ON LOW-CARBON LEVELS

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Annotation. Rapid economic growth leads to severe pollution and excessive resource consumption, which poses challenges to sustainability. To address these issues, green supply chain management was promoted by the government, encouraging firms to enhance their low-carbon reputation and meet the increasing consumer demand for ecofriendly products. This study developed a differential game model for a two-echelon green supply chain that incorporated consumer reference effects on low-carbon levels-a factor that was often overlooked in previous research. By considering retail price, advertising investment, and emission reduction effort as decision variables, the optimal strategies under decentralized and centralized decision making were analyzed. Results show that centralized decision making leads to higher emission reduction, a stronger reputation, and better financial performance, although its efficiency depends on investment intensity. To bridge the gap between decentralized and centralized decision making, we proposed three contract coordination mechanisms: (i) two-way cost-sharing, (ii) revenue-sharing, and (iii) a hybrid contract. Particularly, the hybrid contract best aligns supply chain performance with the centralized optimum, thereby enhancing profitability for manufacturers and retailers. Numerical analysis validated the findings, providing insights for sustainable supply chain management and low-carbon policy formulation.

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Introduction

Since the 1970s, rapid economic growth – driven by the primary consumption of home appliances, food, construction, and automobiles – has led to a significant depletion of natural resources. The heavy reliance on energy and raw materials has intensified resource scarcity, posing substantial challenges to sustainable development. Under the dual constraints of resource limitations and environmental degradation, green development has gradually become a global priority (Linh, 2025). To address environmental and climate challenges, the Chinese government actively promotes carbon emission reduction policies, implementing regulatory measures such as carbon labeling and energy consumption labeling to encourage enterprises to lower emissions and guide consumers toward low-carbon products (Liu *et al.,* 2018; Zhu *et al.,* 2023; Streimikiene, 2021). However, consumer purchasing decisions are not solely influenced by products' current attributes but are also shaped by expectations formed from previous experiences – a psychological phenomenon widely recognized as the reference effect in the academic literature (Ye *et al.,* 2017).

Most existing research assumes that managers make decisions to maximize their own interests. However, firms in the supply chain are closely linked, and an excessive focus on the economic profits of individual firms can lead to a "double marginalization" effect that reduces overall supply chain efficiency. By contrast, optimal performance is achieved through contractual coordination mechanisms, such as transfer payments, which help align individual firm objectives with overall supply chain goals. As a result, supply chain coordination become a critical issue in supply chain management. Various contracts have been explored in the literature to facilitate supply chain coordination, with four primary types being revenue-sharing, cost-sharing, wholesale price, and transfer payment contracts. Jiang *et al.* (2015) examined revenue-sharing coefficient intervals to analyze supply chain coordination. Luo *et al.* (2016) established a revenue-sharing contract to study the effect of emission reduction rates on supply chain decision making under carbon tax policies, demonstrating that such contracts effectively coordinate the supply chain, regardless of whether firms invest in emission reduction technologies. Similarly, Yang *et al.* (2016) designed a revenue-sharing contract to achieve green supply chain coordination in the context of carbon trading. Beyond revenue-sharing, cost-sharing contracts have also been shown to improve supply chain coordination (Zhi *et al.,* 2017). Wang *et al.* (2019) investigated the dynamics of joint emission reduction in a supply chain under government subsidies, finding that cost-sharing contracts influence equilibrium decisions and demand. Variants of cost-sharing contracts have also been explored. Zhou *et al.* (2018) analyzed cooperative advertising and cooperative advertising–abatement cost-sharing contracts, identifying conditions under which supply chain members choose specific contracts. Another study by Zhou *et al.* (2017) introduced an advertising cooperative–abatement cost-sharing contract that accounts for equity concerns, demonstrating its effectiveness in coordinating the supply chain. Wholesale price contracts have also been examined in the literature. Liu *et al.* (2016) explored emission reduction decision making under consumer low-carbon preferences and carbon trading policies, comparing cost-sharing and wholesale price contracts in a low-carbon supply chain. Zhu *et al.* (2017) compared wholesale price and revenue-sharing contracts, proving that under certain conditions, revenue-sharing contracts effectively mitigate the double marginalization effect inherent in wholesale price contracts, leading to better supply chain coordination. Similarly, Wang *et al.* (2017) evaluated three contract models and concluded that restraining the wholesale price and implementing a cost-sharing contract enhance supply chain coordination, with the sharing ratio influencing emission reduction levels. In addition to these contract types, transfer payment contracts have also been extensively studied for supply chain coordination. Furthermore, other contractual mechanisms have been proposed. Li *et al.* (2018) developed a model incorporating three types of rebate-related contract, demonstrating that under certain conditions, selecting appropriate contracts improves overall supply chain performance.

Research on goodwill in the context of supply chains has become a significant area of study for scholars domestically and internationally. Nerlove and Arrow (2014) suggested that advertising investment can be regarded as an investment in a specific asset – namely, the accumulation of brand reputation. Dai *et al.* (2015) analyzed the optimal advertising decisions of monopolistic firms from the perspective of strategic consumers. Liu *et al.* (2015) assumed that advertising is a primary marketing tool and explored how companies with operating and marketing costs should make decisions regarding advertising investment. Lin *et al.* (2014) focused on how firms can strategically place online advertisements in the digital environment. However, most studies, including the aforementioned ones, have focused on individual firm decisions rather than on the cooperative dynamics within supply chains. By contrast, He *et al.* (2020) were the first to quantitatively analyze the determination of vertically cooperative advertising levels within a supply chain. Li *et al.* (2017) argued that advertising cooperation between enterprises not only facilitated dual-channel supply chain coordination but also enabled online and offline channels to achieve a win-win situation. Pnevmatikos *et al.* (2018) examined the influence of price and brand reputation on consumer demand and found that retailers' advertising investment in brand reputation not only enhances supply chain performance but also increases consumer surplus. Yu *et al.* (2018) applied differential game theory (DGT) to study the optimal strategy and profits in a green food supply chain, incorporating the dynamics of food greenness and brand reputation. Taboubi (2019) explored pricing and advertising investment issues in supply chains, using DGT to analyze how retailers should enhance product promotion in a manufacturer-dominated supply chain.

Given the background above, green, low-carbon sustainable development has become an inevitable trend. Simultaneously, the increasing awareness of low-carbon consumption among consumers, along with the growing influence of the reference effect on consumption choices, compels an increasing number of enterprises to emphasize the low-carbon reputation of their products and adopt green operational management strategies.

This study, grounded in green supply chain management and contract coordination theory, constructs a dynamic collaborative decision-making model. The model integrates differential game theory with optimal control methods and considers both the consumer reference effect on low-carbon levels and the product's low-carbon reputation. Under external pressures from the global green transition and internal challenges in balancing emission reduction and advertising investments, manufacturers and retailers dynamically adjust their strategies to achieve both economic and environmental objectives, ultimately optimizing the entire supply chain. This study addresses two progressively layered problems: (1) The pathway formation problem: Drawing on recent theoretical advancements in green supply chain collaborative decision-making and employing dynamic game and contract coordination models, this study investigates how manufacturers' emission reduction efforts and retailers' low-carbon advertising investments interact to form an optimal collaborative pathway, thereby revealing the intrinsic dynamic regulatory mechanisms within supply chain collaboration. (2) The effect evaluation problem: Utilizing optimal control methods and model simulations, this research analyzes the impact of different contractual coordination mechanisms on the overall performance of green supply chain members, thereby assisting enterprises in identifying and selecting the optimal green transformation pathway.

Through systematic research on the collaborative decision-making pathway and effect evaluation problems in green supply chains, this study makes three key contributions. First, the integrated innovation of theoretical perspectives: By overcoming the limitations of previous research that focused solely on either environmental or economic aspects and integrating differential game theory, contract coordination, and consumer behavior, this study establishes a multidimensional and dynamic theoretical framework for green supply chain collaboration, thus enriching the literature in this field. Second, the organic integration of methodologies: In contrast to traditional static or case study approaches, this study employs a comprehensive suite of dynamic game theory, optimal control, and simulation analysis methods to reveal both convergent and divergent phenomena in manufacturers' emission reduction efforts and retailers' advertising investments, thereby enhancing the universality and applicability of the research findings. Third, the in-depth elucidation of practical implications: Against the backdrop of increasing global green transformation and growing consumer preference for low-carbon products, this study offers theoretical foundations and practical insights for enterprises to optimize green operations under different contractual coordination mechanisms, facilitating improved economic performance while ensuring environmental sustainability.

The rest of this paper is structured as follows: Section 2 defines key concepts and outlines the theoretical framework. Section 3 formulates the research questions and hypotheses. Section 4 analyzes green supply chain decision-making and contract coordination, followed by case studies in Section 5. Section 6 discusses the results in relation to the research questions. Finally, Section 7 concludes the study, acknowledges its limitations, and suggests directions for future research.

1. Conceptual Definitions and Theoretical Foundations

1.1 Definition of Concepts

(1) Green Supply Chains

Most scholars have proposed various definitions and concepts of the green supply chain. Siferd *et al.* (2001) defined the primary objective of a green supply chain as ensuring effective environmental protection during production activities. Conversely, Lopomo *et al.* (2011) suggested that companies should integrate low-carbon and environmental considerations throughout the stages of product design, manufacturing, and marketing.

In addition to its environmental significance, a green supply chain can significantly influence consumers' purchasing decisions, making products from such supply chains more likely to gain consumer favor. Plambeck (2012) found that upstream and downstream enterprises in the supply chain can save energy and reduce emissions, thereby enhancing consumer trust in their products, which, in turn, boosts sales and strengthens brand reputation. Building on these findings, several scholars have focused on enterprises within green supply chains to analyze their production and operational decision making, thereby providing insights for improving operational management. Zhao *et al.* (2014) examined the pricing of green products and suggested that consumer preferences affect pricing strategies, advocating for market segmentation to help supply chain participants maximize profits. Similarly, Du *et al.* (2016) incorporated consumer preferences and developed a green production pricing model aimed at optimizing the operations of firms in the supply chain, with product demand as the focal point of their study.

Therefore, building on the broader context of green supply chains, this study further investigated the contractual coordination strategies within green supply chains by simultaneously considering products' low-carbon reputation and consumers' reference low-carbon level effect. The aim was to contribute to the development of green supply chains and provide theoretical guidance for future research in this area.

(2) Low-carbon reputation

Reputation is an intangible asset frequently discussed in the context of changes in a company's overall ownership. Similarly, when a company's products are affected by counterfeits that damage its goodwill, a goodwill appraisal is conducted to assess the appropriate compensation.

With advancements in time and technology, the focus has shifted from solely tangible assets to intangible ones, prompting efforts to identifying the types of intangible asset associated with reputation. As technology's role in manufacturing and products grew, it has evolved into a distinct asset category in the marketplace, with investors beginning to recognize technology as a separate intangible asset. Simultaneously, as product categories expanded, consumers have become increasingly selective, choosing products that best meet their needs. Over time, well-established brands in various sectors have become preferred by purchasing managers. In short, the value of individual intangible assets, such as branding and technology, is now considered separately. Reputation, once viewed as a singular intangible asset, evolved as factors like technology and branding were separated from the overall goodwill value and recognized as individual intangible assets, making the overall value of reputation less clear. The focus of this study is on low-carbon reputation, which reflects how environmentally friendly a company or product is perceived by consumers. Consumers with a preference for low-carbon options are more likely to choose products or brands with a strong low-carbon reputation. To effectively communicate this message, manufacturers must not only invest in emission reduction efforts within their supply chain but also help retailers enhance the low-carbon reputation of their products through targeted publicity, thereby guiding consumers toward low-carbon purchasing decisions.

(3) Supply chain coordination

Supply chain coordination refers to the use of process management techniques in the decision-making and planning processes of supply chain enterprises to create a management mechanism that effectively integrates production, sales, logistics, and other functions. The core element of supply chain coordination is the alignment of goals, ensuring that each enterprise's objectives within the supply chain are consistent with the overall goals of the entire system, thereby fostering cooperation toward a common aim (Guo Z., 2022). In the context of long-term collaboration and emission reduction, the first step is to instill a strong awareness of low-carbon emission reduction across all supply chain members. This ensures that each enterprise actively participates in emission reduction activities, working toward the shared goal of green, low-carbon, and sustainable development. Another crucial aspect of supply chain coordination is the alignment of production operations, which encompasses manufacturing coordination, research and development (R&D) alignment, and the coordination of interests between supply chain enterprises. Thus, during long-term collaboration for emission reduction, implementing measures that align the benefits of each enterprise is essential to prevent the fragmentation of the cooperative emission reduction system due to imbalanced benefits among the participants.

This study examined the "two-way cost-sharing and benefit-sharing" contract mechanism, where the manufacturer sells goods to retailers at a wholesale price significantly lower than the average market price. In return, retailers share a portion of the sales revenue with the manufacturer. In addition, the retailer and the manufacturer bear the costs associated with emission reduction and low-carbon publicity efforts.

1.2 Theoretical Foundations

(1) Reference effect theory

Consumers typically form reference points for the price, quality, and low-carbon level of a product based on their past purchasing experiences. The decision-making influence stemming from these reference points is known as the reference effect. As a significant factor affecting consumer purchasing behavior, the reference effect has garnered increasing academic attention in recent years. Most current research on the reference effect focuses on price and quality. From the perspective of reference price effects, Cao *et al.* (2020) examined how reference price effects influence pricing and inventory decisions within a firm's supply chain. Zhang *et al.* (2018) proposed skimming and penetration pricing strategies for daily necessities, considering the influence of reference price effects on short- and long-term returns. Lin *et al.* (2016) found that the reference price effect not only mitigates the double marginalization effect but also enhances the operational efficiency of the supply chain. Ma *et al.* (2020) explored collaborative management in corporate supply chains under the influence of big data marketing.

From the perspective of the reference quality effect, Zhou *et al.* (2020) introduced a service cost-sharing adjustment agreement to comprehensively assess the influence of the product reference quality effect on market demand. Their study determined the optimal decision-making strategy for each member of the supply chain and effectively eliminated the double marginalization effect. Li *et al.* (2020) examined the influence of the consumer reference quality effect on the product process innovation of monopoly enterprises, finding that a larger consumer "memory parameter" leads to higher investment in process innovation by monopolistic firms. With the growth of the low-carbon economy, the reference effect is increasingly influenced by consumers' heightened environmental awareness. Two key factors are at play: the actual and the reference low-carbon levels. The gap between these levels can significantly affect market demand (Yu *et al.,* 2020).

From the perspective of the reference low-carbon level effect, Zhang *et al.* (2018) and others explored carbon emission reduction and low-carbon publicity strategies in enterprise supply chains, considering with and without reference effects. Using the differential game model, they proposed coordinating enterprise supply chains through the introduction of bilateral cost-sharing contracts. Wang *et al.* (2019) analyzed the influence of the consumer reference effect on market demand, addressing the optimal decision-making problem for each supply chain participant under the carbon cap-and-trade policy.

These previous studies clearly show that the evaluation of the low-carbon level constitutes a comprehensive, objective judgment made by supply chain enterprises based on economic and low-carbon benefits. This evaluation is a critical foundation for assessing the performance of supply chain enterprises from a low-carbon perspective. Therefore, in this study, when analyzing supply chain optimization and coordination, we incorporated the consumer reference low-carbon level effect, assuming that its formation exhibits dynamic characteristics. On this basis, we established a differential game model for green supply chain joint emission reduction.

(2) DGT

Differential games refer to continuous-time strategic interactions among multiple participants, each aiming to optimize their individual and often conflicting objectives. The outcome is a set of time-dependent strategies for each participant that ultimately converge to a Nash equilibrium. As a synthesis of optimal control theory and game theory, differential games offer a novel approach to addressing coordination and control problems. With ongoing advancements in research and applications, the scope of differential game strategies has expanded, and solution methodologies have become increasingly refined. Today, DGT has evolved into a well-established theoretical framework with broad applications in fields such as economics, management, and environmental science. In this study, we applied the differential game approach to construct a dynamic game-based decision model for green supply chain participants. This section provides a brief introduction to DGT and dynamic programming methods.

The theory of differential games originated in the 1940s from research on aerial combat during World War II. Fueled by the strategic and tactical needs of the military, Rufus Isaacs, with the support of the U.S. Air Force and RAND Corporation, investigated pursuit and evasion problems in which both sides could make independent decisions. His research laid the foundational principles of DGT. By the 1960s and 1970s, the theory had matured, leading to significant results, such as the existence of value functions in differential games. In 1965, Isaacs published the first dedicated monograph on the subject, Differential Games, systematically outlining the theoretical framework. By employing differential equations and the concept of saddle points, the theory integrates cooperation and competition constraints to determine optimal outcomes. Differential games are typically analyzed in two key stages: first, differential equations are used to describe the game dynamics and simulate real-world decision-making strategies; second, the players, guided by their individual objective functions, control the system by selecting strategies that maximize their benefits, thereby influencing the evolution of the system. The state equations of the players are used to express the differential game problem, which is specifically represented as follows:

$$\dot{x}(t) = f(x(t), u(x), v(x), t), x(t_0) = x_0 . \quad (1)$$

Where x_0 is the initial state; x, u, and v are the tolerance sets; and f is the state equation.

Each side is independent of the other, minimizing the status indicators:

$$J_1(u,v) = h_1(x(t_f), t_f) + \int_{t_0}^{t_f} g_1(x(t), u(x), v(x), t)dt , \quad (2)$$

$$J_2(u,v) = h_2(x(t_f), t_f) + \int_{t_0}^{t_f} g_2(x(t), u(x), v(x), t)dt . \quad (3)$$

Where for any x_0, there exists a unique solution, that is,

$$u^* = \arg\min_{u \in U} \{q_1^T \cdot f(x, u, v^*, t) + g_1(x, u, v^*, t)\} , \quad (4)$$

$$v^* = \arg\min_{v \in U} \{q_2^T \cdot f(x, u, v^*, t) + g_2(x, u, v^*, t)\} . \quad (5)$$

(3) Contract theory

A supply chain contract is a mechanism designed to strengthen trust among supply chain participants, ensuring that the individual rationality of each party aligns with the overall optimal performance of the supply chain system. A well-structured supply chain contract promotes deeper cooperation among members, facilitates the sharing of benefits and risks, reduces the overall business risk, and leads to a

Pareto improvement in the profits of all participants. Common contract types used in supply chain management include volume discount contracts, cost-sharing contracts, wholesale price contracts, revenue-sharing contracts, and repurchase contracts.

(1) Revenue-sharing contract: A revenue-sharing contract in a supply chain involves an agreement where participants share a portion of the revenue through negotiation. Specifically, the supplier agrees to offer the retailer a lower wholesale price, and in return, the retailer shares a percentage of its revenue with the supplier upon product sale. This arrangement compensates the supplier for the reduced price and aims to align the supply chain's operations, ultimately improving profitability.

(2) Cost-sharing contract: Similar to a benefit-sharing contract, a cost-sharing contract entails an agreement where supply chain participants collectively bear a proportion of the costs. This arrangement is negotiated to ensure fair distribution of financial burdens among parties

(3) Volume discount contract: A volume discount contract is an incentive-based agreement between a supplier and retailer, where the supplier offers price discounts based on the retailer's order quantity. This encourages larger orders from the retailer, ultimately benefiting both parties.

(4) Wholesale price contract: In a wholesale price contract, the supplier sets the price based on market conditions, and the retailer determines the optimal order quantity considering the wholesale price and market demand. However, under this contract, the retailer bears the risk of stockouts or slow-moving products due to market fluctuations, which can lead to an unstable profit margin.

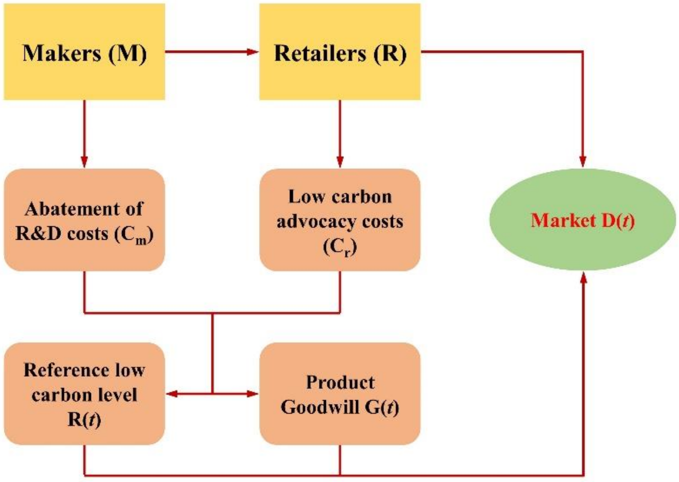
(5) Repurchase contract: A repurchase contract is an agreement in which the supplier agrees to repurchase unsold products from the retailer at the end of the selling season, typically at a price higher than a specified residual value. This arrangement encourages the retailer to order larger quantities. The profit transferred under a repurchase contract is the supplier's profit under the wholesale price contract minus the repurchase value of the unsold products.

Building on previous research, this study combined the benefit-sharing and cost-sharing contracts to address the coordination issues in a green supply chain while considering the effects of reputation and reference low-carbon levels. We proposed a two-way cost-sharing-benefit-sharing contract model and explored its positive incentive effect on green supply chain operations.

2. Problem Description and Model Assumptions

2.1 Problem Description

This study examined a two-tier supply chain system consisting of a single manufacturer and a retailer. The manufacturer focuses on energy-saving and emission reduction initiatives, whereas the retailer concentrates on advertising to establish a collaborative emission reduction model within the supply chain (see *Figure 1*).



Source: created by the authors.

Figure 1. **Structural Diagram of Supply Chain Emission Reduction Activities**

Table 1. Structural diagram of supply chain emission reduction activities

Symbol	Explanation
M, *R*	Manufacturers and retailers
R(t), *G*(t)	Consumer reference low-carbon level effects and goodwill per unit of product at moment t
R(0), *G*(0)	Initial reference low-carbon level effect and initial goodwill, where R(0) = R0, G(0) = G0
E(*t*), *A*(*t*)	Manufacturers' emission reduction efforts and retailers' advertising efforts at moment t
Symbol	Explanation
α, *β*	Coefficients on the effects of manufacturers' abatement efforts and retailers' advertising efforts on product goodwill, α > 0, β > 0
δ	Natural rate of decline of product goodwill, δ > 0
cs	Manufacturers' production costs
D(t)	Moment of product market demand
p(t)	Retail price of the product at moment t
w(t)	Wholesale price of the product at moment t
a	Total market demand for the product, a > 0
b	Price sensitivity to demand, b > 0
ξm, *ξr*	Manufacturers' cost factors for abatement actions and retailers' cost factors for advertising campaigns
η, *γ*	Consumer reference to low-carbon level effect sensitivity and brand preference
Jm, *Jr*, *Js*	Long-term profitability for manufacturers, retailers, and the supply chain system
α, *β*	Coefficients on the effects of manufacturers' abatement efforts and retailers' advertising efforts on product goodwill, *α* > 0, *β* > 0
δ	Natural rate of decline of product goodwill, *δ* > 0
c_s	Manufacturers' production costs
D(*t*)	Moment of product market demand
p(*t*)	Retail price of the product at moment *t*
w(*t*)	Wholesale price of the product at moment *t*
a	Total market demand for the product, *a* > 0
b	Price sensitivity to demand, *b* > 0
ξ_m, ξ_r	Manufacturers' cost factors for abatement actions and retailers' cost factors for advertising campaigns
η, *γ*	Consumer reference to low-carbon level effect sensitivity and brand preference
J_m, J_r, J_s	Long-term profitability for manufacturers, retailers, and the supply chain system

Source: created by the authors.

In this model, the manufacturer acts as the leader, responsible for producing the products, whereas the retailer, as the follower, facilitates the distribution of goods to the market. The manufacturer's emission reduction efforts and the retailer's advertising activities influence the consumer reference low-carbon level effect and product reputation, which, in turn, affect market demand. A summary of the model's key parameters and decision variables is presented in *Table 1*.

2.2 Model Assumptions

Assumption 1: According to Reference (Yu *et al.,* 2020), the emission reduction R&D cost of the manufacturer and the low-carbon advertising cost of the retailer, denoted as C_m and C_r, respectively, are increasing functions of the emission reduction effort and advertising effort. Thus, at time *t*, the emission reduction R&D cost of the manufacturer and the low-carbon advertising cost of the retailer are given by:

$$C_{\mathrm{m}} = \frac{1}{2}\xi_{\mathrm{m}}\mathrm{E}(t)^2, C_{\mathrm{r}} = \frac{1}{2}\xi_{\mathrm{r}}\mathrm{A}(t)^2 \quad . \quad (6)$$

Where $\xi_m > 0$ and $\xi_r > 0$ denote the cost of emission reduction actions by manufacturers and the cost of low-carbon publicity by retailers, respectively. Thus, in this context, ξ_m and ξ_r tend to take larger values.

Assumption 2: The consumer reference low-carbon level effect is defined as the reference point of the product's low-carbon level formed by the consumer over the long-term purchasing activities (Yuan *et al.,* 2018). Specifically, it is represented by the weighted average of the product's past low-carbon levels, which is influenced by the manufacturer's emission reduction efforts and the retailer's advertising efforts, as shown below:

$$\mathrm{R}(t) = \varepsilon\int_0^t e^{\varepsilon(s-t)}\left[\mathrm{E}(t)+\mathrm{A}(t)\right]dt \quad . \quad (7)$$

By taking the derivative of the above equation and applying Leibniz's law, the following equation is obtained:

$$\dot{\mathrm{R}}(t) = \varepsilon\left[\mathrm{E}(t)+\mathrm{A}(t)-\mathrm{R}(t)\right], \mathrm{R}(0) = \mathrm{R}_0 \quad . \quad (8)$$

Where $\varepsilon > 0$ is the memory parameter, indicating that a larger value implies a more short-lived consumer memory of the product's low-carbon level at moment *s*. The value of $\varepsilon > 0$ is the memory parameter.

Assumption 3: The emission reduction efforts of the manufacturer and the advertising efforts of the retailer have a positive effect on the product's reputation. In this study, we referred to the model established by Pietro (2011) and assumed that the manufacturer's emission reduction efforts and the retailer's advertising efforts positively affect the product's reputation, as shown as follows:

$$\dot{\mathrm{G}}(t) = \alpha\left[\mathrm{E}(t)+\beta\mathrm{A}(t)-\delta\mathrm{G}(t)\right], \mathrm{G}(0) = \mathrm{G}_0 \quad . \quad (9)$$

Assumption 4: Price and nonprice factors exhibit a linear relationship with market demand. Through a separable multiplicative form, these factors influence the market demand D(*t*), as expressed below:

$$\mathrm{D}(t) = \mathrm{h}(t)f(R,G) \quad . \quad (10)$$

Where h(*t*) = (a − bp) denotes the price factor, and *f* (R, G) = (*η*R(*t*) + *γ*G(*t*)) denotes the nonprice factor. To ensure a positive demand for the product, the retail price *p* ranges from [0, a/b], and the wholesale price w(*t*) is not usually higher than the retail price in the supply chain; thus, there exists 0 < w(*t*) < p(*t*) < *a*/*b*.

Assumption 5: At any given time, the profit discount factor *λ* is the same for the entire supply chain, and *λ* > 0. The objective functions for the manufacturer, retailer, and the entire supply chain system are represented as follows:

$$\max_{\mathrm{W,E}} \mathrm{J_m} = \int_0^\infty e^{-\lambda t}\left(\left(\mathrm{w}-\mathrm{c_s}\right)\mathrm{D}(t)-\frac{1}{2}\xi_{\mathrm{m}}\mathrm{E}(t)^2\right)dt \quad , \qquad (11)$$

$$\max_{\mathrm{P,A}} \mathrm{J}_r = \int_0^\infty e^{-\lambda t}\left(\left(\mathrm{P}-\mathrm{w}\right)\mathrm{D}(t)-\frac{1}{2}\xi_{\mathrm{r}}\mathrm{A}(t)^2\right)dt \quad , \qquad (12)$$

$$\max_{\mathrm{P,E,A}} \mathrm{J}_s = \int_0^\infty e^{-\lambda t}\left(\left(\mathrm{P}-\mathrm{c_s}\right)\mathrm{D}(t)-\frac{1}{2}\xi_{\mathrm{m}}\mathrm{E}(t)^2-\frac{1}{2}\xi_{\mathrm{r}}\mathrm{A}(t)^2\right)dt \quad . \qquad (13)$$

Where the manufacturer's profit consists of product wholesale revenue, production costs, and **e**mission reduction effort costs; the retailer's profit is composed of product sales revenue, wholesale costs, and low-carbon advertising costs; the overall profit of the supply chain system includes product sales revenue, production costs, emission reduction effort costs, and low-carbon advertising costs.

3. Green Supply Chain Decision Analysis and Contract Coordination Analysis

3.1 Green Supply Chain Decision Analysis

Building on previous research assumptions and explanations, this study applied a differential game model to analyze the coordination of emission reduction strategies between manufacturers and retailers in their long-term pursuit of low-carbon emission reduction. The primary focus is on centralized decision making, where the manufacturer and the retailer collaborate to reduce emissions, with the overall supply chain serving as the standard and the maximization of total profit as the objective. The superscript "C" denotes centralized decision making, and to simplify the notation, the time variable t is omitted in the subsequent derivations.

In the centralized system (denoted by the superscript C), the manufacturer and the retailer act as a unified system. By setting the product's retail price P, advertising effort E, and emission reduction effort A, the goal is to maximize the total profit of the supply chain system. The corresponding optimization problem becomes a standard optimal control problem. Under centralized decision making, the manufacturer's optimal emission reduction effort, the retailer's optimal retail price, and the optimal advertising effort are derived respectively as follows:

$$P^C = \frac{a+bc_s}{2b} \quad , \qquad (14)$$

$$E^C = \frac{(a-bc_s)^2}{4b\xi_m}\left(\frac{\varepsilon\eta}{\lambda+\varepsilon}+\frac{\alpha\gamma}{\lambda+\delta}\right) \quad , \qquad (15)$$

$$A^{C}=\frac{(a-bc_{s})^{2}}{4b\xi_{r}}\left(\frac{\varepsilon\eta}{\lambda+\varepsilon}+\frac{\beta\gamma}{\lambda+\delta}\right). \quad (16)$$

By using the optimal control method, after moment t, the expression for the optimal value function of long-term profit is noted as

$$J_{s}^{C}(R,G)=e^{-\lambda t}V_{s}^{C}(R,G). \quad (17)$$

$V_s^C(R, G)$ satisfies the Hamilton–Jacobi–Bellman (HJB) equation for any R ≥ 0 and G ≥ 0, as shown as follows:

$$\lambda V_{s}^{C}(R,G)=\max\begin{bmatrix}(p-c_{s})D-\frac{1}{2}\xi_{m}E^{2}-\frac{1}{2}\xi_{r}A^{2}+V_{sR}^{C'}(\varepsilon(E+A-R))\\+V_{sG}^{C'}(\alpha E+\beta A-\delta G)\end{bmatrix}. \quad (18)$$

The first-order partial derivatives of Eq. (18) with respect to P, E, and A, set to 0, can be obtained as follows:

$$P=\frac{a+bc_{s}}{2b},E=\frac{\varepsilon V_{sR}^{C'}+\alpha V_{sG}^{C'}}{\xi_{m}},A=\frac{\varepsilon V_{sR}^{C'}+\beta V_{sG}^{C'}}{\xi_{r}}. \quad (19)$$

Substituting Eq. (19) into Eq. (18) and further organizing yield:

$$\lambda V_{s}^{C}(R,G)=(\frac{(a-bc_{s})^{2}}{4b}\eta-\varepsilon V_{sR}^{C'})R+(\frac{(a-bc_{s})^{2}}{4b}\gamma-\delta V_{sG}^{C'})G$$
$$+\frac{\left(\varepsilon V_{sR}^{C'}+\alpha V_{sG}^{C'}\right)^{2}}{2\xi_{m}}+\frac{\left(\varepsilon V_{sR}^{C'}+\beta V_{sG}^{C'}\right)^{2}}{2\xi_{r}}. \quad (20)$$

From the order characterization of the differential equation (Eq. (20)), a conjecture arises that the linear equation on R, G is a solution of the above HJB equation, such that

$$V_{s}^{C}(R,G)=s_{1}R+s_{2}G+s_{3}. \quad (21)$$

where s_1, s_2, and s_3 are constants, which can be obtained by substituting Eq. (21) into Eq. (20), as shown as follows:

$$S_{1}=\frac{\eta(a-bc_{s})^{2}}{4b(\lambda+\varepsilon)}=V_{sR}^{c'},\quad S_{2}=\frac{\gamma(a-bc_{s})^{2}}{4b(\lambda+\delta)}=V_{sG}^{c'},$$
$$S_{3}=\frac{1}{2\xi_{m}}\left(\frac{(a-bc_{s})^{2}}{4b}\right)^{2}\left(\frac{\varepsilon\eta}{\lambda+\varepsilon}+\frac{\alpha\gamma}{\lambda+\delta\lambda}\right)^{2}$$
$$+\frac{1}{2\xi_{r}}\left(\frac{(a-bc_{s})^{2}}{4b}\right)^{2}\left(\frac{\varepsilon\eta}{\lambda+\varepsilon}+\frac{\beta\gamma}{\lambda+\delta}\right)^{2}. \quad (22)$$

Finally, the optimal solution for P, E, and A can be found by substituting Eq. (22) into Eq. (19).

3.2 Contract Coordination Analysis of Green Supply Chain

Centralized decision making requires the entire green supply chain to adopt a unified cooperative strategy. However, in practice, manufacturers and retailers are independent economic entities, making it difficult to mandate that the entire green supply chain operate as a single unit under centralized decision making. Therefore, a cooperation and coordination model must be established among the members of the green supply chain. In this model, each member aims to maximize its own benefit while ensuring that its profit is at least equal to that under decentralized decision making. This incentive motivates each entity to actively participate in the cooperative coordination model. To achieve this, we analyzed the coordination between supply chain enterprises under the two-way cost-sharing contract.

The two-way cost-sharing contract is a pre-agreed contract model. Under this contract, the profits for the retailer and the manufacturer are as follows:

$$\max_{\mathrm{W,E}} \mathrm{J_m^{D1}} = \int_0^{\infty} e^{-\lambda t}\left(\left(\mathrm{w}-\mathrm{c_s}\right)\mathrm{D}(t)-\frac{1}{2}\theta\xi_{\mathrm{m}}\mathrm{E}(t)^2-\frac{1}{2}\tau\xi_{\mathrm{r}}\mathrm{A}(t)^2\right)dt, \quad (23)$$

$$\max_{\mathrm{P,A}} \mathrm{J_r^{D1}} = \int_0^{\infty} e^{-\lambda t}\begin{pmatrix}\left(\mathrm{P\text{-}w}\right)\mathrm{D}(t)-\frac{1}{2}(1-\theta)\xi_{\mathrm{m}}\mathrm{E}(t)^2\\ -\frac{1}{2}(1-\tau)\xi_{\mathrm{r}}\mathrm{A}(t)^2\end{pmatrix}dt. \quad (24)$$

The manufacturer's optimal abatement effort, the retailer's optimal retail price, and the optimal advertising effort are respectively specified as follows:

$$A^{D*} = \frac{\left(\varepsilon\eta(\delta+\lambda)+\beta\gamma(\varepsilon+\lambda)\right)(a-bw)^2}{4b(\delta+\lambda)(\varepsilon+\lambda)\xi_r(1-\tau)}, \quad (25)$$

$$E^{D*} = \frac{\left(\varepsilon\eta(\delta+\lambda)+\alpha\gamma(\varepsilon+\lambda)\right)(a-bw)^2}{4b(1-\theta)(\delta+\lambda)(\varepsilon+\lambda)\xi_m}, \quad (26)$$

$$P^{D*} = \frac{a+bw}{2b}. \quad (27)$$

By using the optimal control method, the expression for the optimal value of long-term profit after moment *t* can be derived as follows:

$$J_m^{D^*}(R,G) = e^{-\lambda t}V_m^{D^*}(R,G), \quad (28)$$

$$J_r^{D^*}(R,G) = e^{-\lambda t}V_r^{D^*}(R,G). \quad (29)$$

V_m^{D*}(R, G) and V_r^{D*}(R, G) satisfy the HJB equation for any R ≥ 0 and G ≥ 0, as shown as follows:

$$\lambda V_m^{D^*}(R,G) = \max\left[\begin{array}{l}(w-c_s)D - \frac{1}{2}\xi_m E^2 - \frac{1}{2}\tau\xi_r A^2 \\ +V_{mR}^{D^*\prime}(\varepsilon(E+A-R)) + V_{mG}^{D^*\prime}(\alpha E + \beta A - \delta G)\end{array}\right], \quad (30)$$

$$\lambda V_r^{D}(R,G) = \max\left[\begin{array}{l}(p-w)D - \frac{1}{2}(1-\theta)\xi_m E^2 - \frac{1}{2}(1-\tau)\xi_r A^2 \\ +V_{rR}^{D\prime}(\varepsilon(E+A-R)) + V_{rG}^{D\prime}(\alpha E + \beta A - \delta G)\end{array}\right]. \quad (31)$$

Taking the first-order derivative of Eq. (31) with respect to P and A and setting it to 0 yields:

$$P = \frac{a+bw}{2b}, A = \frac{\varepsilon V_{rR}^{D\prime} + \beta V_{rG}^{D\prime}}{\xi_r(1-\tau)}. \quad (32)$$

Substituting Eq. (32) into Eq. (30) and taking the partial derivative of Eq. (30) with respect to E and making it equal to 0 yields:

$$E = \frac{\varepsilon V_{rR}^{D\prime} + \alpha V_{rG}^{D\prime}}{\xi_m(1-\theta)}. \quad (33)$$

Furthermore, the corresponding differential equations can be derived by substituting Eqs. (32) and (33) into Eqs. (30) and (31). Based on the order characteristics of these differential equations, a linear equation about R and G can be derived, which serves as the solution to the above HJB equation. Let:

$$V_m^{D^*}(R,G) = m_4 R + m_5 G + m_6, \quad (34)$$

$$V_r^{D^*}(R,G) = r_4 R + r_5 G + r_6. \quad (35)$$

Where m_4, m_5, m_6, r_4, r_5, and r_6 are constants. Substituting them into Eqs. (32) and (33) yields the manufacturer's optimal abatement effort, the retailer's optimal retail price, and the optimal advertising campaign.

The coordinated manufacturer's emission reduction efforts and advertising efforts need to be optimal under centralized decision making, that is, $A^{D*} = A^C$ and $E^{D*} = E^C$. Thus,

$$\frac{\left(\varepsilon\eta(\delta+\lambda)+\beta\gamma(\varepsilon+\lambda)\right)(a-bw)^2}{4b(\delta+\lambda)(\varepsilon+\lambda)\xi_r(1-\tau)} = \frac{(a-bc_s)^2}{4b\xi_r}\left(\frac{\varepsilon\eta}{\lambda+\varepsilon}+\frac{\beta\gamma}{\lambda+\delta}\right), \quad (36)$$

$$\frac{\left(\varepsilon\eta(\delta+\lambda)+\alpha\gamma(\varepsilon+\lambda)\right)(a-bw)^2}{4b(1-\theta)(\delta+\lambda)(\varepsilon+\lambda)\xi_m} = \frac{(a-bc_s)^2}{4b\xi_m}\left(\frac{\varepsilon\eta}{\lambda+\varepsilon}+\frac{\alpha\gamma}{\lambda+\delta}\right). \quad (37)$$

The joint solution is obtained as follows:

$$\theta = \tau = 1 - \frac{(a-bw)^2}{(a-bc_s)^2}, \quad (38)$$

When Eq. (38) is satisfied, the green supply chain under the two-way cost-sharing contract reaches coordination. At this point, the manufacturer's reduction efforts and advertising efforts in the supply chain must achieve the optimal values as under centralized decision making.

In addition, the optimal trajectories of the consumer reference low-carbon level effect and product goodwill are respectively:

$$R^D(t) = \left(R_0 - R^D{}_\infty\right)e^{-\varepsilon t} + R^D{}_\infty, \quad (39)$$

$$G^D(t) = \left(G_0 - G^D{}_\infty\right)e^{-\delta t} + G^D{}_\infty. \quad (40)$$

Particularly,

$$R^D_\infty = \frac{\left(\varepsilon\eta(\delta+\lambda)+\alpha\gamma(\varepsilon+\lambda)\right)(a-bw)^2}{4b(1-\theta)(\delta+\lambda)(\varepsilon+\lambda)\xi_m} + \frac{\left(\varepsilon\eta(\delta+\lambda)+\beta\gamma(\varepsilon+\lambda)\right)(a-bw)^2}{4b(\delta+\lambda)(\varepsilon+\lambda)\xi r(1-\tau)}, \quad (41)$$

$$G^D_\infty = \frac{\alpha\left(\varepsilon\eta(\delta+\lambda)+\alpha\gamma(\varepsilon+\lambda)\right)(a-bw)^2}{4b(1-\theta)(\delta+\lambda)(\varepsilon+\lambda)\xi_m} + \frac{\beta\left(\varepsilon\eta(\delta+\lambda)+\beta\gamma(\varepsilon+\lambda)\right)(a-bw)^2}{4b(\delta+\lambda)(\varepsilon+\lambda)\xi r(1-\tau)}. \quad (42)$$

Under the two-way cost-sharing contract, the profit optimization functions of the manufacturer and the retailer are respectively,

$$J_{\mathrm{m}}{}^{D1} = m_4R_0 + m_5G_0 + m_6, \quad (43)$$

$$J_r{}^{D1} = r_4R_0 + r_5G_0 + r_6. \quad (44)$$

By substituting Eqs. (10) and (25)-(27) into Eq. (23) and Eq. (24) and performing the necessary calculations, the optimal profit functions for the manufacturer and the retailer under the two-way cost-sharing contract can be obtained.

4. Example and Sensitivity Analysis

4.1 Example and Sensitivity Analysis Under Benchmark Game

(1) Example analysis under benchmark game model

In the benchmark model of a green supply chain that considers product reputation and reference low-carbon level effects, specific values were assigned to various parameters. These values are derived from previous research findings (Zu *et al.,* 2017) and serve as the basis for the numerical simulation and analysis of the model. *Table 2* summarizes the parameter values used in the analysis and the related conclusions.

Table 2. Supply chain parameter settings

Symbol	a	b	η	γ	ε	α	β	λ	δ	ξ_m	ξ_r	ϕ	θ	τ
Value	12	2	0.75	0.5	2	0.5	1	0.5	1	1	1	0.2	0.8	0.2

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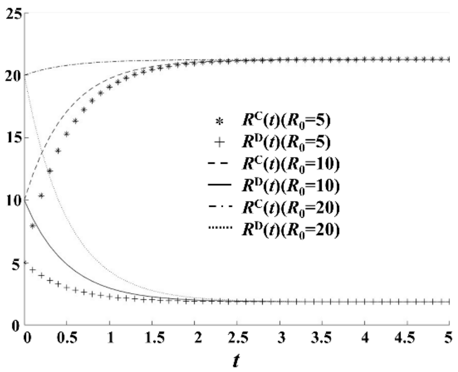
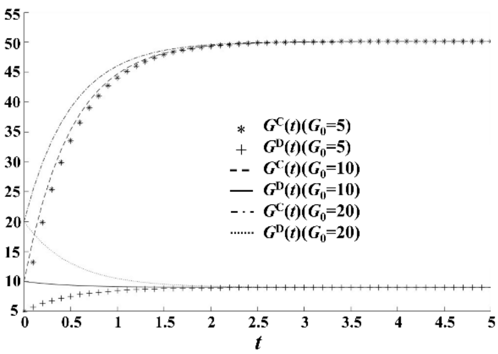


Source: created by the authors.

Figure 2. **Optimal Path of the Reference Low-Carbon Level Effect over Time under Different Initial Values**



Source: created by the authors.

Figure 3. **Optimal Path of Low-Carbon Reputation over Time under Different Initial Values**

Figure 2 illustrates the optimal trajectory of the reference low-carbon level effect over time under varying initial values. The stable value of the reference low-carbon level effect under decentralized decision making is lower than that under centralized decision making, aligning with the conclusion presented in Proposition 3. Moreover, the initial value of the reference low-carbon level effect influences its trajectory over time. Specifically, when the initial value is low, the reference low-carbon level effect increases over time, whereas, for higher initial values, the effect decreases. Despite these variations, the reference low-carbon level effect ultimately stabilizes at the same level after a certain period, regardless of the decision-making scenario.

Figure 3 presents the optimal trajectories of product reputation over time for various initial values. Similar to the findings for the reference low-carbon level effect, under decentralized decision making, the stable value of product reputation is lower than that observed under centralized decision making. Specifically, under decentralized decision making, when R_0 = 5, product reputation increases monotonically over time. However, when R_0 = 10 or R_0 = 20, product reputation decreases monotonically over time. In both cases, the product reputation eventually stabilizes at a certain value. Conversely, under centralized decision making, regardless of whether R_0 = 5, 10, or 20, product reputation increases monotonically over time, ultimately approaching a stable value. These results suggest that although differences in the initial values of the reference low-carbon level effect and product reputation influence their respective trajectories over time, both effects ultimately reach equilibrium. Therefore, under decentralized and centralized decision making, the optimal reference low-carbon level effect and product reputation become time-independent and are primarily influenced by the manufacturer's low-carbon R&D costs and the retailer's advertising costs.

(2) Sensitivity analysis of various reference coefficients

To analyze the influence of product reputation and consumer low-carbon reference effects on the optimal decision making of the green supply chain, a sensitivity analysis was conducted on their respective coefficients. Figure 4 illustrates that under decentralized decision making, the manufacturer's and the retailer's profits increase with the rising influence coefficient, with the retailer's profit exhibiting a faster growth rate. This indicates that the retailer's profit is more sensitive to the influence coefficient. *Figures 5–7* present the sensitivity analysis of the influence coefficients on the manufacturer's low-carbon reduction effort, the retailer's advertising effort, and the overall supply chain profit under various decision-making scenarios. As shown in these figures, whether the decision making is centralized or decentralized, the low-carbon reduction effort, advertising effort, and overall supply chain profit all increase with the low-carbon reference coefficient (η) and product reputation coefficient (γ). This observation is consistent with Proposition 2. The reason for this trend is that as η and γ increase, the unit product reputation and the consumer low-carbon reference effect generate higher market demand, thereby increasing the overall supply chain profit.

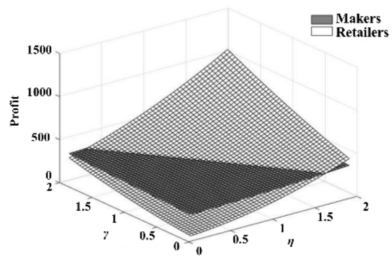


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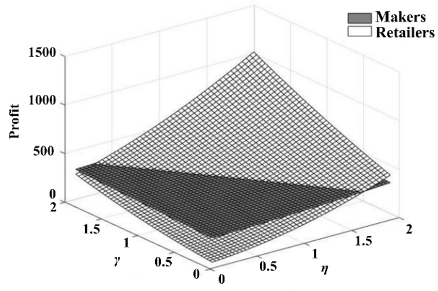
Figure 4. **Manufacturer and Retailer Profits under Decentralized Decision Making**



Source: created by the authors.

Figure 5. **Manufacturers' Low-Carbon Emission Reduction Efforts under Different Decision-Making Scenarios**

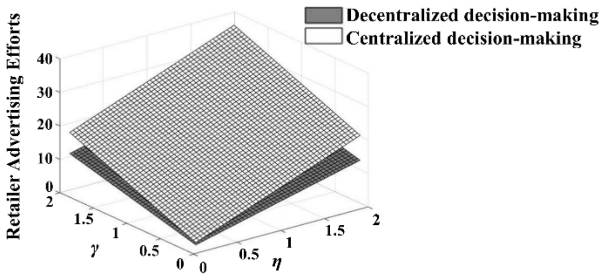


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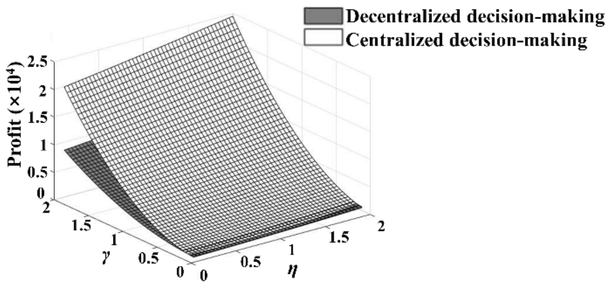
Figure 6. **Retailer Advertising Efforts under Different Decisions**

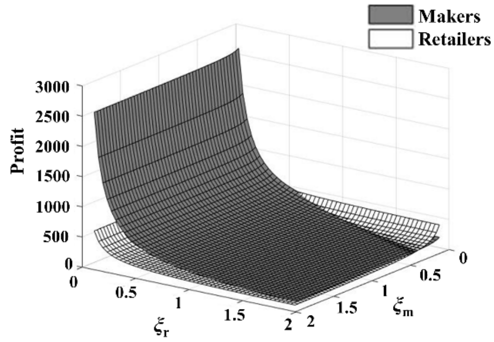


Source: created by the authors.

Figure 7. **Total Profit of Green Supply Chain under Different Decisions**

In addition, under centralized decision making, the optimal low-carbon reduction effort, advertising effort, and overall supply chain profit are all higher than those under decentralized decision making. As the influence parameters grow, the advantages of centralized decision making become more evident. However, when γ approaches 0 (indicating no consumer brand preference), profits under decentralized decision making surpass those under centralized decision making. This scenario, however, does not align with real-world conditions and should be excluded from consideration.

A sensitivity analysis was conducted on the low-carbon investment coefficient and the advertising investment coefficient to examine their effects on the profits of the manufacturer and the retailer under decentralized decision making, as well as the total profit of the green supply chain under different decision-making scenarios. The results are presented in Figures 8 and 9. As shown in Figure 8, under decentralized decision making, the manufacturer's and retailer's profits decrease as the investment coefficients increase. In addition, ξ_r has a greater effect on profits compared to ξ_m. Furthermore, under decentralized decision making, the manufacturer's profit is generally higher, except when ξ_m is low and ξ_r is high, where the retailer's profit exceeds that of the manufacturer. This occurs because the effectiveness of each unit of advertising investment in promoting low-carbon efforts increases with the retailer's advertising cost coefficient, leading to higher profits. Similarly, the effectiveness of each unit of reduction effort decreases with the manufacturer's reduction cost coefficient, resulting in lower profits. Thus, the manufacturer's and retailer's profits move in opposite directions, leading to a situation where the retailer's profit becomes higher.



Source: created by the authors.

Figure 8. **Manufacturer and Retailer Profits under Decentralized Decision Making**

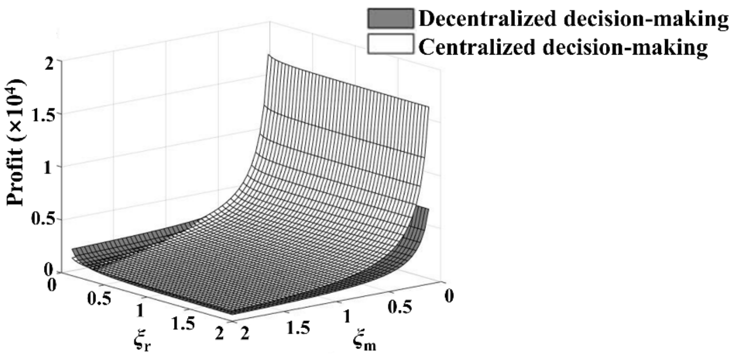


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Figure 9. **Total Profit of Green Supply Chain under Different Decisions**

As shown in *Figure 9*, similar to the conclusion in *Figure 8*, the overall profit of the supply chain under different decision-making scenarios decreases as the investment coefficients increase. Compared to ξ_r, ξ_m has a greater effect on profit. In addition, under centralized decision making, the profit is generally higher than under decentralized decision making, with the exception being when ξ_r is low and ξ_m is high, in which case the profit under decentralized decision making exceeds that of centralized decision making. Therefore, the overall profit of the supply chain under centralized decision making is not always higher than the corresponding value under decentralized decision making. The relative profit levels in both decision-making scenarios depend on the extent of low-carbon investment and advertising investment, and the optimal decision strategy for maximizing overall profit varies based on different parameter ranges.

4.2 Examples and Sensitivity Analysis Under Contractual Games

(1) Two-way cost-sharing and benefit-sharing contract parameter example analysis

This section aims to further validate the conclusions drawn. The parameters involved are listed in *Table 2*. By substituting the corresponding values of these parameters into the supply chain coordination model under the two-way cost-sharing and benefit-sharing contract, the following functions are derived:

$$E^{D*} = \frac{1.5333(1-\varphi)}{1-\theta}, \quad (45)$$

$$J_m d_3 = (25.2+\varphi)R_0 + (1.333+3.333\varphi)G_0 + \frac{25.74+42.42\varphi-68.16\varphi^2+\theta^2\left(16.333+28.311\varphi-44.64\varphi^2\right)+\theta(-44-66\varphi+110\varphi^2)}{\left(-1+\theta\right)^2}, \quad (46)$$

$$J_r d_3 = 0.6(1-\varphi)R_0 + \frac{2}{3}(1-\varphi)G_0 + \frac{(6.7-4.4\theta)(1-\varphi)^2}{1-\theta}. \quad (47)$$

By combining the Pareto improvement analysis, we could derive: $0.2465 < \theta = \tau < 0.3663$, and $0.1456 < \phi < 0.8672$.

The above formulas indicate that the consumer reference low-carbon level effect, product reputation, and the profits of the manufacturer and the retailer all increase with the manufacturer's cost-sharing ratio for emission reduction effort (θ) and advertising effort (τ). Conversely, they decrease as the proportion of sales revenue allocated to the manufacturer by the retailer (ϕ) increases.

According to the value intervals of θ, τ, and ϕ, we set $\theta = \tau = 0.3$, and $\phi = 0.2$. *Table 3* shows the before-and-after comparison results of the related parameters.

Table 3. Before-and-after comparison of the parameters related to contractual harmonization

Decision-making variables	**Pre-covenant harmonization**		**Post-covenant harmonization**
	Decentralized decision making (D)	**Centralized decision making (C)**	**Decentralized decision making (D3)**
R	3.133	13.6	13.6
G	3.4	10.53	10.53
E	3.067	6.133	6.133
Q	1.867	7.467	7.467
D	8.1	30.93	30.93
J_m	36	/	50
J_r	9.27	/	26
J_s	45.27	76	76

Source: created by the authors.

As shown in *Table 3*, contract coordination leads to significant improvements in key indicators, including the consumer reference low-carbon level effect, product low-carbon reputation, manufacturer's emission reduction effort, and retailer's advertising effort. These indicators align with the optimal outcomes under centralized decision making. Moreover, the profits of the manufacturer and the retailer have substantially increased, resulting in the overall supply chain profit matching the centralized decision-making scenario, thereby achieving supply chain profit optimization. This further validates the feasibility of the model.

(2) Two-way cost-sharing and revenue-sharing contract parameter sensitivity analysis

Based on the parameter settings, the value range of $\theta = \tau$ is obtained. Within this range, the influence of $\theta = \tau$ on the profits of supply chain entities was analyzed, as shown in *Figure 10*.

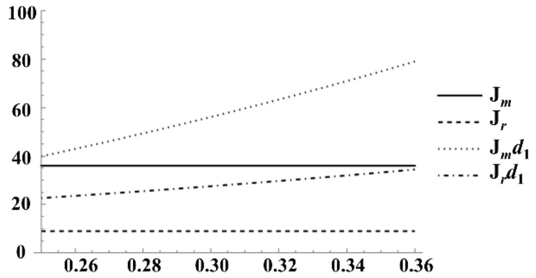

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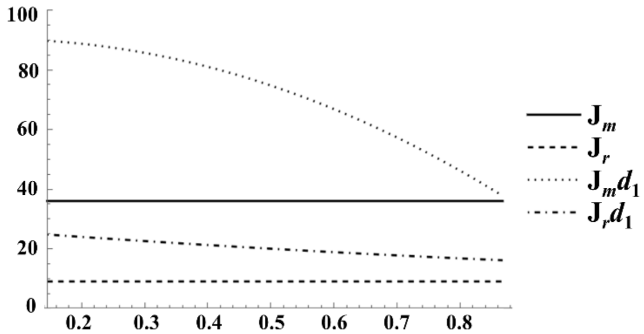
Figure 10. **Influence of θ on the Profits of the Manufacturer and the Retailer before and after Contract Coordination**

As shown in *Figure 10*, when $0.2465 < \theta = \tau < 0.3663$, the profits of the manufacturer and the retailer after coordination show an increasing trend as the value of θ increases, with higher profit values observed after coordination.



Source: created by the authors.

Figure 11. **Influence of ϕ on the Profits of the Manufacturer and the Retailer before and after Contract Coordination**

As shown in *Figure 11*, when $0.1456 < \phi < 0.8672$, after coordination, the profits of the two supply chain entities gradually decrease as the value of ϕ increases. When ϕ reaches its maximum value, the profits of the manufacturer before and after coordination become equal.

5. Discussion

This study examines green supply chain decision-making and contract coordination by integrating dynamic optimization and contract theory with environmental and consumer behavior considerations. It begins by defining key concepts, including green supply chains, low-carbon reputation, and supply chain coordination. Based on these foundations, a differential game model is developed to capture the dynamic interactions between a manufacturer—focused on emission reduction—and a retailer—dedicated to low-carbon advertising. Initially, a centralized decision-making framework is formulated, where both parties operate as a single entity to maximize overall supply chain profit. Using optimal control techniques, the study derives optimal strategies for emission reduction, pricing, and advertising by solving the Hamilton–Jacobi–Bellman equations, establishing baseline trajectories for the consumer reference low-carbon level and product goodwill. Recognizing that real-world supply chains consist of independent entities, the study introduces a two-way cost-sharing contract to facilitate decentralized coordination. This contract ensures that cooperative profits exceed those from non-cooperation. Analytical results demonstrate that when cost-sharing parameters meet certain conditions, decentralized decisions align with centralized optimal outcomes, mitigating the double marginalization problem common in supply chain operations. Additionally, the analysis underscores the significance of the consumer's reference low-carbon level, which, along with product reputation, influences market demand and shapes the strategic decisions of both manufacturers and retailers. The findings suggest that targeted emission reduction and advertising efforts not only enhance the supply chain's green performance but also strengthen its competitive positioning in an environmentally conscious market.

In summary, this study provides insights into how contractual coordination mechanisms can bridge the gap between decentralized decision-making and centralized optimal performance in green supply chains. While contributing to the theoretical advancement of sustainable supply chain management, further empirical validation and model extensions – such as multi-tier supply chains or competitive market structures – are recommended for future research.

Conclusions

This study examines the dynamic decision-making problem in a green supply chain by integrating product reputation and consumer reference low-carbon level effects. By adopting a dynamic perspective and applying DGT, the study explores optimal emission reduction strategies, advertising strategies, and profit maximization. Retail price, retailer advertising effort, and manufacturer emission reduction effort are considered decision variables, whereas consumer reputation and reference low-carbon level effects serve as state variables under decentralized and centralized decision making. The key conclusions are as follows:

(1) Under decentralized and centralized decision making, the manufacturer's optimal emission reduction effort and the retailer's optimal advertising effort exhibit a positive correlation with consumer sensitivity to low-carbon reference effects and brand preference. Conversely, these efforts are negatively correlated with the natural decay rate of product reputation, memory parameters, profit discount rate, manufacturer's emission reduction cost coefficient, and retailer's advertising cost coefficient. Notably, the levels of consumer low-carbon reference effect and product reputation remain time-independent, as they are primarily influenced by the manufacturer's low-carbon R&D expenditure and the retailer's advertising investment. In the decentralized decision-making scenario, the manufacturer's and retailer's profits increase with higher consumer reference sensitivity and brand preference, with the retailer's profit displaying greater sensitivity to these factors.

(2) Compared to decentralized decision making, centralized decision making results in higher optimal emission reduction efforts, advertising efforts, retail prices, low-carbon reference effects, and product reputation. As the influence of key parameters intensifies, the benefits of centralized decision making become more pronounced. However, the overall supply chain profit under centralized decision making does not always surpass that under decentralized decision making. The relative profitability of both strategies depends on the magnitude of low-carbon and advertising investments, leading to different optimal decision-making strategies for maximizing total profit. Moreover, emission reduction investment exerts a more significant effect on overall profitability than advertising investment.

(3) The implementation of a bidirectional cost-sharing and profit-sharing contract enables effective supply chain coordination. Under this contract, the manufacturer supplies products to the retailer at a reduced wholesale price, whereas the retailer shares a portion of the sales revenue with the manufacturer at a predetermined ratio. Both parties jointly bear the costs of emission reduction and low-carbon advertising while maintaining product quality. This coordination mechanism ensures that the consumer reference low-carbon effect, product reputation, manufacturer's emission reduction efforts, retailer's advertising efforts, and market demand align with the optimal outcomes observed under centralized decision making. Consequently, the profits of the manufacturer and the retailer significantly improve, and the overall supply chain profit reaches the level achieved under centralized decision making, thereby optimizing total supply chain profitability and promoting sustainable development.

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EKOLOGIŠKOS TIEKIMO GRANDINĖS SPRENDIMŲ PRIĖMIMAS IR SUTARČIŲ KOORDINAVIMAS: PREKĖS ŽENKLO REPUTACIJOS IR VARTOTOJŲ ORIENTACINIS POVEIKIS MAŽO ANGLIES DIOKSIDO KIEKIO LYGIUI

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Santrauka. Spartus ekonomikos augimas lemia didelę taršą ir besaikį išteklių naudojimą, o tai kelia iššūkių tvarumui. Siekdama išspręsti šias problemas, vyriausybė diegė žaliosios tiekimo grandinės valdymą skatindama įmones gerinti savo mažai anglies dioksido į aplinką išskiriančių produktų reputaciją ir patenkinti didėjančią vartotojų paklausą ekologiškiems produktams. Šio tyrimo metu buvo sukurtas diferencijuotas dviejų ešelonų žaliosios tiekimo grandinės žaidimo modelis, į kurį buvo įtrauktas vartotojų orientacinis poveikis mažo anglies dioksido kiekio lygiui – kaip veiksnys, kuris dažnai buvo nepastebėtas ankstesniuose tyrimuose. Atsižvelgus į mažmeninę kainą, investicijas į reklamą ir išmetamųjų teršalų mažinimo pastangas kaip sprendimų kintamuosius, išanalizuotos geriausios decentralizuoto ir centralizuoto sprendimų priėmimo strategijos. Rezultatai atskleidžia, kad centralizuotas sprendimų priėmimas lemia didesnį išmetamųjų teršalų kiekio mažinimą, geresnę reputaciją ir geresnius finansinius rezultatus, nors jo efektyvumas priklauso nuo investicijų intensyvumo. Siekiant panaikinti atotrūkį tarp decentralizuoto ir centralizuoto sprendimų priėmimo pasiūlyti trys sutarčių koordinavimo mechanizmai: i) abipusis išlaidų pasidalijimas, ii) pajamų pasidalijimas ir iii) mišri sutartis. Hibridinė sutartis geriausiai suderina tiekimo grandinės našumą su centralizavimu ir taip padidina gamintojų ir mažmenininkų pelningumą. Skaitinė analizė patvirtino išvadas ir suteikė įžvalgų apie tvarų tiekimo grandinės valdymą ir mažo anglies dioksido kiekio technologijų politikos formavimą.

Reikšminiai žodžiai: orientacinis vartotojų efektas; mažai anglies dioksido į aplinką išskiriančių produktų reputacija; ekologiškos tiekimo grandinės valdymas; diferencinė žaidimo teorija; sutarčių koordinavimas.