ISSN 1392-5016. ACTA PAEDAGOGICA VILNENSIA 2015 35
DOI: http://dx.doi.org/10.15388/ActPaed.2015.35.9191

How to promote text comprehension with pupils of grades 1–6 when teaching to solve combinatorial problems

Elfrīda Krastiņa
Professor
Faculty of Education and Management,
Daugavpils University
Address: Vienības Street 13, Daugavpils,
LV-5401, Latvia
E-mail: elfridak@inbox.lv

Anita Sondore
Dr. math., docent
Faculty of Natural Sciences and Mathematics,
Daugavpils University
Address: Vienības Street 13, Daugavpils,
LV-5401, Latvia
E-mail: anita.sondore@du.lv

Elga Drelinga
Mg.paed., lecturer
Faculty of Education and Management,
Daugavpils University
Address: Vienības Street 13, Daugavpils,
LV-5401, Latvia
E-mail: elga.drelinga@du.lv

Abstract. *It is essential to train pupils in working with diverse information resources and making use of various strategies for solving mathematical problems. To determine the probability of various events, knowledge of the strategies for solving combinatorial problems as well as adequate reading competence are essential.*

Research problem– how to promote text comprehension of pupils of Grades 1–6 when teaching to solve combinatorial problems in the Latvia. Research questions – Which mathematical problems are difficult for pupils in national diagnostic tests in Grades 3 and 6? What is the teachers' opinion about pupils' ability to solve mathematical word problems (focus on pupils in Grades 1 to 6)? What is the teachers' opinion about the use of strategies for solving problems with elements of combinatorics?

To answer the research questions, an evaluative case study design was created. The authors conducted analysis of relevant educational documents, performed content analysis of combinatorial problems which pupils in Grades 1–6 have to master, carried out a survey of 130 primary school teachers, analyzed the results of national diagnostic tests of Grade 3 and Grade 6 (2014–2015) in mathematics and in the Latvian language and performed observation of problem solving strategies used in primary school.

This paper examines some typical primary school pupils' difficulties in solving combinatorial problems. Suggestions are proposed for drafting relevant methodological recommendations for teachers for teaching to solve non-standard problems.

Key words: *elements of combinatorics, reading comprehension, problem solving strategies.*

Introduction

Sustainable education is unimaginable without creative thinking to solve various non-standard problems (Gerretson et al., 2010). But it is based on basic skills, basic competences, which pupils acquire already in primary school (Cedefop, 2012). One of the basic competences is competence in mathematics (Eurydice, 2012), which includes problem solving competence (Eurydice, 2011). Word problems with elements of combinatorics for Grades 1–6 ought to be considered as problem tasks, since "*combinatorial reasoning is not restricted to solving verbal combination and arrangement problems, but that it includes a wide range of concepts and problem-solving abilities*" **(**Batanero et al., 1997, 251). Combinatorial problems require looking at different cases, creatively visualizing situations and making new decisions.

Yet, in teaching to solve combinatorial problem tasks, text comprehension is crucial, and its essential component is the acquisition of key notions (Vigotskis, 2002). Text comprehension depends on the level of reading competence (Fišers, 2005 a, b) and is related to transversal skills (Cedefop, 2012) – for instance, ability to learn, to think critically, to assume self-initiative, meta-cognitive skills, the skill of using digital and mass media tools.

The problem that motivated the research is Latvian pupils' achievement in solving word problems with elements of combinatorics (mathematical competence in relation to reading competence**)**. This is confirmed by the results of the 5th cycle of the Program for International Student Assessment (PISA 2012): the lowest achievement in mathematics among Latvian pupils (15 year olds) is demonstrated in topics such as probability and statistics – by 12 points lower than the average OECD result, but Latvian pupils achievement in reading is slightly below the OECD average, the difference is statistically significant (Geske et al., 2013). There is a low pupils' achievement in combinatorial exercise in the National Diagnostic Test for Grade 3 in mathematics in 2014 and also in 2015 (VISC, 2014a, 2014b, 2015a, 2015b), because only approximately one third of all pupils could solve a non-standard problem with elements of combinatorics. The 3rd grade pupils' achievement in Latvian language in Exercise 3 from Diagnostic test with a combined learning content in 2014 allows for identifying the acquisition of literacy levels. The deliberate reading experience of 3rd grade pupils is still insufficient, as only 37% of pupils (N = 11829) have been able to find and underline the sentence in the text which expresses the main idea. It should be noted that achievement for Exercise 3.2. compared to the other exercises is the lowest (VISC, 2014c, 6). From the exercises (VISC, 2015c) and results of the National Diagnostic Test in Latvian Language for Grade 6 in 2015 (in education programs with Latvian as the language of instruction) (VISC, 2015a) we have established that 42% of pupils are able to analyze information and make conclusions (see Exercise 8), but only 24% of pupils are able to justify the attitude of a literary hero by a sentence or phrase (see Exercise 3). If only approximately one fourth of Grade 6 pupils (N = 11350) could substantiate their reasoning on the basis of the story fragment that they had read in their native language, we cannot expect the results to be better in solving the mathematical problems that involve text comprehension.

The given facts actualize the necessity to evaluate the opportunities for raising primary school teachers' methodological competence in teaching to solve word problems and combinatorial problems.

Research aim – to determine the necessary methodological assistance for helping teachers promote text comprehension of pupils of Grades 1–6 when teaching to solve word problems and combinatorial problems.

Research questions – Which mathematical problems are difficult for pupils in national diagnostic tests in Grades 3 and 6? What is the teachers' opinion about pupils' ability to solve mathematical word problems (focus on pupils in Grades 1 to 6)? What is the teachers' opinion about the use of strategies for solving problems with elements of combinatorics?

Theoretical background

The theoretical background of the study is underpinned by the competence approach (de Corte, 2010) and constrictive learning (Hofmeister, 1998; Bruner, 1977), emphasizing a person's activity in information processing and knowledge creation (Irvine, 2015; Bono, 2012; Fišers, 2005 a, b; Piaget, 1974). The creative solution of problems that feature elements of combinatorics requires critical thinking and divergent thinking (Savery, 2006; Collins and Amabile, 1999; Dewey, 1993; Kolb, 1984). Traditionally, we are used to the idea that "*mathematics is a strictly structured network of ideas*" (Fišers, 2005a, 246). "*At the same time, we live in a world which lacks certainty, and probability gives an opportunity to measure the uncertainty"* (Fišers, 2005 a, 254). De Bono speaks about a creative probability which has a crucial role in science (de Bono, 2012).

"*In the modern world, people in almost every area of activity find it necessary to solve problems of a combinatorial nature"* (Roberts et al., 2005, 1). Combinatorics is a branch of mathematics concerning the study of finite or countable discrete structures, enumerative combinatorics include counting the structures of a given kind and size (Anderson et al., 2004; Plocki, 2004). There are three basic problems of combinatorics – the existence problem (is there at least one arrangement of a particular kind?), the counting problem (how many arrangements are there?) and the optimization problem – to choose the best according to some criterion arrangement (Roberts et al., 2005; Batanero et al., 1997). Pupils of Grades 1–6 generally have to solve counting problems, i.e. in how many ways a certain choice can be made. Already in primary school it is possible to gradually introduce the notions that are related to more or less probable events. The description of the event is included in the text.

As several authors of methodological aids point out (Mencis, 1984/2014; Fišers 2005a, b), in a mathematical text, the correct perception of the logical structure of the text is essentially important (cause and effect, conditions and question); afterwards, text comprehension is then divided into content comprehension and notion comprehension, which promotes getting information from the text and interpretation and visualization of the acquired information (model, schema). Evaluation of the content of the text is related to finding key words and ability to express one's own opinion.

As Fisher argues, reading has two main components – "*deciphering words and comprehension of the meaning of the text*" (Fišers, 2005a, 229). Vygotsky (Vigotskis, 2002) distinguishes two levels of notion development. On the first level, notions develop in a spontaneous manner, underpinned by the rich experience. The top level involves scientific notions which rely on the use of language and learning. Piaget's viewpoint is that notional structures are the very grounds for skills and experience (Piaget, 1974).

The communication in the mother tongue and the mathematical competence are two important key competences at school in Europe (Eurydice, 2012). Communication in the mother tongue involves reading competence. In the fifth cycle of research of the Program for International Student Assessment, reading competence is defined as the ability to comprehend, use and evaluate written texts to achieve one's aims, improve knowledge and potential, and participate in the social life (Geske et al., 2013). Reading competence involves reading various kinds of related text (for instance, description, narration, interpretation, argumentation, instruction) and variously structured documents (Geske et al., 2013). Whereas, as Geske and co-authors point out (Geske et al., 2010), reading competence involves a multitude of cognitive skills – from recognition of written text, knowledge of words, grammar, structure of language and text to knowledge of the world in general. It also includes metacognitive skills – the use of different kinds of suitable strategies when working with a text.

In the contemporary teaching and learning process (especially as regards natural sciences and mathematics), it is crucial to prepare pupils for working with various sources of statistical and graphic information which combine text, numbers and other mathematical symbols that can be incorporated in tables, diagrams, graphs, schemas, etc.

Theoretical literature summarizes the strategies for solving combinatorial problems. Batanero et al. (1997) discuss two essential components in the teaching and assessment of combinatorics (basic combinatorial concepts and models) and five combinatorial procedures. From the combinatorial problem solving strategies which are summarized in theoretical literature, the following procedures are and can be used in working with pupils of Grades 1–6:

- **Logical** procedures: guess – check, predict – check – prove, modeling with counting material, systematic enumeration.
- **Graphical** procedures: tree diagrams, graphs.
- **Numerical** procedures: addition and multiplication principles.
- **Tabular** procedures: constructing a table.

Untraditional approaches in the development of reasoning are suggested in a publication by de Bono (2012), which opens a wide scope for improvement in the mathematical methodology as well.

Research methodology

To answer the research question, an evaluative case study design was created (Pipere, 2011b; Geske & Grīnfelds, 2006) to determine the necessary actions to be taken to solve the problem of teaching elements of combinatorics in primary school.

Document and combinatorial problem content analysis, observation and survey were used for data gathering.

A qualitative analysis (Pipere, 2011a) of educational documents (Latvian National Standard for Primary Education and Mathematics Curriculum Samples for Grades 1–9, European Union documents) was performed in order to assess the opportunities for improving the mathematics curricula and methodologies for teaching a propaedeutic course on elements of combinatorics in Grades 1–6 in Latvia. Since the National Centre for Education (NCE) of the Republic of Latvia administrates the state examination system in Latvia from the drawing up of the tasks to the descriptive analysis of the national test results, we used the statistics summarized by the NCE about the results of diagnostic tests in mathematics and native language for Grades 3 and 6 in years 2014–2015 in regard to pupils' ability to solve combinatorial problems and their text comprehension in the exercises where the Latvian language reading competence is tested.

We performed a content analysis (Pipere, 2011a) of the word problems with elements of combinatorics from different mathematics competitions in Latvia and textbooks that are meant for pupils of Grades 1–6, in order to discover the notions that are used in combinatorial problems. We selected mathematical competitions from the period 2012–2015: 1. *Professor Digit's Club* (mathematical competition for pupils up to Grade 9, including); 2. *Young Mathematicians' Competition* (mathematical competition for pupils up to Grade 7, including); 3. An Olympiad *"So much or… how much?"* for Grade 4; 4. *Latvian Open Mathematics Olympiad* for pupils of Grades 5–12 (more information on the webpage of A. Liepa's Extramural Mathematics School http://nms.lu.lv), and 5. *The International Mathematics Competition "Kangaroo"* for pupils of Grades 2–12 (http://kengurs.lv/index.php/lv/konkurss).

Observation was used for gathering the research data (Pipere, 2011a). It is a research method that permits to observe the strategies for solving combinatorial problems that the pupils use from the perspective of "external onlooker" (Pipere, 2011a, 182). To ensure the credibility of the results, an observer (teacher Elita Skrimble) was specifically trained for the purpose. She used the results obtained from observation in a more comprehensive form in her bachelor paper "Integrated combinatorial problems to develop 4th Grade pupils' mathematical skills". The observation was organized in an environment that is familiar to the pupils: in the classroom, during a school lesson. The observer did not get involved in the proceedings, choosing the role of "spectator" (Geske & Grīnfelds, 2006).

Survey **(**Pipere, 2011a**)** was another research method used in the survey. In the period from March to August, 2015, it involved 130 primary school mathematics teachers from different regions of Latvia. The authors conducted the survey during teachers' further education courses. This survey was organized with a view to assessing teachers' opinions on what causes difficulties to pupils in teaching to solve mathematical word problems, as well as determining the teacher's understanding of the strategies for solving combinatorial problems in mathematics.

The obtained data were analyzed qualitatively (Pipere, 2011c). Some quantitative measures were used only to reveal separate tendencies (Geske & Grīnfelds, 2006).

Research results

In the Latvian National Mathematics Curriculum for Grades 1–9 (MK, 2014), section two “Application of Mathematics in the Analysis of Natural and Social Processes” contains a sub-section “Elements of Information Processing, Statistics and Probability Theory”. Yet, in the mathematics syllabi for Grades 1–6, emphasis is laid on the elements of information processing and statistics. Only in Grade 9 there is a topic that relates to combinatorics – *Probability. Combinatorics. Variation, Combination*. Thus, elements of combinatorics are not incorporated in the national mathematics curriculum for Grades 1–6 in a detailed manner, the use of the mathematical language and notions is not envisaged. For instance, for Grade 3 there is a topic – comparing objects, sorting according to a characteristic, but the decision on what content to include in this topic remains in the competence of textbook authors and teachers.

To discover the concepts and notions used in combinatorial problems, which pupils of Grades 1–6 have to master, the content analysis of problems was performed. The competitions that took place in the academic years 2012–2015 in Latvia were examined. The most frequent question from competitions in the academic years 2012–2015 in the combinatorial problems for Grades 1–6 was – *in how many different ways…?* The notions that can be found there are: *all possible variants, for each part; exactly the same color, different colors; at least one of three; all; just one; like–dislike, exactly once; all the same elements have to be divided by choice into several equal piles (at least two)*, *different ways; all the remaining, except two, for each of the…; cut each into three parts; have to be nearby,* etc.

The content of combinatorial problems for Grades 1–6 is related to situations from the pupils and their families’ daily life. Therefore, it is important for the pupils to understand the notions most frequently used in daily life. These notions are also used in textbooks and problem compilations (Ģingulis, 2012; Mencis & Būmeistere, 2001; Bettner & Dinges, 2009): *often – rare, always – never, sometimes; some, none; most, except …; and, or; if…, then…; for…to …; different, in a different way, differently, how many different options?; is it possible?; each with each, every two, every following; in a different order; different groupings; the least or greatest possible.* Pupils often pay no due attention to the use of these concepts, which causes problems in the process of solving combinatorial tasks.

In combinatorial problems, the requirement is to examine different choice options, to determine their number, to perform sampling procedures of elements (with replacement and with order, with replacement and without order, without replacement and with order, without replacement and without order), to form rows of elements, and the like. During the teaching and learning process, pupils’ attention ought to be drawn to various problem formulations, how the solution of the problem changes depending on the formulation of words. Although problems with elements of combinatorics are not incorporated in the national mathematics curriculum for Grades 1–6 in a detailed manner, they can be found, albeit scarcely, both in various mathematical competitions and in textbooks in Latvia.

Considering the authors' experience in teaching to solve combinatorial problems, observation results about combinatorial problem solving strategies used by the primary school pupils and drawing on the summary of conclusions found in theoretical literature (Batanero et al., 1997), let us turn to some typical pupils' difficulties in solving combinatorial problems.

The pupils can confuse two different models: the selection model – a sample of m elements (a sample of a particular kind) must be taken from a given set of *n* elements, the distribution model – a given set of *n* elements must be divided into *m* subsets. It ought to be emphasized to the pupils: either a specific number of elements is selected from the group of elements (the selection model) or all elements must be divided into sub-groups (it is important to understand whether empty subsets are possible in the distribution model). Of course, challenges of the distribution model are difficult for pupils of Grades 1–6. For example, the problem: *Laila drew three different drawings, she decided to share these drawings with her three friends. In how many different ways can Laila share the drawings?* Some pupils consider only six different ways in which the set of three drawings can be divided into three subsets because they consider the type of partition that only one drawing can be shared with each friend, but forget that distribution does not necessarily mean that each friend must be given exactly one drawing, because some friend can take nothing.

Non-systematic enumeration. Writing all the possibilities (making full enumeration), pupils do not use a particular system in order not to lose or count some possibility repeatedly. Graphical (tree diagrams, graphs) and tabular procedures help to create a systemic approach in determining the different variants. For instance, the following exercise: *Tince wants to learn how to read. She has letter cards with letters - a, d, l, u, h, i, k. How many different syllables can Tince assemble using two letters?* Two different solutions to this problem are presented in Figure 1 (observer E. Skrimble). Pupil 1 of Grade 4 constructed a table but Pupil 2 of Grade 4 performed non-systematic enumeration, as evidenced by the lost syllables.

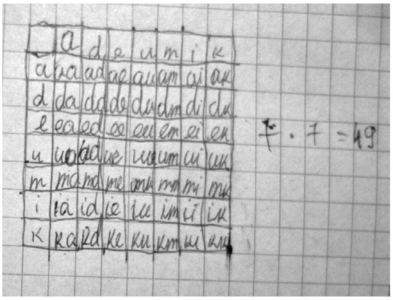
Incorrect use of graphical procedures. Yet, the use of tables, especially constructing tree diagrams or graphs, can lead to an erroneous solution if one does not understand how to construct tree diagrams or graphs.

Error of order. This mistake consists of mixing two basic sampling procedures – with order and without order.

Error of repetition. The pupil repeats the elements when there is no possibility of doing so or the pupil does not see that the elements can be repeated. For instance, the problem: *In how many different ways is it possible to choose a group leader and cashier in a 6-person tourist group?* Pupils do not realize that the group leader and the cashier can be the same person.

In the recent years, in these tests for Grade 6, problems with elements of combinatorics are not included. Pupils' academic achievement in national tests in mathematics in 2015 for Grade 6 reveals (VISC, 2015a; 2015d) that the average result for solving word problems was 63% (total number of pupils N = 15973). Let us indicate the comprehension of specific notions, adding the percentage of correct answers in brackets: *by how much increased*

Pupil 1:



Pupil 2:

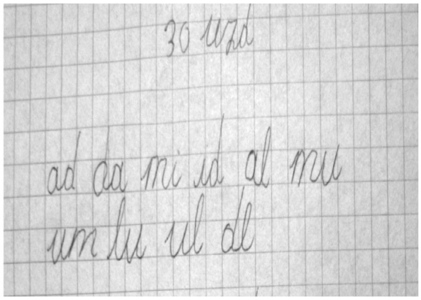


Fig. 1. ***Two solutions to the same problem (constructing a table and using non-systematic enumeration)***

(81%); *how many times longer* (55%), *by how much more* (43%), *distance, speed, time–motion problems* (43%); *course of pouring water* (37.65%). The skill of working with graphical information sources is evidenced by the following results: if the data are given in a table (fractions problems), the problems were solved by 36–37% of pupils of Grade 6; if the data were given in a sector diagram (percentage problems), 43–44% of solutions were correct. These results underscore teachers' lack of attention to working with various information sources and text comprehension.

Since 2014, the Diagnostic test in mathematics for Grade 3 had a combinatorial problem. In 2014 and in 2015 it was the last problem, Exercise 9; in addition, two answers were possible (VISC, 2014b; 2015b).

Exercise 9 in 2014. *There are 20* ***different*** *mushrooms in a basket. The number of king boletes* ***is the smallest****. The number of russulas is* ***greater by 4*** *than that of chanterelles. How many mushrooms of* ***each kind can there be*** *in the basket?*

Exercise 9 in 2015. *Rudolfs has some 50 cent, 20 cent and 10 cent coins. He bought a book for 2 euros. He paid for the purchase with 9 coins.* ***How many of each type*** *of coin did Rudolfs spend on purchase?*

Figure 2 summarizes the academic achievement in Exercise 9 from the diagnostic test in mathematics for Grade 3 in 2014 and 2015 (VISC, 2014a; 2015a). In 2014, the diagnostic test in mathematics was taken by N = 16767 of Grade 3 pupils. The total success rate in this test was 76.49%. Only 30.64% of pupils could solve the combinatorial problem (both answers), while one answer is given by 41.52% of pupils. In 2015, the diagnostic test in mathematics was taken by N=16973 of Grade 3 pupils. The total success rate in this test was 77.54%. Both answers were found by 37.89% of pupils, but only 33.06% wrote down the calculations correctly, while one answer was found by 50.73% of pupils (45.71% wrote down calculations correctly). The reason for the low achievement in solving combinatorial problems could be the fact that this type of problems is not sufficiently included in textbooks in Latvia. This actualizes the need to create a compilation of combinatorial problems for Grades 1–6.

To determine primary school teachers' opinions, in 2015 a teachers' survey (pilot

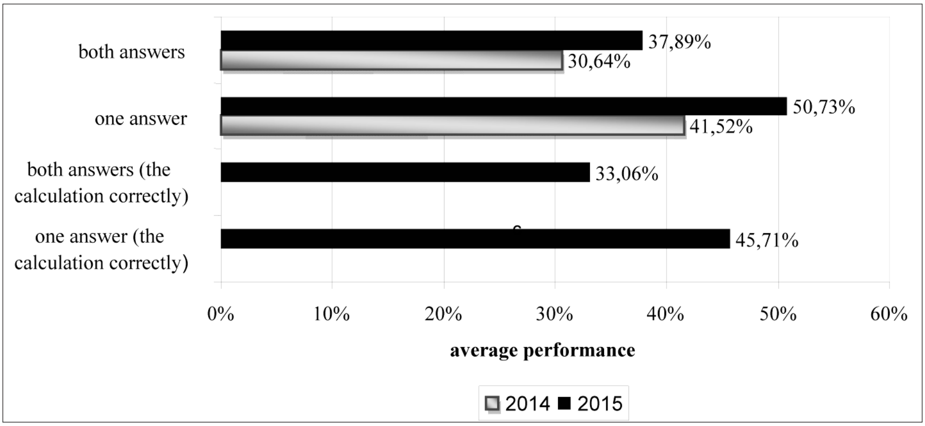


Fig. 2. ***Academic achievement in Exercise 9 from diagnostic test in mathematics for Grade 3 in 2014 (N = 16 767) and in 2015 (N = 16 973).***

research) was conducted; N = 130 primary school mathematics teachers from different Latvian schools took part in it. The distribution of answers to the question *What poses difficulties to pupils in solving mathematical word problems?* is presented in Figure 3. The answers indicate that as the main cause of difficulties in solving word problems 85% of the teachers mentioned the perception of the text. In addition, more than a half of the teachers (60%) claimed that what caused difficulties was the inability to draw up a plan for solving a word problem. Drawing up a plan for solving the problem, in its turn, is related to the comprehension of the mutual interconnections among the quantities.

Strategies for solving word problems that are used by teachers working with pupils are presented in Figure 4. Analysis of teachers' opinions confirms the tendency that teachers are familiar with various strategies for solving word problems of which the most frequently used is drawing problem conditions or displaying them schematically (90%). On the other hand, teachers are less familiar with specific strategies for solving combinatorial

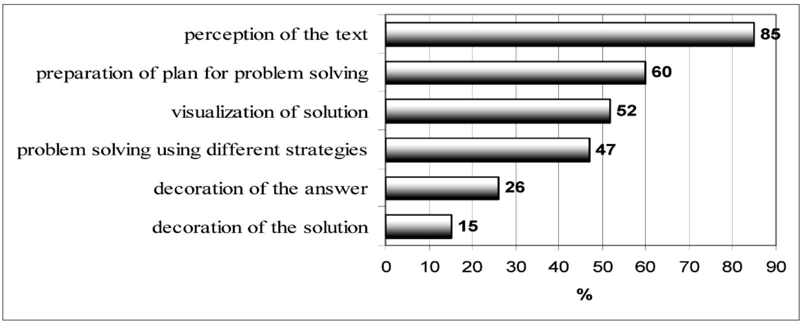


Fig. 3. ***Distribution of teachers' answers to the question "What poses difficulties to pupils in solving mathematical word problems?" (N = 130)***

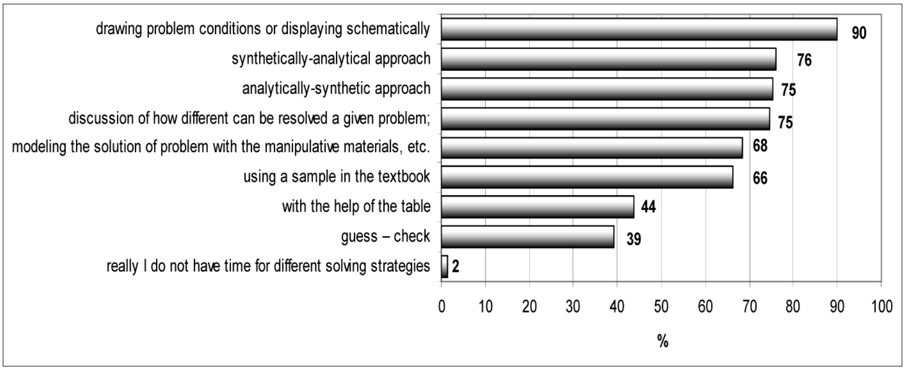


Fig. 4. ***Percentage of teachers who were familiar with the indicated strategies for solving word problems working with pupils (N = 130)***

problems – 44% of teachers use the table strategy, while only 39% of teachers presented to their pupils the guess-and-check strategy.

The following survey question elicited the percentage of teachers who were familiar with the indicated special strategies for solving combinatorial problems. Figure 5 shows that from the 5 suggested specific strategies for solving a combinatorial problem, teachers mostly know, and thus use, tables (62%), but such techniques as the construction of trees and enumeration of possible variants are familiar to less than 50% of the questioned 130 teachers, whereas only 26% of teachers know about the graphs strategy. 2% of teachers have noted that they are also familiar with other strategies.

77% of teachers support the idea that the methodology for teaching mathematics to Grades 1–6 ought to be supplemented with strategies for solving combinatorial problems which would be adequate for junior schoolchildren, thus implementing a systematic and sequential approach.

Discussion and conclusion

The conducted research confirms that a complex evaluation of the mathematics standards, curricula, teaching aids and results of diagnostic tests, drawing on the

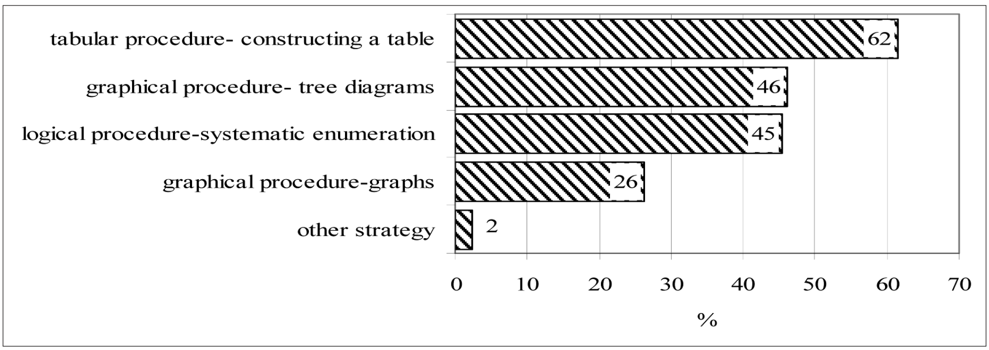


Fig. 5. ***Percentage of teachers who are familiar with the indicated strategies for solving a combinatorial problem (N = 130)***

international research and determining teachers' opinions enable us to find opportunities for solving methodological problems in mathematics and reading skills in Grades 1–6. More specifications are required about the elements of combinatorics in the mathematics standards and curricula for Grades 1–6. This would help textbook authors and teachers to be more familiar with the requirements regarding this issue. A methodological outline of combinatorial topics in a textbook would help pupils master specific problem-solving strategies.

To promote pupils' text comprehension in teaching to solve combinatorial problems, specific attention ought to be paid to the notions incorporated in the text and the problem-solving strategies, in addition, by encouraging to solve the same problem with different strategies (techniques).

In the non-standard problem-solving process, pupils' meaningful activity is crucial, learning how to substantiate their idea. Considering the fact that different strategies are possible for problems with elements of combinatorics, teachers ought to encourage pupils to demonstrate various solutions. When organizing reflection after the completed independent work, it is necessary to involve pupils in discussion about the mistakes that were made, encouraging the explanation of the causes of mistakes and finding the correct solution. The most complicated option – to generalize the answer to the problem for another possible number of elements or for any number of elements.

The analysis of teachers' opinions in a pilot research confirms the tendency that teachers know different strategies for solving mathematical word problems, but it turned out that among the least familiar were the specific strategies for solving combinatorial problems. This actualizes the need to elaborate a compilation of combinatorial problems for junior schoolchildren as well as to propose methodological recommendations to teachers about the acquisition of the elements of probability and combinatorics in Grades 1–6, in order to introduce them into the Latvian school practice.

When teaching to solve combinatorial problems, teachers require knowledge not only of the most characteristic strategies of combinatorial problems, but also of the methodological techniques of notion acquisition and how they can be implemented in the teaching and learning process:

- how to read the text (find key words), grasp the main idea,
- how to visualize the mathematical text, model the content with visual aids, how to search for the interconnections of quantities;
- how to project the problem solution;
- how to check the accuracy of the solution, how to search for another solution strategy, in what other problems this solution strategy could be used;
- how to teach pupils to analyze the causes of their mistakes, how to search for new problem solutions.

The further development of problem solution and text perception skills requires an interdisciplinary study about the correlation between the reading skill and mathematical competence.

REFERENCES

Anderson, J. A.; Lewis, J.; Dale Saylor, O. (2004). *Discrete Mathematics with Combinatorics*. 2nd ed. Upper Saddle River: Pearson.

Batanero, C.; Godino, J. D.; Navarro-Pelayo, V. (1997) Combinatorial Reasoning and its assessment. *The Assessment Challenge in Statistics Education.* IOS Press, 239–252. *Online:* https://www.stat.auckland.ac.nz/~iase/publications/assessbk/ chapter18.pdf [Retrieved 1.04.2015].

Bettner, M.; Dinges E. (2009). *Stochastik in der Grundschule (Kombinieren, schätzen, Daten erfassen und auswerten)*. Buxtehude: Persen. Online: https://books.google.lv/books?id=5UIFK9SPmlAC&pg=PA3&lpg=PA3&dq=kombinatorik+4+klasse&source=bl&ots=PWgnk8pUJI&sig=nYa6zURU3D79CvSjwEjZ6zaQzZg&hl=lv&sa=X&ved=0CC0Q6AEwAmoVChMIk-GlxPWKyAIVyEQUCh30dwt4#v=onepage&q=kombinatorik%204%20klasse&f=false [Retrieved 22.06.2015].

de Bono, E. (2012). *Domā! Kamēr nav par vēlu* [Think! Before It's Too Late]. Rīga: Zvaigzne ABC.

Bruner, D. (1977). Psihologija poznanija: Za predelami neposredstvennoi informacii [Psychology of inquiry: beyond the obvious information]. Moskva: Progress.

Cedefop (2012). *Rethinking Education***:** *Investing in Skills for Better Socio-economic Outcomes***,** Strasbourg. Online: http://www.cedefop**.**europa.eu/files/com669_en.pdf

Collins, M. A.; Amabile, T. M. (1999). Motivation and creativity. In R. J. Sternberg (ed.), Handbook of Creativity. Cambridge, UK: cambridge University Press, 297–312.

de Corte, E. (2010). Historical developments in the understanding of learning. In Dumont, H.; Istance, D.; Benavides, F. *The Nature of Learning: Using Research to Inspire Practice.* OECD: Paris. 199–216. Online: http://www.educ.ethz.ch/pro/litll/oecdbuch.pdf [Retrieved 20.09.2015].

Dewey, J. (1993). *How We Think: A Restatement of the Relation of Reflective Thinking to the Educative Process*. Boston: Houghton Mifflin Company.

Eurydice (2011). Mathematics Education in Europe: Common Challenges and National Policies. *Eurydice Report. Education, Audiovisual and Culture Executive Agency.* Online: http://eacea.ec.europa.eu/education/ eurydice/documents/thematic_reports/132EN.pdf

Eurydice (2012). Developing Key Competences at School in Europe: Challenges and Opportunities for Policy. *Eurydice Report.* Luxembourg: Publications Office of the European Union. Online: http://eacea.ec.europa.eu/ education/eurydice/documents/thematic_reports/145EN.pdf

Fišers, R. (2005a). *Mācīsim bērniem domāt* [Teaching Children To Think]. Rīga: Raka.

Fišers, R. (2005b). *Mācīsim bērniem mācīties* [Teaching Children To Learn]. Rīga: Raka.

Gerretson, H.; Iliško, Dz.; Fortino, C. (2010). Sustaining self-regulated students' learning through inquiry-driven mathematics and science instruction. A. Skrinda (ed.). *Discourse and Commuication for Sustainable Education*, 1, 3-17.

Geske, A.; Grīnfelds, A. (2006). *Izglītības pētniecība* [Educational research]. Rīga: LU Akadēmiskais apgāds.

Geske, A.; Grīnfelds, A.; Kangro, A.; Kiseļova, R. (2013). *Latvija OECD Starptautiskajā skolēnu novērtēšanas programmā 2012 – pirmie rezultāti un secinājumi* [Latvia OECD Program for International Student Assessment 2012 – the first results and conclusions]. Rīga: LU Izglītības pētniecības institūts. Online: http://www.ipi.lu.lv/uploads/media/Latvija_SSNP_2012_pirmie_rezultati_un_secinajumi.pdf [Retrieved 1.04.2015].

Geske, A.; Grīnfelds, A.; Kangro, A.; Kiseļova, R. (2010*). Ko skolēni zina un prot - kompetence lasīšanā, matemātikā un dabaszinātnēs. Latvija OECD valstu Starptautiskajā skolēnu novērtēšanas programmā 2009.* [What pupils know and can do – competence in reading, mathematics and natural science. Latvia in the OECD Programme for International Student Assessment 2009]. Rīga: LU Izglītības pētniecības institūts.

Online: http://www.ipi.lu.lv/uploads/media/OECD_SSNP_2009.pdf [Retrieved 23. 09. 2015].

Ģingulis, E. (2012). *489 spici atjautības un pacietības uzdevumi matemātikā, 7–12 gadi*. [489 cool ingenuity and patience problems in mathematics, 7–12 years]. Rīga: Zvaigzne ABC.

Hofmeister, A. (1998). *Zur Kritik des Bildungsbefriffs aus subjektwissenschflicher Perspektive. Diskursanalytische Untersuchungen.* Hamburg: Argument.

Irvine, J. (2015). Problem solving as motivation in mathematics: just in time teaching. Journal of Mathematical Sciences, Vol. 2, 106–117. Online: http://bettyjonespub.com/math/4JMS20150307-1.pdf [Retrieved 20.09.2015].

Kolb, D. (1984). *Experiential learning: experience as the source of learning and development.* Englewood Cliffs, NJ: Prentice Hall. Online: http://academic.regis.edu/ed205/kolb.pdf [Retrieved 1.04.2015].

MK (2014). *Ministru kabineta 2014. gada 12. augusta Nr. 468 noteikumi „Noteikumi par valsts pamatizglītības standartu, pamatizglītības mācību priekšmetu standartiem un pamatizglītības programmu paraugiem".* [Regulations Nr.468 of August 12, 2014 on national primary education standard, primary education academic subject standards and primary education curricula samples]. Online: http://likumi.lv/doc.php?id=268342

Mencis, J. (1984/2014). *Matemātikas metodika pamatskolā* (*Mathematical Methods in elementary school).* Rīga: Zvaigzne/Zvaigzne ABC.

Mencis, J.; Būmeistere, P. (2001). *Cietie riekstiņi: riekstkožiem no 6 līdz 9 gadiem*. [Hard nuts for nutcrackers aged 6 to 9]. Rīga: Zvaigzne ABC.

Piaget, J. (1974). *To Understand Is to Invent: the Future of Education.* New York: Viking.

Pipere, A. (2011a). Datu ieguves metodes pētījumā un to analīze [methods of data gathering for the research and its analysis] in Martinsone, K. (ed.). *Ievads pētniecībā: stratēģijas, dizaini, metodes. [Introduction in research: strategy, design and methodology]* Rīga: Raka, 157–192.

Pipere, A. (2011b). Kvalitatīvo pētījumu dizainu veidi [Types of qualitative research design] in Martinsone, K. (ed.) *Ievads pētniecībā: stratēģijas, dizaini, metodes. [Introduction in research: strategy, design and methodology]*. Rīga: Raka, 90–105.

Pipere, A. (2011c). Datu analīze kvalitatīvajā pētījumā [Data analysis in qualitative research] in Martinsone, K. (ed.) *Ievads pētniecībā: stratēģijas, dizaini, metodes. [Introduction in research: strategy, design and methodology]* Rīga: Raka, 220–243.

Plocki, A. (2004). *Prawdopodobieństwo wokól nas* [Probability around us]. Wilkowice: Dla szkoly.

Roberts, F. R.; Tesman, B. (2005). *Applied Combinatorics*. 2nd ed. Upper Saddle River: Prentice-Hall.

Savery, J. R. (2006). Overview of Problem-based Learning: Definitions and Distinctions. *Journal of Problem-based* Learning, Vol. 1(1). *Online:* http://docs.lib.purdue.edu/ijpbl/vol1/iss1/3/ [Retrieved 1.04.2015].

Vigotskis, Ļ. (2002). *Domāšana un runa* [Thoutght and language]. Madona: EVE.

VISC (2014a, 2015a). *Valsts pārbaudes darbi (norises statistika un rezultātu raksturojums) matemātikā un latviešu valodā 3. un 6. klasei 2013./2014.m.gadā un 2014./2015.m.gadā* [National tests (statistics of process and characteristics of results) in mathematics and in the Latvian language for Grades 3 and 6 in 2013/2014 and 2014/2015]. Online: http://visc.gov.lv/vispizglitiba/eksameni/statistika/2014 (2015).

VISC (2014b, 2015b). *2013./2014.m.g., 2014./2015.m.g Diagnosticējošais darbs ar kombinētu mācību saturu 3. klasei (matemātika)* [Diagnostic test with a combined learning content for grade 3 (mathematics) in 2013/2014, 2014/2015]. Online: http://visc.gov.lv/vispizglitiba/eksameni/dokumenti/uzdevumi/ 2014(2015)/3klase/3kl_mat_lv.pdf

VISC (2014c). *Diagnosticējošais darbs sākumskolā2013./2014.m.g.: rezultātu analīze un ieteikumi. Metodisks materiāls.* [Diagnostic test in primary school in 2013/2014: analysis of results and recommendations. Methodological material]. Online: *http://visc.gov.lv/vispizglitiba/eksameni/dokumenti/metmat/2013–_2014_ darbs_sakskola_analize.pdf*

VISC (2015c). *2014./2015. m.g. Diagnosticējošais darbs latviešu valodā 6. klasei* [Diagnostic Test in Latvian Language for Grade 6 in 2014/2015]. Online: http://visc.gov.lv/vispizglitiba/eksameni/dokumenti/uzdevumi/2015/6klase/6kl_latv_val.pdf

VISC (2015d). *2014./2015. m.g. Diagnosticējošais darbs matemātikā 6. klasei* [Diagnostic test in mathematics for Grade 6 in 2014/2015]. Online: http://visc.gov.lv/vispizglitiba/eksameni/dokumenti/uzdevumi/2015/6klase/6kl_mat_lv.pdf

KAIP PALENGVINTI PIRMŲ–ŠEŠTŲ KLASIŲ MOKINIŲ TEKSTO SUVOKIMĄ MOKANT JUOS SPRĘSTI KOMBINATORIKOS UŽDAVINIUS

Elfrīda Krastiņa, Anita Sondore, Elga Drelinga

Santrauka

Mokinius svarbu išmokyti dirbti su skirtingais informacijos ištekliais ir naudotis įvairiomis matematikos uždavinių sprendimo strategijomis. Įvairių įvykių tikimybei nustatyti būtina išmanyti kombinatorikos uždavinių sprendimo strategijas ir būti pasiekus tam tikrą skaitymo kompetenciją, nes, sprendžiant tokius uždavinius, reikia suprasti ne tik skaitinę informaciją, bet ir žodinį tekstą. Deja, vaikams dažnai ne taip lengva įveikti žodinius uždavinius. Uždavinių sprendimo kompetenciją lemia sąvokų išmokimo ir skaitymo gebėjimai.

Tyrimo problema – kaip palengvinti Latvijos pirmų–šeštų klasių mokinių teksto supratimą mokant juos spręsti kombinatorikos uždavinius. Tyrimo klausimas – kokios trečiai ir šeštai klasėms skirtos nacionalinių diagnostinių testų užduotys mokiniams atrodo sunkios, kokia yra mokytojų nuomonė apie mokinių gebėjimą spręsti matematinius žodinius uždavinius (daugiausia dėmesio skiriant pirmos ir šeštos klasės mokiniams) ir ką mokytojai mano apie strategijų, skirtų spręsti uždavinius su kombinatorikos elementais, naudojimą.

Tyrimas grindžiamas kompetencijų ir konstruktyvaus mokymosi teorine prieiga, kuria pabrėžiamas paties asmens aktyvumas apdorojant informaciją ir kuriant žinias.

Siekiant atsakyti į tyrimo klausimą, sukurta vertinamoji atvejų tyrimo schema. Tyrimo autorės atliko svarbių švietimo dokumentų analizę, kombinatorikos uždavinių, kuriuos turi mokėti spręsti pirmų–šeštų klasių mokiniai, turinio analizę, apklausė 130 pradinių mokyklų mokytojų, išanalizavo trečiai ir šeštai klasėms skirtų Latvijos nacionalinių diagnostinių matematikos ir latvių kalbos testų rezultatus (2014–2015) ir stebėjo, kaip pradinėse mokyklose taikomos uždavinių sprendimo strategijos.

Nacionalinių matematikos testų (2014–2015) rezultatai rodo žemus trečios klasės mokinių pasiekimus sprendžiant kombinatorikos uždavinius ir tai, kad šeštos klasės mokiniams sunkiai sekasi statistikos uždaviniai. Latvijos pradinių klasių mokytojų nuomonių analizė patvirtina tendenciją, kad mokytojai būna susipažinę su įvairiomis žodinių uždavinių sprendimo strategijomis, bet turi mažiau žinių apie kombinatorikos uždavinių sprendimo strategijas. Pradinių mokyklų mokytojai pritaria minčiai, kad jiems reikia metodinės pagalbos mokant spręsti nestandartinius uždavinius ir aiškinant temą „kombinatorikos elementai“, ypač siekiant palengvinti skaitomo teksto supratimą ir uždavinių sprendimo strategijų išmokimą.

Šiame straipsnyje nagrinėjama keletas tipiškų sunkumų, su kuriais spręsdami kombinatorikos uždavinius susiduria pradinių mokyklų mokiniai. Pateikiami siūlymai, kaip parengti atitinkamas metodines rekomendacijas mokytojams, kurie moko spręsti nestandartinius uždavinius. Šios rekomendacijos parengtos remiantis svarbiomis įžvalgomis ir rekomendacine įvairiose šalyse paskelbta medžiaga. Jos atskleidžia, koks būtinas Latvijos pradiniam ugdymui yra propedeutikos kursas, kad pirmų–šeštų klasių mokiniai būtų parengti septintoje–devintoje ir aukštesnėse vidurinės mokyklos klasėse dėstomai temai „Tikimybių teorija ir kombinatorikos elementai“.

Pagrindiniai žodžiai: kombinatorikos elementai, skaitymas, uždavinių sprendimo strategijos.

Įteikta: 2015 10 02
Priimta: 2015 11 10