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Measurement in Latvian School Mathematics Education

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This article covers some aspects related to teaching measurement in school mathematics, the reflection of the issues of mathematical measurements in regulatory documents and national curricula, peculiarities of teaching the topic and opportunities for improving the teaching methods, as well as the experience gained during the development of the study project on teaching measures to future teachers of mathematics.

Keywords: *measures, measuring, standards, programmes, projects.*

Introduction

Scientific measurement has been defined as "rules for assigning numbers to objects in such a way as to represent quantities of attributes" (Nunnally, 1978, p. 3). Measurement "consists of rules for assigning symbols to objects so as to (1) represent quantities of attributes numerically (scaling) or (2) define whether the objects fall in the same or different categories with respect to a given attribute (classification)" (Nunnally & Bernstein, 1994, p. 3).

Measurement is best taught not as a simple skill – rather, it is a complex combination of concepts and skills. Complex knowledge includes the following knowledge and skills: understanding of measures, ability to see measures in the surroundings, understanding and knowledge of units of measurement, ability to measure, calculation of measures, finding a value by using a formula, evaluating measures, converting smaller units of measurement into larger ones and vice versa, operations with units of measurement, comparing measures. It is also important to know how to analyse changes in measures if measuring data are changed.

School mathematics is commonly structured into arithmetic, algebra and geometry. This raises the issue of where to place ideas within the topic of measurement since some aspects of measurement (such as measuring length or area) have a geometrical component, while other aspects of measurement (such as time) are about number. Furthermore, when actual measures are unknown, relationships between measures can be expressed – and this is one of the roots of algebra. Measurement occurs across the topics that comprise school mathematics.

The complexity of the topic of measurement is affected by several conditions. One of these is the fact that the approach

necessary for learning algebra and geometry differs from the approach used when learning measures (Knuth, 2002). The logic of geometry and algebra is characterised by a deductive approach, while measuring is based on a completely different, empirical approach and practical activity, which is then followed by a transition to a language of abstract symbols.

While a portion of units of measurement is based on the decimal counting, there are many units of measurement which are based on non-decimal counting, for example, systems with the base of 4, 7, 12, 60, and others. Converting old, although frequently used units of measurement to the SI base units and vice versa may be rather complicated. Different measures have also a different principle of measurement: scales are used for measuring length and temperature, while area and volume are measured by using separate units of measurement.

The available evidence indicates that the principles of measurement are difficult for many pupils, require more attention in school than is usually given, and that the transition from informal to formal measurement needs considerable time and care.

Some researches (Owens, Outhred, 2006; Batista, 2002) suggest that pupils and the mathematics teachers find it difficult to understand measures (especially the area and volume) and the connection between various units of measurement. Several researches point out the necessity to improve the understanding of pupils regarding measurements (Smith, Silver, Stein, Boston and Henningsen, 2005).

The research problem: although the knowledge and skills for learning measurements are developed by practising arithmetic, algebra, and geometry, these activities do not guarantee emergence of complex knowledge and skills.

The goal of the research is to study the possibilities for improving the learning of measurements.

The tasks of the research:

1) to analyse the complex knowledge of pupils and next mathematics teachers in the field of measurements;
2) to analyse the reflection of the topic of measures in regulatory documents – the standard and study programmes for Years 1–12 in Latvia;
3) to study the possibilities for improving the teaching of measurements at school.

The research instruments are tests for assessing the knowledge and skills, analysis of documents regulating the field of education, feedbacks by students and pupils, interviews with mathematics teachers, and observations during classes.

This article analyses the reflection of the topic of measures in the regulatory documents drafted by the Ministry of Education and Science of the Republic of Latvia and describes the experience in improving the topic of teaching measurements for the future mathematics teachers.

Measures in School Mathematics Curriculum in Latvia

The knowledge and skills to be taught in Latvia are covered by two documents – the educational standard and the study programmes.

The minimum requirements for learning measurements at primary school are established in the valid Cabinet Regulation No. 468 of 12 August 2014 “Regulations

on the State Basic Education Standard, Basic Education Subject Standards and Basic Curriculum" of the Republic of Latvia.

The chapter "Creating a Set of Mathematical Instruments" of the basic requirements for mathematic skills upon graduation from Year 3 establishes that a pupil must be able to measure the length of a line segment and to draw a line segment of a particular length. The chapter "Using Mathematics in Analysing the Processes in the Nature and Community" states that a pupil must be able to make a distinction between comparable and incomparable measures, to characterise measures by using numbers, to use the units of measuring time, mass, temperature, length, money correctly, to perform exact measurement of time and length, to convert larger units of measurement into smaller ones by solving practical tasks, to sort measures expressed in natural numbers in ascending or descending order, and to write down the results of comparing.

As specified in the chapter "Using Mathematics in Analysing the Processes in the Nature and Community", the basic requirements for graduates of Year 6 are ability to use units of measuring area, volume, and speed, to use the results of direct measurements for deriving other measures, to evaluate the size of covered geometric figures by eye, in some cases to convert smaller units of measurement to larger ones by solving practical tasks, to sort measures expressed in rational numbers in ascending or descending order.

The basic requirements for graduates of Year 9, included in the chapter "Creating a Set of Mathematical Instruments", state that a pupil must be able to measure the values necessary for calculating the area and volume of figures (right prism, regular prism, pyramid, regular pyramid, cylinder, cone, ball) and to calculate the area and volume. In addition, the chapter "Using Mathematics in Analysing the Processes in the Nature and Community" mentions that a pupil must be able to find a specified value by using a formula, to measure and compare amounts by using appropriate units of measurement and instruments, to convert one unit of measurement to another by solving practical tasks, and to sort measures expressed in rational numbers in ascending or descending order.

The minimum requirements for learning measures at secondary school have been established in Cabinet Regulation No. 281 of 21 May 2013 "Regulations on the State General Secondary Education Standard, General Secondary Education Subject Standards and Curriculum" of the Republic of Latvia. The chapter "Mathematical Models" of this document states that a graduate of Year 12 must be able to use the properties of geometric figures for calculating the values of area and volume of geometric figures and their elements.

The standard requirements are first included in the mathematics programme. Let us analyse the programme offered by the National Centre for Education of the Republic of Latvia (Matemātika 1-9 klasei. Mācību priekšmeta programmas paraugs). Measures and measuring, as well as connections between them are covered by the chapter "Using Mathematics in Analysing the Processes in the Nature and Community". The process of learning measures has been summarised in the table below.

The other programme, offered by the National Centre for Education of the Republic of Latvia, is a sample programme for the education subject "Natural

Table 1. ***Learning measures within the mathematics programme.***

Year 1 Length (m, dm, cm), mass (kg), time (year, month, week, day, h, min), volume (l), money (cnt, euro).	Year 2 Length (km, mm), mass (kg, g), time (second), money.	Year 3 Length (km, m, dm, cm, mm), mass (kg, g), time (h, min, s), volume (l). Relations among different units of measurement of one measure.
Year 4 Units of measurement and their abbreviations. Relations among different units of measurement of one measure.	Year 5 Area (ha, a, m^2), mass (t, c), speed (km/h, m/s).	Year 6
Year 7 Finding a value by using a formula.	Year 8	Year 9 Converting larger units of measurement to smaller ones and vice versa.
Year 10	Year 11 Prism, its elements, calculating the area and volume of elements.	Year 12 Pyramid and truncated pyramid, its elements, calculating the measures, area, and volume of elements. Rotating bodies (cylinder, cone, truncated cone, ball, sphere), their elements, calculating the measures, area, and volume of elements. Combinations of geometric figures (ball and prism, cone, or cylinder, cone and cylinder or pyramid, cylinder and prism). Mutual position and properties of figures and bodies, size, area, volume, of elements, and their relations.

Sciences and Mathematics" for Years 7–9 (Matemātika. Mācību priekšmeta programmas paraugs projekta „Dabaszinātnes un matemātika" vietnē). In this programme, measures have been mentioned in Years 8 and 9.

Year 8: area. Units of measuring area, properties of area, formulae for calculating the area of a triangle and a circle, geometric bodies (right prism, pyramid, cone), their elements and properties, nets and areas of a right prism and a cylinder, the volume and units of measuring volume and formula for calculating the volume of a right prism and a cylinder.

The attainable results state that after covering this topic a pupil is able to apply the properties of area for the calculation and proving tasks, to understand the notion "equivalent figures", to create a mathematical expression for calculating the area of a figure, to use formulae for calculating the area of a triangular and a circle, as well as the volume of a right prism and a cylinder.

Year 9: perimeter and area of combined figures.

A comparison of the current standards with the previous ones developed at the beginning of the 1990s suggests that the current standards are much more detailed.

For example, Guidelines and Standard for Primary Education (Sākumizglītības vadlīnijas un standarti, 1992), when speaking about measurements and measuring, state that pupils must gain understanding about measurements (length, time, volume, mass, area, speed) and measuring and assessment thereof, and understanding about the scale and the relations among various measuring units, as well as they must be able to transform and compare measurements and solve arithmetic tasks by using measurements. In addition, the Primary Education Standard (Pamatizglītības standarts. Matemātika, 1992) states that pupils must be able to calculate the area of a figure and volume of geometric figures by applying the necessary constructions and measurements.

The analysis of the standard and programmes shows that school mathematics covers nine measures — length, area, volume, mass, time, money, temperature, the size of an angle, and speed. It is planned to cover all these measures during the basic education stage from Year 1 until Year 9. Length, time, temperature, money, mass, speed, and the size of angle are covered by Year 6, while the area and volume – by Year 9. Thus, learning measures is planned for 9 years, which is quite a long period of time; however, the knowledge and skills to be covered are rather scattered. A systematic summary of the topic of area is presented in Year 8. Most often, the subject is not separated from other subjects, and a summary is missing in general. Learning of measurements is also hindered by gaps, for example, in Years 6, 8, and 10. It is necessary to make more precise and improve the topic of measurements also in the standard and programmes. Over the last years, several historic units of measurement used in some other EU countries (inches, pints, yards, feet, pounds, etc.) previously not used in Latvia have entered the everyday life. These units of measurement have been included neither in the standards nor in the programmes.

Accordingly, measurement is both everywhere and nowhere in the primary and secondary mathematics curriculum. That is, this measurement occurs across the topics that comprise primary and secondary school mathematics, but the ideas of measurement are so scattered that the teaching of measurement in secondary school mathematics may well lack some focus that might store up problems for learners as they progress with mathematics.

The project “Measures” for the future mathematics teachers

The analysis of standards and programmes shows that the topic of measurements is rather scattered and that a concise summary is needed at the end thereof. Year 9 is the most suitable time for giving this summary, since all the separate elements of the topic are more or less covered by that time. The summary can be provided in various forms, and the remaining part of the article is devoted to one of them — classes at teaching stations.

A study – a project on measurement in mathematics – was carried out in the Liepaja University for future mathematics teachers. The aim of the study was to explore the possibility of creating a compact vision for the measurement in school mathematics course and incorporating it into the learning process of future mathematics teachers. In 2015, 15 students of the programme “Teacher of Basic Edu-

cation" implemented the project for two weeks. The future mathematics teachers had to prepare and run six various activities at each of the five measurement stations (length, area, volume, mass, time).

Each of the five measurement stations included six various activities:

1. Systematization and visualization of historic units of length, area, volume and weight. The pupils got to know new units of measurement and their relation with other familiar units of measurement; they learned how to measure by using pounds, feet, inches, yards, etc.

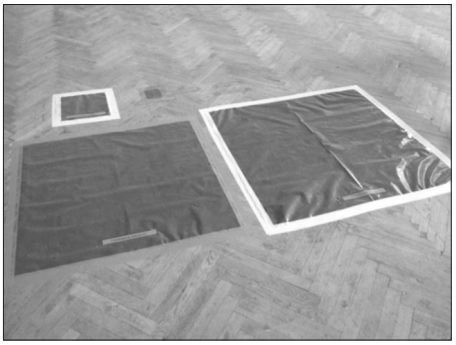

Figure 1. ***Learning new units of measurement***

2. Preparation of exercises of base units of the International System (SI) and metric prefixes. The pupils had to find the unit of measurement proper for the size of the object.

Figure 2. ***Activity on the SI units of measurement***

3. Activities for comparing and evaluating different measures. Each of the covered measurements can be found also in

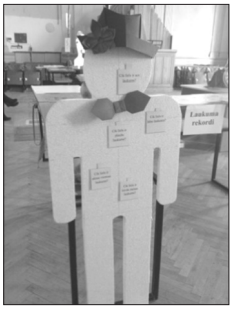

Figure 3. ***Activity on the measures in a human body***

the human body. In order to answer the questions of the self-control activity, the pupils had to evaluate and compare different measures. The answer was hidden in a human model.

4. Activities for improving the measuring skills. The pupils got to know new methods for performing measurements, for example, how to measure the reaction time by using a ruler or how to measure the volume of an irregular body, for example, arm, by using water or sand, etc.

Figure 4. ***Learning methods of measuring***

5. Preparation of educational experiments related to measurement. During the educational experiments, pupils learned the relations among measures, for example, between the perimeter and the area, mass and volume.

Figure 5. ***Experiments with measures***

6. Exercises on the boundary values of measures. Within this activity, the pupils had to evaluate the minimum and maximum value of a measure, for example, the states with the largest and smallest area, the tallest and the shortest person, etc.

Figure 6. ***Activity on the boundary values of measures***

During the project, students developed presentations and sets of interactive tasks involving length, area, volume, time, and mass to help building understanding about measurements of the future mathematics teachers.

The research instrument of project results was students' feedbacks. All students evaluated the preparatory stage and the gained compact knowledge positively.

During the project, the tasks drafted were tested during classes for pupils of Years 4–9. Each class was 60 minutes long, and pupils participated in activities covering 5 different measures.

Figure 7. ***A class for students***

In total, 723 pupils participated, out of which 489 pupils wrote a voluntary feedback on the class. In total, 64% of pupils evaluated the class as interesting, 34% – as very interesting, while 2% of students found the class to be boring. Almost all pupils participated in the class with enthusiasm and interest, and the activity of boys should be pointed out especially. According to the qualitative analysis of feedbacks, pupils learned many new things during the class and developed their measuring skills, as well as gained a better understanding of measurements.

Conclusions

The analysis of regulatory documents – the standard and programmes – has shown that it is necessary to precise, define, and improve the planning of the topic of measurements for Years 1–9 in order to develop it in line with the complex knowledge and skills connected with measurements.

During the research, didactic materials for teaching five measurements (length, area, volume, mass, time) and measuring thereof were developed and tested; these materials are suited for the final stage of teaching measurements. This work should be continued by developing didactic materials about the connections among measurements and connections among values.

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MATAVIMAI MOKANT MATEMATIKOS LATVIJOS MOKYKLOSE

Vaira Karklina

S a n t r a u k a

Straipsnyje aptariama vienos iš matematikos mokymo sričių – matavimų – situacija Latvijos mokyklose. Matematikos mokymo specialistai sutinka, kad mokant matų ir matavimų svarbu, jog mokiniai suprastų bendrų matavimo vienetų būtinumą ir universalumą, kai yra poreikis išmatuoti kokį nors objektą ir reikia nustatyti, kiek tam tikrų matavimo vienetų į jį telpa. Matavimas – ne tik paprastas gebėjimas, kuris yra formuojamas matematikos pamokose, bet veikiau sudėtingas sąvokų ir įgūdžių derinys, į kurią įeina gebėjimas suvokti aplinkoje esančių daiktų dydį: ilgį, plotį, aukštį, tūrį, gebėjimas pasitelkiant skirtingus matus ir matavimo vienetus bei matavimo įrankius išmokti pamatuoti įvairius kasdienio naudojimo objektus.

Paprastai matematikos disciplinos, kurių mokoma mokykloje, struktūruojamos į aritmetiką, algebrą ir geometriją. Kuriant bendrąsias ugdymo programas kyla klausimas, į kurį iš minėtų dalykų turėtų būti įtraukiama matai ir matavimų veiklos sritis? Ilgio ir ploto matavimai dažniausiai susiję su geometriniu matematikos aspektu, o laiko matavimas su skaičiavimu.

Pasak Knuth (2002), matavimų tematikos kompleksiškumas yra veikiamas kelių sąlygų. Prieiga, taikoma algebros ir geometrijos mokymuisi, skiriasi nuo tos, kurios reikia mokantis matavimų, nes geometrijos ir algebros mokymosi pagrindas yra dedukcinis. O matavimų mokymuisi reikia visai kitos empirinės prieigos, kurioje svarbu praktinė veikla, matavimų atlikimo pratybos, po kurių vyksta perėjimas į matematikos abstrakčių simbolių kalbą.

Straipsnyje pateikiama atlikta Latvijos švietimo dokumentų ir bendrojo ugdymo programų analizė, kuri parodė, kad Latvijos mokyklose matavimų mokiniai pradeda mokytis nuo pat matematikos mokymosi pradžios, nuo pirmos klasės ir baigia 12 klasėje, tačiau duomenys rodo, kad matavimų temų išdėstymas matematikos programoje nėra idealus, šiai sričiai reikia daugiau dėmesio norint gerinti mokinių matavimo gebėjimus. Straipsnyje taip pat trumpai pristatomi projekto, skirto būsimų matematikos mokytojų matavimo mokymo kompetencijai tobulinti, rezultatai.

Pagrindiniai žodžiai: matai, matavimai, standartai, programos, projektai.

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