ELECTRICITY PRICE FORECASTING USING MONTE CARLO SIMULATION: THE CASE OF LITHUANIA

Andrejus Ngujen Tat*

Vilnius University, Lithuania

Abstract. *The main purpose of this article is to determine the practical use of the Monte Carlo simulations in electricity markets for forecasting future prices. First, we review the structure of the electricity markets – how they work, what implications do they have and how they've evolved during the last decades. Second, we disco-ver that there are only few researches that have been made on this topic as well as there haven't being made any researches regarding the Lithuanian electricity market. Then, we will carry out an analysis on how to use a Monte Carlo simulation approach in electricity markets. A Mean-Reverting process method will be introduced, which, at first, was used to predict oil prices. Also, we analyze the essence of price spikes and find a solution on how to predict them.*

Keywords: *Monte Carlo simulation, mean-reversion, electrify market, price spikes, forecasting.*

Introduction

In 20th century, electricity markets have experienced significant changes. Market liberalization covered all areas of energy sector. From centralized vertical structures, markets moved to horizontal ones. In Europe, a large number of directives and regulations were introduced. Instead of a monopoly market with fixed pricing models, power exchanges appeared, creating new opportunities for businesses.

Electricity is a unique commodity that has specific attributes. First of all, electricity is a non-storable product, i.e., it must be consumed once it was produced. It leads to a necessity to balance the entire power system in any period of time. Recently, there have appeared more and more new technological innovations to mitigate or even solve the non-storability issue (house batteries connected to a distribution network, house-scale systems synchronized with electric vehicles that help absorb any production surplus or lend capacities when there is a production deficit). However, at this moment, these new technologies are at the project stage and were tested on a small scale.

* Corresponding author:
Faculty of Economics and Business Administration, Vilnius University,
Sauletekio Ave. 9-II, LT-10222 Vilnius, Lithuania
Email: andrejus.ngujentat@gmail.com

Second, to trade electricity on the markets, it requires a physical infrastructure that connects electricity producers with electricity consumers. Infrastructure can be divided into two levels:

- Electricity transmission: a 110–800 kV transmission grid through which electricity flows from medium and large power plants to medium and large electricity consumers, such as mills or factories. Also, it is used to import or export electricity from other countries or other transmission systems.
- Electricity distribution: electricity is distributed to the households or small-medium businesses.

Third, electricity demand is totally inelastic from the economic point of view. It means that consumption stays the same in short and medium terms despite any price shocks. For example, if the wholesale electricity prices increase by 50% for some period, it doesn't mean that consumers will decrease their consumption or even stop to consume. Generally, the largest electricity consumers are large businesses related to manufacturing. They won't shut down their factories and mills due to increased prices. Another cause of demand elasticity is consumption variation in the different times of a day. Basically, consumption during the night significantly decreases comparing the day (directly related with working day hours).

These peculiarities of the electricity market create a lot of difficulties for forecasting its future prices. The power markets are still very young comparing not only with financial markets but with other commodity markets, such as oil or agricultural produce. At this moment, there is not so much scientific literature regarding electricity forecasting; therefore, this article is intended for a brief introduction of the application of the Monte Carlo models in electricity markets.

Literature Review

Liberal electricity markets, where trading is executed through power exchanges, can be characterized as a very volatile. Their strong volatility enforces market participants to explore solutions to manage high volatility risks. Therefore, in academic literature, researchers pay most of their attention to these two areas –predicting electricity prices and managing price-related risks.

S. Pinedo and A. J. Conejo (2012), in their work paper, described the use of derivatives for managing market risks hedging against price volatility. They applied a multi-stage, stochastic model to simulate an optimal portfolio of instruments. They tried to prove that options are more effective than usual forward contracts. Another work paper on financial derivatives as the main tool of risk management was written by S. J. Deng and S. S. Oren (2001). They conducted a comprehensive analysis of the existing hedging

instruments and its pricing. Authors concluded that the hedging derivatives are able to mitigate price volatility and must be used by market participants.

A. Giovanni and A. Gigli (2001) tried to explore the causes behind the sudden prices spikes. They made an empirical analysis of the Nordic wholesale power market. A Garch-Earji jump model was employed covering all available related statistical data to identify price jumps. The results showed that the nature of price jumps depends on market structure. Another empirical research regarding price jumps was made by C. Blanco and D. Soronow (2001). They declared that electricity spot prices do not jump but rather spike. These price spikes occur for a very short period of time. Later, prices instantly return to the long-term average. The authors suggested that the prices can spike in both directions (up and down). F. E. Benth, R. Kiesel and A. Nazarova (2011) made an empirical analysis of three mathematical models. The first model was a jump-diffusion model filled with all available data that directly or indirectly affect electricity prices. The second model was proposed by A. Roncoroni (2002) and later complemented by A. Roncoroni and H. Geman (2006). It was an Ornstein-Uhlenbeck process driven by a Brownian motion together with a Poisson process to catch the price spikes. The third model was proposed by F. E. Benth et al. (2008) as a model to price forward contracts. An empirical analysis showed that all three models indicated a strong sensitivity to the market structure and penetration.

To sum up, we can say that the majority of researchers underlined the high volatility and significance of the market structure. Price spikes make it difficult to forecast as well as evaluate the prices of derivatives.

Methodology

To begin with, electricity prices and their formation differ from financial markets. These have two major attributes – mean reversion and price spikes. The mean reversion can be explained as spot price fluctuations around its long-term average. As any commodity, electricity has its cost of generation, which differs depending on generation type in a specific country as well as the diversification of generation. Countries where most of the generation capacity is based on fossil fuels will have different production costs in comparison with countries where renewables dominate the market. Price spikes refer to a significant increase or decrease in spot prices for a very short time. They occur when the whole power system faces either foreseen or unforeseen changes in generation capacity, consumption or outage in the transmission grids.

Mean reversion and price spikes require that the stochastic forecasting model should also fluctuate around the long-term average as well as be able to catch price spikes if specific conditions are met. R. Weron (2005) introduced a forecasting model that was called the Mean-Reverting Jump-Diffusion process. This model covered both mean reversion

and price spike aspects. To catch spikes, the Poisson jump process was introduced. It supposed that price spikes occur suddenly – so the question is, how often?

In this article, the mean-reverting model will be introduced; however, unlike Weron's (2005) model, we won't use a Poisson process, making an assumption that price spikes can be caught if specific conditions are met.

First, we need to define a mean reversion process. It can be defined as a stochastic differential equation:

$$dlogP_t = \beta(\mu - logP_t)dt + \sigma dR_t \qquad (1)$$

where P_t is a spot price at moment t and β is a measure of defining the speed of reversion.

The above equation is also known as an Ornstein-Uhlenbeck process, which leads us to the further expression $\{S_t, t \geq 0\}$, where $S_t \equiv logS_t$:

$$S_t = S_0 * e^{-t} + \mu \int_0^t e^{-(t-m)} dm + \sigma \int_0^t e^{-(t-m)} dBm \qquad (2)$$

where $\{S_t, t \geq 0\}$ and $0 \leq m \geq t$.

It follows that the conditional variance and expectation with the probability measure γ and filtration φ should be

$$E_\gamma[S_t|\varphi_m] = \mu + (S_0 - \mu) * e^{-t} \qquad (3)$$

and

$$\sigma_\gamma^2[S_t|\varphi_m] = (1 - e^{-2t}) * \frac{\sigma^2}{2} \qquad (4)$$

Now, we have a Mean-Reverting model:

$$dlogP_t = \beta(\mu - logP_t)dt + \sigma dR_t \qquad (5)$$

where μ is equal to long run mean, σ is long run standard deviation. Both are calculated from the historical dataset. The mean reversion criteria β is calculated as:

$$\beta = \frac{\sum_{t=1}^n (S_t - \bar{S})((S_{t+1} - S_t) - (\overline{S_{t+1} - S_t}))}{\sum_{t=1}^n (S_t - \bar{S})^2} \qquad (6)$$

If the estimated criteria β is positive, there is no mean reversion and changes must be made. If the criteria β is negative, then β is positive which indicates the presence of mean reversion. In our case β is equal to 0.395. It means that a mean reversion exists. To make sure, a t-test might be performed (in our case, the t-test repeatedly proved the existence of mean reversion).
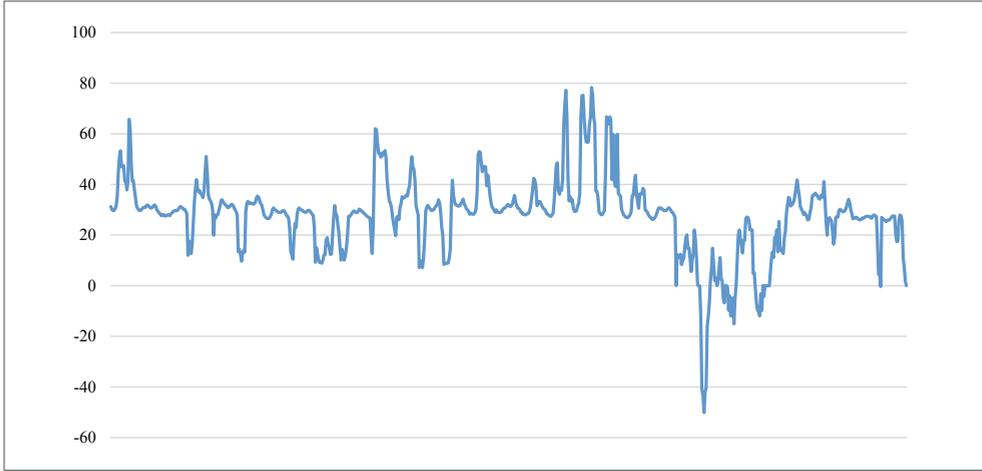
FIG. 1. **Spot prices during 2017 December in the Danish bidding zone at NordPool exchange, Eur/MWh**

The next step is to express a function that defines price spikes and its sizes. It must be noted that electricity prices can be negative (Fig. 1). This occurs when supply exceeds demand. In that case, the market operator (power exchange) can employ additional measures for balancing the system.

The possibility of having negative electricity prices must be included into the model scope. As it was mentioned above, price formation basically relies on the demand-supply and infrastructure capacity, which are represented by electricity production and import on one side and by electricity consumption with export on the other, connected via transmission grid. Any sudden changes in these variables create price spikes. Therefore, the price spike factor ω can be expressed as a function:

$$\omega = f(C; G; E_x; I_m) \tag{7}$$

where

    C – forecasted consumption;
    G – forecasted generation;
    Ex – forecasted export;
    Im – forecasted import.

An Ornstein-Uhlenbeck process shows that prices will always fluctuate around its average, which means every (7) equation's variable has its equilibrium value when the prices are equal to the long-term mean (Fig. 2). Making this assumption allows us to say that any deviation from the equilibrium can trigger prices to go down or up with the different level of significance.[1]

---

[1] This assumption is made by eliminating seasonality and long-term infrastructure projects, which can affect the equilibrium.
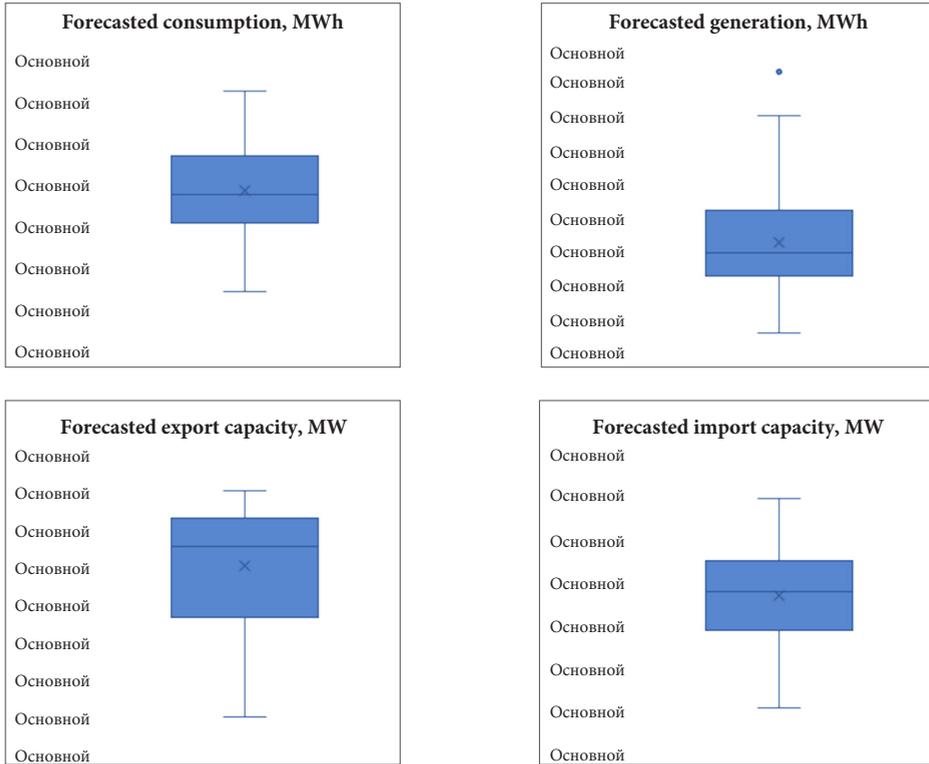
Fig. 2. **Distribution and equilibrium values of consumption, generation, export and import**

Therefore, the price spike factor can be expressed as:

$$\omega_t = a_1\,C_t + a_2 G_t + a_3 Ex_t + a_4 Im_t + \varepsilon \tag{8}$$

where a_1, a_2, a_3, a_4 weight coefficients and C_t, G_t, Ex_t, Im_t – standardized values at period t which vary in (-1;1) interval depending on deviation from an equilibrium value.

Finally, we can derive the equation of our model:

$$dlogP_t = \omega(\beta(\mu - logP_t)dt + \sigma dR_t) \tag{9}$$

Variance Reduction for Improving Accuracy

In statistics, when we use the methods of the Monte Carlo simulations, variance reduction is an indispensable part of calculations, which is used to increase the precision of the estimates that can be obtained for a given simulation or computational effort (Botev, Ridder 2017). Every randomly generated output variable using the simulations is related to a variance that directly affects the precision of the simulation results. Therefore, variance reduction techniques are used to obtain greater precision, reduce confidence intervals, make the calculations faster and shorter. It makes simulations statically more

efficient instead of using brute force by increasing the number of iterations. The most common variance reductions techniques are random numbers, antithetic variates, control variates, importance sampling and stratified sampling. In this article, we take a look at two techniques – moment matching and antithetic variates.

Moment Matching

The moment matching method is also one of the most popular techniques, sometimes known as a quadratic resampling. The essence of this method consists of making adjustments in a sample taken from a population with a normal distribution; thereby, the first, second and possibly subsequent moments are matched. Suppose there are samples S_i taken from a normal distribution with a mean of zero and the standard deviation equal to 1. The next step is to calculate sample's mean $\bar{S}$ and its standard deviation σ to match moments, defined by the following expression:

$$S_i^* = \frac{S_i - \bar{S}}{\sigma} \tag{10}$$

It allows us to get a new set of samples which have adjusted mean and standard deviation. These samples should be used in further calculations.

Antithetic Variates

Suppose we use the Monte Carlo simulations. The calculated averages of quantities $f(P_i)$ result in randomness within the algorithm, and it leads to error cancellation. Using antithetic sampling, we try to get more cancelation. An antithetic sample is a sample that is composed of opposite values of $f(X_i)$. This technique allows us to derive the $f(X_{av})$ as an average of two opposite functions.
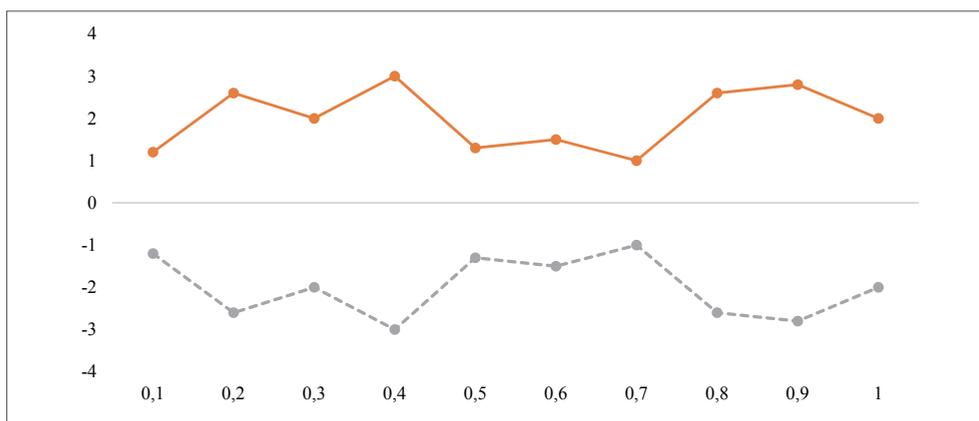

FIG. 3. **The chart shows a random trajectory of several points connected via solid lines along with its antithetic reflection.**

Now, suppose we want to calculate a specific day's electricity spot price S_i. To do that, we select samples B_i from the normal standard distribution. Simultaneously, we can calculate another value $\bar{S}_i$ using B_i and changing the sign of B_i. Now, to get a new value of S_{av}, we need to compute an average:

$$S_{av} = \frac{1}{n} \sum_{i=1}^{n} \frac{S_i + \bar{S}_i}{2} \tag{11}$$

A new estimator S_{av} is unbiased and can be used to in further calculations. The advantages of using antithetics is that the antithetic pairs B_i and $-B_i$ are distributed more regularly than a sample of $2n$ (Boyle, Broadie and Glasserman 1997).

Since we know that

$$Var\left(\frac{S_i + \bar{S}_i}{2}\right) = Var(S_{av}) = \frac{1}{2}(Var(S_i) + Var(\bar{S}_i) + cov(S_i, \bar{S}_i)) \tag{12}$$

Therefore,

$$Var(S_{av}) \leq Var(\bar{S}_i) \text{ if } cov(S_i, \bar{S}_i) \leq Var(S_i) \tag{13}$$

However, S_{av} uses $2n$ replications unlike $\bar{S}_i$, so

$$2Var(S_{av}) \leq Var(\bar{S}_i) \tag{14}$$

It comes up with the following condition:

$$cov(S_i, \bar{S}_i) \leq 0 \tag{15}$$

It allows us to run only $\frac{n}{2}$ S_{av} simulations instead of $n\bar{S}_i$ simulations. Therefore, S_{av} is a reasonable choice for us if the condition is met:

$$Var(S_{av}) \leq Var(\bar{S}_i) \tag{16}$$

It will reduce the variance as well as decrease the number of iterations.

Results

This model was applied to the Lithuanian day-ahead power market, which is part of the Nordic-Baltic market. The observation period was chosen from June 1, 2017 to November 28, 2017. The model was recalculated for 180 times, every day, making 20 000 iterations a day, predicting the next day's prices.

First, the simulation was made using only an Ornstein-Uhlenbeck process without catching price spikes. As you can see, the forecasted prices distributed in the 35–40 Eur/
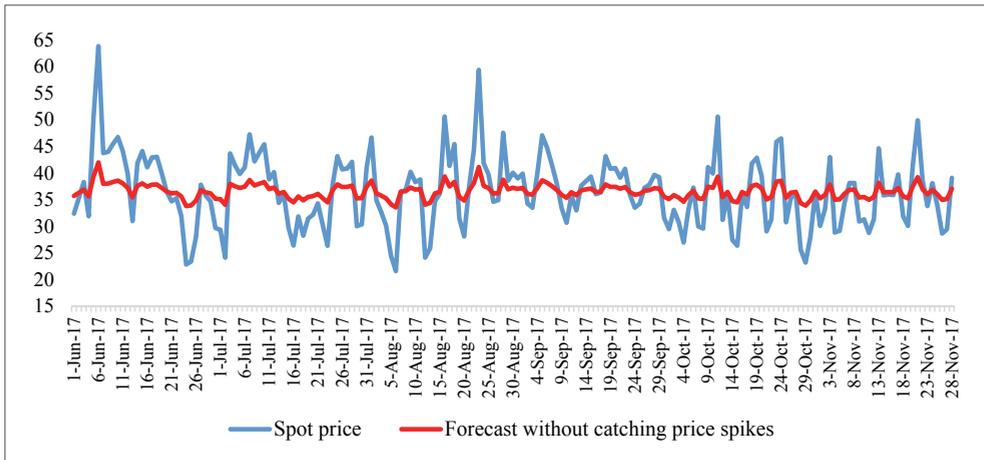
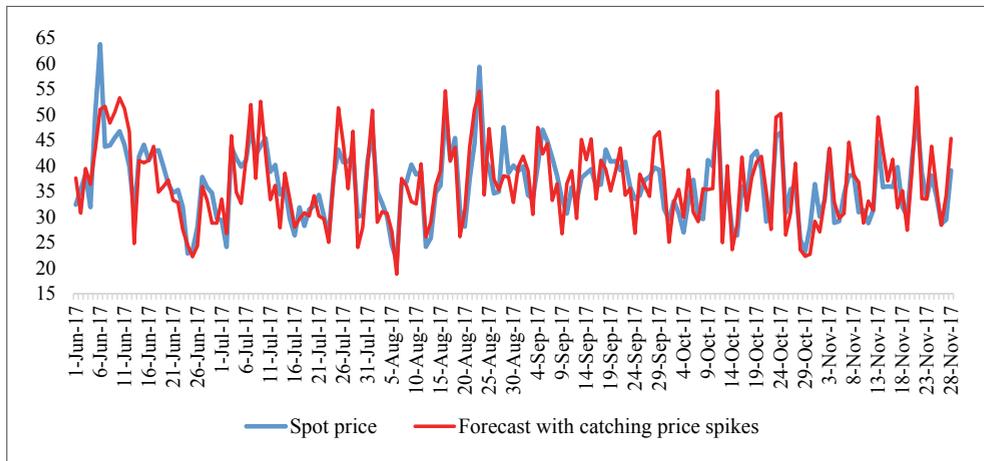FIG. 4. **A comparison of spot and forecasted electricity prices without catching price spikes, Eur/WMh.**

FIG. 5. **A comparison of spot and forecasted electricity prices with catching price spikes, Eur/MWh.**

MWh interval, iterating only the general trend. The obtained correlation coefficient is 0.83, which shows us that a strong relation does exist. The model was not able to catch price spikes; therefore, the achieved results cannot be precise.

The second simulation was run by adding the price spikes factor to the model. The results showed significant changes in our forecast (Fig. 5). The model allowed us to track not only the general trend, but it was able to catch most of the price spikes. Calculated correlation coefficient was 0.81, which also shows us a strong relation. As you can see on Fig. 5, there were several price spikes in June. The model was able to catch them due to significant changes in import and export capacities. Market supply was decreased and it caused prices to go up. It happened because of the outages on the interconnections.

As we can see, the model catches not only the price spikes that went up but also the sudden downward spikes. It is very important for forecasting, because frequently, researches concentrate only on prices spiking up, which, of course, is more important to buyers if we look at the problem from the market perspective. However, downward spikes create great risks to sellers that do not hedge for a long period and have difficulties with cash flow management.

Conclusion

To sum up, we can say that the Monte Carlo simulation techniques can be applied to forecast electricity prices. Due to its nature, electricity spot prices are very volatile. Arising price spikes increase uncertainty and make predictions difficult. Mathematical models based on the concept of mean-reversion – the assumption that prices fluctuate around their long-term average – is a good choice for forecasting electricity prices, as its pricing is based on cost of generation. The results of the empirical analysis showed that a mean-reversion model with the price spikes factor, used to forecast electricity price in the Lithuanian power market, can give us a statistically accurate prediction. The model was able to predict the general trend as well as to catch most of the price spikes.

REFERENCES

Amelin M. (2004). On Monte Carlo Simulation and Analysis of Electricity Markets. Doctoral Thesis. Royal Institute of Technology.

Benth, F.E., Cartea, Á. and Kiesel, R. (2008). Pricing forward contracts in power markets by the certainty equivalence principle: Explaining the sign of the market risk premium. Journal of Banking & Finance, 32(10), 2006-2021.

Benth, F.E., Kiesel, R. and Nazarova, A. (2011). A critical empirical study of three electricity spot price models. Energy Economics.

Blanco, C. and Soronow, D. (2001). Jump diffusion process-energy prices processes used for derivatives pricing & risk management.

Botev, Z.; Ridder, A. (2017). "Variance Reduction".

Boyle, P. (1977). Options: a Monte Carlo approach. Journal of Financial Economics, 4, 323-338.

Chan Shung Tak (2009). Identifying Risk Factors in the Generating Section of the Power Plants. Master thesis. Maastricht School of Management.

Deng, S. J. and Oren, S. S. (2006). Electricity derivatives and risk management. Energy, 31(6-7), 940-953.

Deng, S.J., Johnson B. and Sogomonian, A. (2001). Exotic electricity options and the valuation of electricity generation and transmission assets. Decision Support Systems, 30 (3), 383-392.

Geman, H. and Roncoroni, A. (2006). Understanding the fine structure of electricity prices. Journal of Business, 79 (3), 1225-1261.

Giovanni, A. and Andrea Gigli, A. (2002). Electricity Derivatives. Working Paper, National Center of Competence in Research, Università della Svizzera Italiana.

Hoff A. (2010). An analysis of energy prices and market efficiency at Nord Pool: Master Thesis. Norwegian School of economics and Business Administration. Area of specialization: Financial Economics.

Kristiansen T. (2004). Risk Management in Electricity Markets Emphasizing Transmission Congestion. Master thesis.

Määttä T., Tom-Filip Johansson (2011). The System Price of Electricity on Nord Pool: Bachelors's thesis. Umeå University, Department of Economics.

Nobaza L. (2012). Efficient Monte Carlo Methods for Pricing of Electricity Derivatives: Master thesis. University of the Western Cape.

Pavlovitch Kolos (2005). Risk Management in Energy Markets. Dissertation. The University of Texas at Austin.

Pinedo, S. and Conejo, A.J. (2012). Managing the financial risks of electricity producers using options. Energy economics, 32(7), 1268-1276.

Roncoroni, A. (2002). Essays in Quantitative Finance: Modelling and Calibration in Interest Rate and Electricity Markets. PhD dissertation., University Paris Dauphine.

Voronin S. (2013). Price spike forecasting in a competitive day-ahead energy market: PhD dissertation. Diss. Lappeenranta University of Technology. ISBN 978-952-265-461-8, ISBN 978-952-265-462-5 (PDF), ISSN-L 1456-4491, ISSN 1456-4491.

Weron, R. (2005). Heavy tails and electricity prices.

Yan, X. (2009). Electricity market clearing price forecasting in a deregulated electricity market: Master thesis, University of Saskatchewan.