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Negative Real Balance Effects in the Presence of Involuntary Unemployment

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Abstract. We examine the existence of negative real balance effects (or so-called Pigou effects) by falls in the nominal wage rate and the prices of the goods in situations where there is involuntary unemployment using a three-generations overlapping generations model, with a childhood period and pay-as-you go pension system for the older generation of consumers. We will show that if the net savings of the younger generation of consumers are larger than their debts due to consumption in their childhood period, there exists a positive real balance effect, and employment increases by a fall in the nominal wage rate; on the other hand, if the net savings of the younger generation of consumers are smaller than their debts, there exists a negative real balance effect, and employment decreases by a fall in the nominal wage rate.
Keywords: negative real balance effects, involuntary unemployment, three-generations overlapping generations model.

1. Introduction

In this paper we examine the existence of negative real balance effects (or so-called Pigou effects) by falls in the nominal wage rate and the prices of the goods in situations where there is involuntary unemployment. The positive real balance effect works when the real assets of consumers increase due to falls in the nominal wage rate and the prices promote their consumption. Then, involuntary unemployment, due to lack of demand, could be reduced naturally. However, as Kalecki (1944) said in his comments to Pigou (1943), the positive real balance effect may not work or may work in reverse when people's debts are greater than their assets.

Involuntary unemployment is a phenomenon when workers are willing to work at the market wage or just below but are prevented by factors beyond their control, mainly, the deficiency of aggregate demand. Umada (1997) derived an upward-sloping labor demand curve from the mark-up principle for firms and argued that such an upward-sloping labor demand curve leads to the existence of involuntary unemployment without wage rigidity.

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But his model of firm behavior is ad-hoc[1]. Otaki (2009) assumes the indivisibility of labor supply and has shown the existence of involuntary unemployment using efficient wage bargaining according to McDonald and Solow (1981). The arguments of this paper do not depend on bargaining. If labor supply is indivisible, it may be 1 or 0. On the other hand, if it is divisible, it takes a real value between 0 and 1. As discussed in Otaki (2015) (Theorem 2.3) and Otaki (2012), if labor supply is divisible and very small, no unemployment exists[2]. However, we show that even if labor supply is divisible, unless it is so small, there may exist involuntary unemployment. We consider consumers' utility maximization and firms' profit maximization in an overlapping generations (OLG) model under monopolistic competition according to Otaki (2007, 2009, 2011, 2015). We extend Otaki's model to a three-generations OLG model with a childhood period, and we also consider the pay-as-you-go pension system for the older generation of consumers. We show that in such a model there can be a positive or negative real balance effect in situations where there is involuntary unemployment.

In the next section we explain the model and analyze consumers' utility maximization and firms' profit maximization. In Section 3 we show the following results (Proposition 1).

1. If the net savings of the younger generation of consumers are larger than their debts due to consumption in their childhood period, there exists a positive real balance effect, and employment increases by a fall in the nominal wage rate.
2. If the net savings of the younger generation consumers are smaller than their debts, there exists a negative real balance effect and, employment decreases by a fall in the nominal wage rate.

In Section 4, we discuss the role of fiscal policy in a situation where COVID-19 is prevalent.

2. The model and behaviors of agents

2.1. Consumers' utility maximization

We consider a three-periods (0: childhood, 1: younger or working, and 2: older or retired) OLG model under monopolistic competition. It is a re-arrangement and an extension of the model put forth by Otaki (2007, 2009, 2015). The structure of our model is as follows.

1. There is one factor of production, labor, and there is a continuum of perishable goods indexed by $z \in [0,1]$. Good z is monopolistically produced by firm z with constant returns to scale technology.
2. Consumers consume the goods during the childhood period (Period 0). This consumption is covered by borrowing money from (employed) consumers of the younger generation and/or scholarship. They must repay these debts in their

[1] Lavie (2001) presented a similar analysis.

[2] About indivisible labor supply also please see Hansen (1985). In Tanaka (2020) involuntary unemployment under indivisible labor supply is analyzed.

Period 1. However, unemployed consumers cannot repay their own debts. Therefore, we assume that unemployed consumers receive unemployment benefits from the government, which are covered by taxes on employed consumers of the younger generation.

3. During Period 1, consumers supply l units of labor, repay the debts and save money for their consumption in Period 2. They also pay taxes for the pay-as-you go pension system for the older generation.
4. During Period 2, consumers consume the goods using their savings carried over from their Period 1 earnings and receive the pay-as-you go pension, which is a lump-sum payment. It is covered by taxes on employed consumers of the younger generation.
5. Consumers determine their consumptions in Periods 1 and 2 and the labor supply at the beginning of Period 1. We assume that their consumption during the childhood period is constant.

Further we make the following assumptions.

Ownership of the firms. Each consumer inherits ownership of the firms from the previous generation. Corporate profits are distributed equally to consumers of the younger generation.

Zero interest rate. We assume a zero interest rate and that the repayment of debts of consumers is assured. Consumers' borrowing in the childhood period is constant. If the savings of consumers in the younger period are insufficient for the borrowing, the government lends the scholarship to consumers in the childhood period. Consumers in the younger period are indifferent between lending money to childhood period consumers and savings by money.

Due to the existence of pay-as-you-go pension, the savings are likely to be insufficient for borrowing when consumption by consumers in the childhood period is not so small.

Notation

We use the following notation.

C_i^e: consumption basket of an employed consumer in Period i, $i = 1,2$.

C_i^u: consumption basket of an unemployed consumer in Period i, $i = 1,2$.

$c_i^e(z)$: consumption of good z of an employed consumer in Period i, $i = 1,2$.

$c_i^u(z)$: consumption of good z of an unemployed consumer in Period i, $i = 1,2$.

D: consumption basket of an individual in the childhood period, which is constant.

P_i: the price of consumption basket in Period i, $i = 1,2$.

$p_i(z)$: the price of good z in Period i, $i = 1,2$.

$\rho = \frac{P_2}{P_1}$: (expected) inflation rate (plus one).

W: nominal wage rate.
R: unemployment benefit for an unemployed individual. $R = D$.
$\widehat{D}$: consumption basket in the childhood period of a next generation consumer.
Q: pay-as-you-go pension for an individual of the older generation.
Θ: tax payment by an employed individual for the unemployment benefit.
$\hat{Q}$: pay-as-you-go pension for an individual of the younger generation when he retires.
Ψ: tax payment by an employed individual for the pay-as-you-go pension.
Π: profits of firms which are equally distributed to each consumer.
l: labor supply of an individual.
$\Gamma(l)$: disutility function of labor, which is increasing and convex.
L: total employment.
L_f: population of labor or employment in the full-employment state.
y: labor productivity.

We assume that the population L_f is constant. We also assume that the nominal wage rate is constant in this section. We examine the effects of a change in the nominal wage rate in Section 3.

We consider a two-step method to solve the utility maximization of consumers, such that:

1. Employed and unemployed consumers maximize their utility by determining consumption baskets in Periods 1 and 2 given their income over two periods.
2. Then, they maximize their consumption baskets given the expenditure in each period.

Since the taxes for unemployed consumers' unemployment benefits are paid by employed consumers of the same generation, $D(= R)$ and Θ satisfy

$$D(L_f - L) = L\Theta.$$

This means

$$L(D + \Theta) = L_f D.$$

The price of the consumption basket in Period 0 is assumed to be 1. Thus, D is the real value of the consumption in the childhood period of consumers.

Also, since the taxes for the pay-as-you-go pension system are paid by employed consumers of the younger generation, Q and Ψ satisfy the following relationship:

$$L\Psi = L_f Q$$

The utility function of employed consumers of one generation over three periods is

$$u(C_1^e, C_2^e, D) - \Gamma(l).$$

We assume that $u(\cdot)$ is a homothetic utility function. The utility function of unemployed consumers is

$$u(C_1^u, C_2^u, D).$$

The consumption baskets of employed and unemployed consumers in Period *i* are

$$C_i^e = \left(\int_0^1 c_i^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}},\ i = 1,2,$$

and

$$C_i^u = \left(\int_0^1 c_i^u(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}},\ i = 1,2.$$

σ is the elasticity of substitution among the goods, and $\sigma > 1$.

The price of consumption basket in Period *i* is

$$P_i = \left(\int_0^1 p_i(z)^{1-\sigma} dz\right)^{\frac{1}{1-\sigma}},\ i = 1,2.$$

The budget constraint for an employed consumer is

$$P_1 C_1^e + P_2 C_2^e = Wl + \Pi - D - \Theta + \hat{Q} - \Psi.$$

The budget constraint for an unemployed consumer is

$$P_1 C_1^u + P_2 C_2^u = \Pi - D + R + \hat{Q} = \Pi + \hat{Q}\ \ (\text{since } R = D).$$

Let

$$\alpha = \frac{P_1 C_1^e}{P_1 C_1^e + P_2 C_2^e},$$

and

$$1 - \alpha = \frac{P_2 C_2^e}{P_1 C_1^e + P_2 C_2^e}.$$

Since the utility functions $u(C_1^e, C_2^e, D)$ and $u(C_1^u, C_2^u, D)$ are homothetic, α is determined by the relative price $\frac{P_2}{P_1}$, and do not depend on the income of the consumers. Therefore, we have

$$\alpha = \frac{P_1 C_1^e}{P_1 C_1^e + P_2 C_2^e} = \frac{P_1 C_1^u}{P_1 C_1^u + P_2 C_2^u},$$

and

$$1 - \alpha = \frac{P_2 C_2^e}{P_1 C_1^e + P_2 C_2^e} = \frac{P_2 C_2^u}{P_1 C_1^u + P_2 C_2^u}.$$

From the first order conditions and the budget constraints for employed and unemployed consumers we obtain the following demand functions for consumption baskets.

$$C_1^e = \alpha \frac{Wl + \Pi - D - \Theta + \hat{Q} - \Psi}{P_1}, \tag{1}$$

$$C_2^e = (1-\alpha) \frac{Wl + \Pi - D - \Theta + \hat{Q} - \Psi}{P_2},$$

$$C_1^u = \alpha \frac{\Pi + \hat{Q}}{P_1},$$

and

$$C_2^u = (1-\alpha)\frac{\Pi+\hat{Q}}{P_2}.$$

Solving maximization problems in Step 2, the following demand functions of employed and unemployed consumers are derived[3].

$$c_1^e(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{\alpha(Wl+\Pi-D-\Theta+\hat{Q}-\Psi)}{P_1},$$

$$c_2^e(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma}\frac{(1-\alpha)(Wl+\Pi-D-\Theta+\hat{Q}-\Psi)}{P_2},$$

$$c_1^u(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{\alpha(\Pi+\hat{Q})}{P_1},$$

and

$$c_2^u(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma}\frac{(1-\alpha)(\Pi+\hat{Q})}{P_2}.$$

From these analyses we obtain the indirect utility functions of employed and unemployed consumers as follows:

$$V^e = u\left(\alpha\frac{Wl+\Pi-D-\Theta+\hat{Q}-\Psi}{P_1},(1-\alpha)\frac{Wl+\Pi-D-\Theta+\hat{Q}-\Psi}{P_2},D\right)-\Gamma(l),$$

and

$$V^u = u\left(\alpha\frac{\Pi+\hat{Q}}{P_1},(1-\alpha)\frac{\Pi+\hat{Q}}{P_2},D\right).$$

Let

$$\omega = \frac{W}{P_1},\ \rho = \frac{P_2}{P_1}.$$

Then, since the real value of D in the childhood period is constant, we can write

$$V^e = \varphi\left(\omega l+\frac{\Pi-D-\Theta+\hat{Q}-\Psi}{P_1},\rho\right)-\Gamma(l),$$

and

$$V^u = \varphi\left(\frac{\Pi+\hat{Q}}{P_1},\rho\right).$$

ω is the real wage rate. Denote

$$I = \omega l+\frac{\Pi-D-\Theta+\hat{Q}-\Psi}{P_1}.$$

[3] Calculations of the maximization problems in Step 2 are standard using Lagrange multiplier method. Please see Appendix abut details.

The condition for maximization of V^e with respect to l given ρ is

$$\frac{\partial\varphi}{\partial I}\omega - \Gamma'(l) = 0, \tag{2}$$

where

$$\frac{\partial\varphi}{\partial I} = \alpha\frac{\partial u}{\partial C_1^e} + (1-\alpha)\frac{\partial u}{\partial C_2^e}.$$

Given P_1 and ρ the labor supply is a function of ω. From (2) we get

$$\frac{dl}{d\omega} = \frac{\frac{\partial\varphi}{\partial I}+\frac{\partial^2\varphi}{\partial I^2}\omega l}{\Gamma''(l)-\frac{\partial^2\varphi}{\partial I^2}\omega^2}.$$

If $\frac{dl}{d\omega} > 0$, the labor supply is increasing with respect to the real wage rate ω.

2.2. Firms' profit maximization

Let $d_1(z)$ be the total demand for good z by younger generation consumers in Period 1. Then,

$$d_1(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{\alpha\left(WLl+L_f\Pi-L_fD+L_f\hat{Q}-L_fQ\right)}{P_1}.$$

This is the sum of the demand of employed and unemployed consumers. Note that $\hat{Q}$ is the pay-as-you-go pension for younger generation consumers in their Period 2. Similarly, their total demand for good z in Period 2 is written as

$$d_2(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma}\frac{(1-\alpha)\left(WLl+L_f\Pi-L_fD+L_f\hat{Q}-L_fQ\right)}{P_2}.$$

Let $\overline{d_2(z)}$ be the demand for good z by the older generation. Then,

$$\overline{d_2(z)} = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{(1-\bar{\alpha})\left(\bar{W}\bar{L}\bar{l}+L_f\bar{\Pi}-L_f\bar{D}+L_fQ-L_f\bar{Q}\right)}{P_1},$$

where $\bar{W}$, $\bar{\Pi}$, $\bar{L}$, $\bar{l}$, $\bar{D}$ and $\bar{Q}$ are the nominal wage rate, the profits of firms, the employment, the individual labor supply, the debt of an individual, and the pay-as-you-go pension, respectively, during the previous period. $\bar{\alpha}$ is the value of α for the older generation. We assume $\bar{\alpha} = \alpha$. Q is the pay-as-you-go pension for consumers of the older generation themselves. Let

$$M = (1-\alpha)\left(\bar{W}\bar{L}\bar{l}+L_f\bar{\Pi}-L_f\bar{D}+L_fQ-L_f\bar{Q}\right).$$

This is the total savings or the total consumption of the older generation consumers including the pay-as-you-go pensions they receive in their Period 2. It is the planned consumption that is determined in Period 1 of the older generation consumers. Net savings is the difference between M and the pay-as-you-go pensions in their Period 2, as follows:

$$M - L_fQ.$$

Their demand for good z is written as $\left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{M}{P_1}$. Government expenditure constitutes the national income as well as the consumptions of the younger and older generations. It is financed by the tax on the younger generation consumers. Then, the total demand for good z is written as

$$d(z)=\left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{Y}{P_1}, \tag{3}$$

where Y is the effective demand defined by

$$Y=\alpha\left(WLl+L_f\Pi-T-L_fD+L_f\hat{Q}-L_fQ\right)+G+L_f\hat{D}+M.$$

Note that $\hat{D}$ is consumption in the childhood period of a next generation consumer. G is the government expenditure, except for the pay-as-you-go pensions, scholarships and unemployment benefits, and T is the tax revenue for the government expenditure (except for the pay-as-you-go pensions and unemployment benefits). See Otaki (2007, 2015) about this demand function.

Let L and Ll be employment and the "employment × labor supply" of firm z. The output of firm z is Lly. At the equilibrium $Lly = d(z)$. Then, we have

$$\frac{\partial d(z)}{\partial(Ll)}=y.$$

From (3)

$$\frac{\partial p_1(z)}{\partial d(z)}=-\frac{p_1(z)}{\sigma d(z)}.$$

Thus,

$$\frac{\partial p_1(z)}{\partial(Ll)}=-\frac{p_1(z)y}{\sigma d(z)}=-\frac{p_1(z)y}{\sigma Lly}.$$

The profit of firm z is

$$\pi(z)=p_1(z)Lly-LlW.$$

The condition for profit maximization is

$$\frac{\partial\pi(z)}{\partial(Ll)}=p_1(z)y-Lly\frac{p_1(z)y}{\sigma Lly}-W=p_1(z)y-\frac{p_1(z)y}{\sigma}-W=0.$$

Therefore, we obtain

$$p_1(z)=\frac{1}{(1-\frac{1}{\sigma})y}W=\frac{1}{(1-\mu)y}W,\ \mu=\frac{1}{\sigma}.$$

This means that the real wage rate is

$$\omega=(1-\mu)y.$$

Since all firms are symmetric,

$$P_1 = p_1(z) = \frac{1}{(1-\mu)y} W.$$

3. Positive or negative real balance effect with involuntary unemployment

The (nominal) aggregate supply of the goods is equal to

$$WL + L_f \Pi = P_1 Lly.$$

The (nominal) aggregate demand is

$$\alpha\left(WL + L_f\Pi - T - L_f D + L_f\hat{Q} - L_f Q\right) + G + L_f\widehat{D} + M$$
$$= \alpha\left[P_1 Lly - L_f D + L_f\hat{Q} - L_f Q\right] + G + L_f\widehat{D} + M.$$

Since they are equal in the equilibrium,

$$P_1 Lly = \alpha\left[P_1 Lly - T - L_f D + L_f\hat{Q} - L_f Q\right] + G + L_f\widehat{D} + M. \tag{4}$$

Ll cannot be larger than $L_f l$. However, it may be strictly smaller than $L_f l$. We then have $L < L_f$ and involuntary unemployment exists.

We assume a balanced budget, that is, $G = T$, and consider the steady state where the employment, the output, the nominal wage rate, and the prices of the goods are constant. In such a state we can assume $\widehat{D} = D$ and $\hat{Q} = Q$. Then, from (4) the savings of the younger generation consumers are

$$(1-\alpha)\left[P_1 Lly - G - L_f D\right] = -L_f D + L_f\widehat{D} + M = M.$$

This means that the steady state with constant employment and prices is maintained by the balanced budget.

Now suppose that corresponding to the existence of involuntary unemployment the nominal wage rate and the prices of the goods fall at the rate $\rho - 1 < 0$. Let L' be the employment in this case. If the price changes are correctly predicted by the consumers and the government, we can assume $\hat{Q} = \rho Q$ and $\widehat{D} = \rho D$. Then, (4) is rewritten as

$$\rho P_1 L'ly = \alpha\left[\rho P_1 L'ly - G - L_f D + L_f\rho Q - L_f Q\right] + G + L_f\rho D + M\cdot$$

The savings of the younger generation consumers are

$$(1-\alpha)\left[\rho P_1 L'ly - G - L_f D + L_f\rho Q - L_f Q\right] = (\rho - 1)(L_f D + L_f Q) + M. \tag{5}$$

We find that if

$$(1-\rho)(M - L_f D - L_f Q) > 0, \tag{6}$$

(5) is larger than ρM. Then, since the marginal propensity to consume, α, is constant, we have $L' > L$, that is, the employment increases by a fall in the nominal wage rate. On the other hand, if

$$(1-\rho)(M-L_fD-L_fQ)<0, \tag{7}$$

(5) is smaller than ρM. Then, $L' < L$, and the employment decreases by a fall in the nominal wage rate. (6) means that the net savings of the younger generation consumers, $M - L_fQ$, are larger than their debts, L_fD. In this case there exists positive real balance effect. On the other hand, (7) means that the net savings of the younger generation consumers are smaller than their debts. In that case there exists negative real balance effect.

The results of this paper are summarized in the following proposition.

1. If the net savings of the younger generation consumers are larger than their debts due to consumption in their childhood period, we have $M - L_f D - L_f Q > 0$, and there exists a positive real balance effect, and employment increases by a fall in the nominal wage rate.
2. If the net savings of the younger generation consumers are smaller than their debts, we have $M - L_f D - L_f Q < 0$, and there exists a negative real balance effect, and employment decreases by a fall in the nominal wage rate.

4. Fiscal Policy

(4) implies

$$P_1Lly=\frac{G+L_f\hat{D}+M-\alpha[T+L_fD-L_f\hat{Q}+L_fQ]}{1-\alpha}.$$

This means that the aggregate demand is determined by the government expenditure *G* given the savings of the older generation consumers, taxes, pay-as-you go pensions and so on. With COVID-19 prevalent in many countries around the world, economic activity is likely to remain sluggish, and involuntary unemployment is likely to increase. With a negative real balance effect, a decline in wage rates will not help the economy to recover. In such a situation, an effective policy to restore depressed economic activity is to increase aggregate demand through an aggressive fiscal policy using government debt or seigniorage.

5. Concluding Remarks

Using a three-generations overlapping generations model, we have shown that whether the real balance effect is positive or negative depends on whether the net savings of consumers are greater or less than their debts. In this paper we assume that the goods are produced by only labor. In the future research we want to analyze real balance effects in situations where the goods are produced by capital and labor and there are investments by firms.

Appendix: Calculations of the second step of consumers' utility maximization

Lagrange functions in the second step for employed and unemployed consumers are

$$\mathcal{L}_1^e = \left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}} \\ -\lambda_1^e\left[\int_0^1 p_1(z)c_1^e(z)dz - \alpha(Wl + \Pi - D - \Theta + \hat{Q} - \Psi)\right], \tag{A.1}$$

$$\mathcal{L}_2^e = \left(\int_0^1 c_2^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}} \\ -\lambda_2^e\left[\int_0^1 p_2(z)c_2^e(z)dz - (1-\alpha)(Wl + \Pi - D - \Theta + \hat{Q} - \Psi)\right],$$

$$\mathcal{L}_1^u = \left(\int_0^1 c_1^u(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}} - \lambda_1^u\left[\int_0^1 p_1(z)c_1^u(z)dz - \alpha(\Pi + \hat{Q})\right],$$

and

$$\mathcal{L}_2^u = \left(\int_0^1 c_2^u(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}} - \lambda_2^u\left[\int_0^1 p_2(z)c_2^u(z)dz - \alpha(\Pi + \hat{Q})\right].$$

λ_1^e, λ_2^e, λ_1^u and λ_2^u are Lagrange multipliers.

The first order condition for (A.1) is

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{1}{\sigma-1}} c_1^e(z)^{-\frac{1}{\sigma}} - \lambda_1^e p_1(z) = 0. \tag{A.2}$$

From this

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{-1} c_1^e(z)^{\frac{\sigma-1}{\sigma}} = (\lambda_1^e)^{1-\sigma} p_1(z)^{1-\sigma}.$$

Then,

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{-1} \int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}} dz = (\lambda_1^e)^{1-\sigma} \int_0^1 p_1(z)^{1-\sigma} dz = 1,$$

It means

$$\lambda_1^e \left(\int_0^1 p_1(z)^{1-\sigma} dz\right)^{\frac{1}{1-\sigma}} = 1,$$

and so

$$P_1 = \frac{1}{\lambda_1^e}.$$

From (A.2)

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{1}{\sigma-1}} c_1^e(z)^{\frac{\sigma-1}{\sigma}} = \lambda_1^e p_1(z) c_1^e(z).$$

Then,

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{1}{\sigma-1}}\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz=\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{\sigma}{\sigma-1}}$$
$$=C_1^e=\lambda_1^e\int_0^1 p_1(z)c_1^e(z)dz=\frac{1}{P_1}\int_0^1 p_1(z)c_1^e(z)dz.$$

Therefore,

$$\int_0^1 p_1(z)c_1^e(z)dz=P_1C_1^e.$$

Similarly,

$$\int_0^1 p_2(z)c_2^e(z)dz=P_2C_2^e.$$

Thus,

$$\int_0^1 p_1(z)c_1^e(z)dz+\int_0^1 p_2(z)c_2^e(z)dz=P_1C_1^e+P_2C_2^e=Wl+\Pi-D-\Theta+\hat{Q}-\Psi,$$

and from (1) we obtain

$$P_1C_1^e=\alpha(Wl+\Pi-D-\Theta+\hat{Q}-\Psi).$$

By (A.2)

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{\sigma}{\sigma-1}}c_1^e(z)^{-1}=C_1^ec_1^e(z)^{-1}=(\lambda_1^e)^\sigma p_1(z)^\sigma=\left(\frac{p_1(z)}{P_1}\right)^\sigma.$$

From this we get

$$c_1^e(z)=\left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{\alpha(Wl+\Pi-D-\Theta+\hat{Q}-\Psi)}{P_1}.$$

$c_2^e(z)$, $c_1^u(z)$ and $c_2^u(z)$ are similarly obtained.

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