CONSTRUCTING AN OPTIMAL INVESTMENT PORTFOLIO FOR THE BANK OF LITHUANIA

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Abstract. *The main goal of this article is to illustrate the strategy, devised to improve the effectiveness of utilizing the financial assets, or in this case, the official international reserves, belonging to the Bank of Lithuania. In Lithuania, the value of financial assets as a percentage of total state assets has doubled in the span of 10 years. Moreover, a strong correlation between the real GDP growth and the Bank of Lithuania's financial assets/profitability implies that the effectiveness of financial assets management has a nationally wide impact. Unfortunately, the Bank's profit/invested value indicator has reached a record low in 2012–2013, which resulted in the whole bank's profit being absorbed into the state's budget (as opposed to 70 % of it). Such signs meant that the previous investment strategy has become ineffective and needed changes.*

To highlight the necessary changes, the authors conduct a practical research and construct the optimal investment portfolio, according to the goals and variables given by the guidelines, proposed by Bank of Lithuania. The size of the portfolio is 4,14 bn euros, and the maximum loss per year (VaR) allowed is -100 M euro/year, as stated by the Bank of Lithuania's risk budget limit. The authors also focus on the issue of increased currency risk after investing in volatile share indices and whether hedging against it with Forex spot transactions is beneficial.

The result of the research is an optimal portfolio, consisting of 9,85 percent of risk-free assets and 90,15 percent of risky assets. Hedging against currency risk in this case is an ultimately beneficial course of action, yielding an increase of annual returns by 0,3 percent, which translates to +12,3 mln euros. Finally, the portfolio is flexible and simple to reshape into a less risky variant, if the institution predicts the dangers of possible future economic downfalls.

This research was further used in a broader paper whose goal was to analyse and assess the effectiveness of currently employed assets' management strategies in Lithuania.

Keywords*: financial assets, investment strategy, Bank of Lithuania, Harry Markowitz, efficient frontier, VaR, Sharpe ratio.*

Introduction

State owned assets are considered to be public goods that enable and catalyse an increase in the quality of life of the state's citizens. The effective use of state owned assets is often the priority of economic policies, the basis of economic prosperity, social safety, political

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stability and citizen welfare. Since the usage of state assets is related to economic growth, many may question whether the ineffective use of assets ultimately leads to an increasing gap between developed and developing EU countries. Indeed, in Lithuania's case, the asset management inefficiency problem was highlighted in 2010, when the State Government, reacting to global financial crisis, admitted that the country of Lithuania did not manage its assets profitably (The Annual Review, 2010), and therefore it is required to implement a new centralized state-owned asset management strategy.

According to Lithuanian asset management and disposition law, the state-owned assets here consist of 1) long-term and short-term real assets, 2) intangible assets and 3) financial assets. This article strictly focuses on financial asset management and the Bank of Lithuania's role in it, supervising the largest investment portfolio in the country – the official international reserves. The responsibility of formulating the investment strategy falls on the Bank of Lithuania's Banking Service's Investment Management Department (IMD) branch, which has devised and publicly disclosed guidelines of asset allocation in investment portfolio, which is predicted for the year 2016.

Joining the euro zone in 2015 signified major changes in the Bank's investment strategy, such as increasing the non-euro investments in the portfolio, following the European banks example and adding volatile stocks to the portfolio, and taking higher risks in general. The "safe" treasury bill oriented portfolio proved to be rather ineffective in the years 2012–2013. Such changes supplement the management models with additional risks, the most significant one being the currency risk. Global financial specialists, such as IMF, BlackRock Inc., and MSCI researchers conclude that, according to historical evidence, hedging against the currency risk has been rather beneficial for European investors, especially in USA and Japanese markets. Although, in some cases the results can be inconclusive (BlackRock Inc., 2014; IMF, 2010; MSCI, 2009/2011).

The objective of this article is to analyse the Bank of Lithuania's financial assets management policy, to construct an optimal investment portfolio and to compare the results with the theoretical guidelines proposed for year 2016.

The main tasks of this article are: 1) to analyse the dynamics of state owned financial assets and their part relatively to total state owned assets; 2) to analyse the dynamics of Bank of Lithuania's investments profitability and determine whether the assets were managed effectively; 3) to measure the strength of correlation between investment profitability and the real GDP growth in Lithuania; 4) to select the securities for the optimal portfolio and collect empirical data of daily returns/prices; 5) to construct an optimal portfolio as proposed by Harry Markowitz's portfolio selection model, maximizing the Sharpe ratio; 6) to calculate the portfolio's value at risk (VaR) and make sure it meets the risk requirements, and if not – to diversify the portfolio with a risk-free asset; 7) to compare the risk of specific assets to the optimized portfolio's risk and to assess the benefits of risk diversification; 8) to compare the structure of the optimized

portfolio to theoretical guidelines; 9) to discuss and assess the alternative strategies; 10) to propose final recommendations for a new investment strategy.

Scientific methods used in the research:

- **Dynamic** analysis of state-owned financial assets;
- **Empirical** research/data collection of selected securities;
- **Statistical** analysis – calculating basic data needed for constructing the portfolio: rate of returns, standard deviations, correlations and co-variations;
- **Risk** analysis – RiskMetrics standards, evaluating VaR and Sharpe ratios;
- **Optimization** methods – Harry Markowitz's model;
- **Structural** and **comparative** analysis of the final portfolio.

Research assumptions

The modelling of financial assets portfolio is carried out as realistically as possible, following the guidelines and rules of the Bank of Lithuania, without disclosing any sensitive, non-public information. The research follows these assumptions:

1. The portfolio is created for the end of year 2016 – the end of the transitionary period, as stated in the Bank of Lithuania's official international reserves management policy. This period marks a successful entry into the euro zone, transforming the portfolio into a more profitable one, by periodically increasing the percentage of share investments, and also foreign currency investments, relatively to the euro part;
2. The size of the invested value equals to 4,14 billion euros, which was the publicly disclosed size of the Bank of Lithuania's investment portfolio in 2013 (M. Vaičiulis' interview to „Verslo Žinios", 2013);
3. The non-euro part of the portfolio will be constructed from IMF's SDR basket, which consists of four major global currencies: USD, EUR, JPY and GBP (Antweiler, 2013). Such currencies are considered to be a safe investment

TABLE 1. **SDR structure 2011–2015**

SDR basket 2011–2015	
Currency	**Weight**
US dollars	41,9%
European euros	37,4%
Japanese yen	9,4%
Great Britain pounds	11,3%

Source: compiled by authors

4. The portfolio will consist of all security classes named in the Bank of Lithuania's strategic investment guidelines

TABLE 2. **Bank of Lithuania's strategic investment guidelines for the transitionary period**

Date	Risk-free asset (German Treasury Bills), %	Euro zone Government bonds, rated AAA-A, %	Quasi-government bonds, rated AAA-AA, %	Corporate bonds, rated AAA-BBB-, %	World share indices, %
2013-11-30	68,0	19,0	7,5	5,0	0,5
2014-02-28	66,2	19,0	8,1	5,6	1,1
2014-05-31	64,3	19,0	8,8	6,3	1,6
2014-08-31	62,5	19,0	9,4	6,9	2,2
2014-11-30	60,8	19,0	10,0	7,5	2,7
2015-02-28	56,5	21,6	10,6	8,1	3,2
2015-05-31	51,7	24,4	11,3	8,8	3,8
2015-08-31	47,2	27,2	11,9	9,4	4,3
2015-11-30	42,7	30,0	12,5	10,0	4,8
2016-02-29	34,0	37,5	13,1	10,0	5,4
2016-05-31	25,3	45,0	13,8	10,0	5,9
2016-08-31	16,6	52,5	14,4	10,0	6,5
2016-11-30	8,0	60,0	15,0	10,0	7,0

Source: compiled by authors, using data from Bank of Lithuania, 2013

5. In addition to the securities above, the authors include foreign currencies (USD, JPY, GBP) as spot exchanges conducted in the FOREX market, as an additional hedging factor;
6. The optimizer will be set to operate on certain conditions for each asset's weight to meet realistic constraints of the Central bank (i.e., no negative weights for bonds and shares allowed; the weight of quasi-government bond sector must not be higher than the government sector; the weight of a single country's bonds should not be higher than 50 percent);
7. As stated in the Bank of Lithuania's risk budget, the critical limit of negative returns per year is -100 M euros, or 2 % of total official international reserves value, so the constructed portfolio must not be riskier than that (Bank of Lithuania, 2014)

Besides providing an optimal portfolio, the authors' research will additionally help answer these following questions: how much is advised to invest in volatile stocks? How much can portfolio risk be diversified? Is it beneficial at all to hedge security investments to buy/sell currencies in the FOREX market?

The dynamics and impact of financial assets in Lithuania

To perform a sophisticated and accurate state-owned asset analysis in Lithuania's case is rather complicated, for the following reasons:

- Short timespan of data: the state asset reports, gathered in the official statistics database, cover the timespan of 2001 – 2011. Later reports are not published in any statistical database because of the still unfinished Government's goal to launch a new unified state-owned asset informative online search system (or, in Lithuanian, *VTIPS*);
- The lack of data and mismatched data: until 2004 the monetary value of the whole state-owned assets was not even calculated, since the value of geothermal resources and state land fund's data was not expressed in monetary terms. Moreover, the data tends to differ in different reports: i.e., in 2012's report the value of state assets in 2010 is stated to be 170,143 M LTL, while in 2011's report it's 163,449 M LTL, making the 7,000 M LTL difference – such facts prove the reliability of these reports to be quite doubtful.

Following these reasons, the authors limit the research to a short dynamic analysis, to indicate the financial assets' growth tendencies in the last 10 years.

TABLE 3. **The value of state-owned assets and financial assets of Lithuania (thousands of LTL); financial assets expressed as percentage from total assets, 2001–2011. Source: compiled by authors, using data from state–owned asset reports, gathered by the official statistics portal of Lithuania**

Year	State-owned assets value (thousands of LTL)	State-owned financial assets value (thousands of LTL)	
2001	*Insufficient data*	16487,6	
2002	*Insufficient data*	15940,9	**Financial assets**
2003	*Insufficient data*	15548,7	**% of total assets**
2004	157286	15369	**9,77%**
2005	171433,1	15750,5	9,19%
2006	183996,1	19055,5	10,36%
2007	200734,6	19754,7	9,84%
2008	219700,6	21762,1	9,91%
2009	166934,7	24129,1	14,45%
2010	170143,4	31335,1	18,42%
2011	169990,8	31174,5	**18,34%**

Noticeably, in the span of 8 years the value of financial assets has almost doubled relatively to total assets. However, the real values remain unclear, if we consider the lack of data and the mismatched data.

Furthermore, we take a look at the dynamics of Bank of Lithuania's financial assets (official international reserves) and investment profitability.

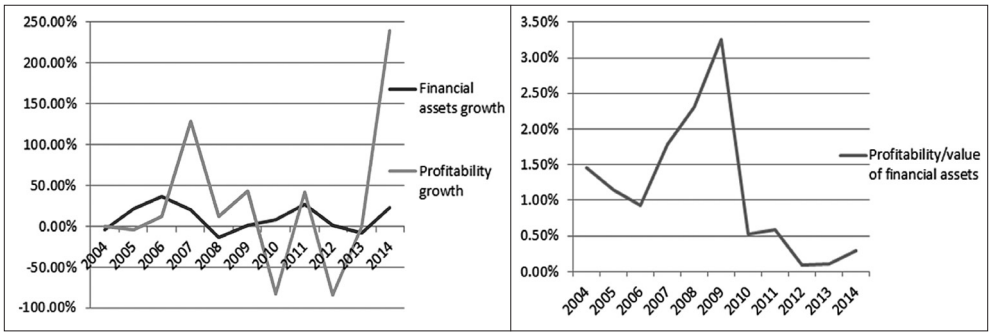

FIGURE 1. **Bank of Lithuania's (LB) financial assets and profitability growth (on the left), LB profitability/value of assets ratio (on the right) 2004–2014**

Source: compiled by authors, using data from Bank of Lithuania's statistics and profit – loss accounts

The financial assets, managed by the Bank of Lithuania (official international reserves) displays a constant growth tendency with cyclical ups and downs. Since 2003, the reserves have grown 1,6 times (from 2,760 to 7,176 M EUR). On the other hand, the Bank's profitability shows sharp volatility that can be partially explained as a consequence of economic boom and the financial crisis. It is worth noting 130 % profit growth in 2007, 42 % more in 2009, and also 80 % fall in 2010 (the lagging crisis effect), a slightly over 40 % surge in 2011 (after the safe treasury bills investment strategy was chosen) and finally, 84 % slump in 2012. Continuing to 2013, those were the years when the Bank's profits reached record low (6 M EUR) and were fully adapted to the state budget (as opposed to partially). In 2014, the profitability rose by a record 240 %, as a response to the new aggressive investment strategy, which is deeply analysed in this research.

In 2007–2009, the ratio of profits/value invested was the highest and reached 3,26 %. However, in 2012–2013 it only managed to make 0,1 %, which led to obvious conclusions about the ineffectiveness of financial asset management. A change of strategy would surely increase this ratio, although as of 2014 it was still low (0,3 %).

The questions, whether the Bank's profitability and effective asset management ratios have a connection with overall economic growth in the country, and whether it's worth fixing, could be answered by measuring the correlation between the variables.

The economic growth of Lithuania displays a moderate correlation (ratio enters the interval of 0,4–0,7) with the growth of Bank of Lithuania's financial assets. The bank's net profits and income from investments, naturally, possess a perfectly accurate correlation, however, it weakly correlates with financial assets growth (interval of [0,2–0,4)) (Kasnauskienė, 2010).

Also, it is worth noting that the Bank's profitability and income correlation with the country's economic growth could almost be considered as "strong" (close to the value

TABLE 4. **The correlations of Lithuania's real GDP growth, Bank of Lithuania's financial asset growth, net profits and income from investments, 2005–2014**

CORRELATIONS	Real GDP growth	LB financial assets growth	LB net profit (M EUR)	LB income from investments (M EUR)
Real GDP growth	1			
LB financial assets growth	0,525517925	1		
LB net profit (M EUR)	-0,680606057	-0,216119221	1	
LB income from investments (M EUR)	-0,692221065	-0,240293138	0,995082876	1

Source: compiled by authors, using data from official statistics portal of Lithuania and Bank of Lithuania

of 0,7) and is also negative. This could be explained by the fact that the Bank's profit and income are **lagging** indicators – as the economic boom started, the profit/income grew one period later, and when the crisis began in 2009, those indicators reached their highest values.

Overall, the main conclusion from this analysis would be that it is ultimately advised to increase the effectiveness of financial asset management strategy, because the results up until 2014 were rather lacking, and the correlation shows their significant connection with the country's overall economic growth.

The general portfolio structure

The securities for the optimal portfolio were carefully selected according to assumptions discussed in the ***Research Assumptions*** chapter. The portfolio includes all security classes from the Bank's guidelines, and by authors' choice is supplemented by FOREX spot exchange deals. The selection set is simplified, choosing assets from only four main world regions which operate on four SDR basket currencies – therefore, the portfolio will only be compiled of USA, UK, Japanese and euro zone's securities.

According to the Bank of Lithuania's safety regulations, the selected securities must meet the credit rating passing the investment grade criteria (not lower than BBB) and be equally weighted.

The collected daily returns/price data covers the time period of 18 years (1997–2014). Such a long time period was selected accordingly to the dates the securities were first issued, and also with the goal of including more than one global crisis, such as the Dot. com bubble in 1995–2000, the financial crisis in 2007 and the euro zone recession in 2009.

The final list of selected indices and other assets (total: 19)

1. **USA bonds**:
   i. *BofA Merrill Lynch US Treasury Index*. AAA+ rated Government sector bonds. Minimal fixed coupon: 1 bn USD. Bloomberg code: G0Q0
   ii. *BofA Merrill Lynch US Agency Index*. Investment grade Quasi-government sector bonds. Minimal fixed coupon: 250 M USD. Bloomberg code: G0P0
   iii. *BofA Merrill Lynch US Corporate Index*. Investment grade corporate sector bonds. Minimal fixed coupon: 250 M USD. Bloomberg code: C0A0
2. **Japanese bonds**:
   iv. *BofA Merrill Lynch Japan Government Index.* AAA+ rated Government sector bonds. Minimal fixed coupon: 200 bn JPY. Bloomberg code: G0Y0;
   v. *BofA Merrill Lynch Japan Quasi-Government Index.* Investment grade Quasi-government sector bonds. Minimal fixed coupon: 20 bn JPY. Bloomberg code: JQ00;
   vi. *The BofA Merrill Lynch Japan Corporate Index.* Investment grade corporate sector bonds. Minimal fixed coupon: 20 bn JPY. Bloomberg code: JC00;
3. **UK bonds**:
   vii. *BofA Merrill Lynch UK Gilt Index.* AAA+ rated Government sector bonds. Minimal fixed coupon: 500 M GBP. Bloomberg code: G0L0;
   viii. *BofA Merrill Lynch Sterling Quasi-Government Index.* Investment grade Quasi-government sector bonds. Minimal fixed coupon: 100 M GBP. Bloomberg code: UQ00;
   x. *BofA Merrill Lynch Sterling Corporate Index.* Investment grade corporate sector bonds. Minimal fixed coupon: 100 M GBP. Bloomberg code: UR00;
4. **Euro zone bonds**:
   x. *BofA Merrill Lynch Euro Government Index.* Long-term investment grade Government sector bonds of the euro zone member countries. Minimal fixed coupon: 1 bn EUR. Bloomberg code: EG00;
   xi. *BofA Merrill Lynch Euro Large Cap Quasi-Government Index.* Investment grade Quasi-government sector bonds. Minimal fixed coupon: 500 M EUR. Bloomberg code: EQL0;
   xii. *BofA Merrill Lynch Euro Corporate Senior Index.* Investment grade corporate sector bonds. Minimal fixed coupon: 250 M EUR. Bloomberg code: ERS0;
5. **Stock indices**:
   xiii. *Standard & Poor's 500.* 500 stock indices, encompassing all main industries in USA. Bloomberg code: SPX;
   xiv. *TOPIX.* Also known as Tokyo Stock Price Index, it encompasses the shares of top listed Japanese companies, representing 33 industries. Bloomberg code: TPX;

xv. *FTSE 350.* UK stock indices, reflecting roughly 90 % of UK's capital market. Bloomberg code: NMX;

xvi. *Bloomberg EURO 500.* Shares of 500 companies from the euro zone countries that have the highest capitalization. Bloomberg code: BE500;

6. **Currencies**:

xvii. US dollar. Spot EUR-USD rate, calculated by FOREX market;

viii. Japanese yen. Spot EUR-JPY rate, calculated by FOREX market;

xix. GB pound sterling. Spot EUR-GBP rate, calculated by FOREX market.

Further data analysis and the construction of the optimal portfolio require calculating[1] the following indicators: moving annual return[2] for each asset, co-variations between assets, expected return of the whole portfolio and portfolio variation.

$$\textbf{\textit{Moving annual return}}\ E_r = \frac{R_n}{R_{n-252}} - 1\ or\ \frac{P_n}{P_{n-252}} - 1$$

(it is assumed that there is a difference of 252 work days between annual prices/returns)

$$\textbf{\textit{Expected return of the portfolio}}\ E_{r\,p} = \sum_{1}^{n} (E_{r1,\dots,n} \times w_{1,\dots,n})$$

$$\textbf{\textit{Portfolio variation}}\ \sigma^2 = \sum_{1}^{n} (\sigma^2_{1,\dots,n} \times w^2_{1,\dots,n} + 2 \times Cov(X_{1,\dots,n}, X_{1,\dots,n-1}))$$

$$\textbf{\textit{Covariation}}\ cov(X_{1,\dots,n}, X_{1,\dots,n-1}) = \sigma_{1,\dots,n} \times \sigma_{1,\dots,n-1} \times \rho_{1,\dots,n}$$

Optimal portfolio selection theory by Harry Markowitz

The core of this research is the construction of an optimal investment portfolio, according to Harry Markowitz's model. H. Markowitz (1927) is an American scientist, who majored in economics and studied under the famous monetarist M. Friedman. Markowitz introduced mathematical methods into analysing stocks and other securities, and for such fundamental discoveries he received a Nobel Prize of Economics, and is also titled to be the father of modern investment portfolio theory. In his article "Portfolio Selection", published in The Journal of Finance in 1952, the scientist introduced such widely used terms as portfolio risk, risk diversification and optimization. Moreover, he was the first one to prove that asset diversification decreases the portfolio risk (Cibulskienė, Grigaliūnienė, 2007).

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[1] In these formulas R_n – bond indice's returns on the *n-th* day; P_n – stock indice's or currency's last price on the *n-th* day; $w_{1,\dots,n}$ – n securities weights in the portfolio; $E_{r\,1,\dots,n}$ – n securities moving annual returns; $\sigma_{1,\dots,n}$ – n securities standard deviations; ρ – correlation between two securities

[2] This research assumes the investment horizon to be one year and unchangeable through the whole period (as limited by the Markowitz model); however, it is important to note that the current investment horizon for the Bank of Lithuania has been changed to three years

The model is not without drawbacks – it assumes market normality, does not take transaction costs into consideration and has a single-period perspective (whereas the investors might want to plan for a multiple-period horizon); however, it is still the most widely used in practice to this day.

The economist also illustrated the link between portfolio returns (y) and risk (x), by drawing an efficient frontier curve. The curve represents only optimal portfolios, which are 1) the most profitable with the given risk level or 2) the least risky with the given profitability level. However, since all the portfolios on the curve are optimal choices, Markowitz does not provide a singular answer what should the investor choose – in reality, the choice depends on the investor's risk aversion level and external requirements. It's only natural for conservative institutional investors to save their capital and highly restrict the risk (which is also the case for the Bank of Lithuania). In the end, the selection of optimal portfolio according to preferred risk or return levels, is a subjective **investment decision**, whereas the **financial decision** (Kancerevyčius, 2004) – the decision how to split your investment weighs on your selected securities to achieve the best results. In this case, Markowitz's model can and will provide us with a clear answer.

To optimize the portfolio, it is required to calculate and keep in mind these ratios:

Portfolio's **expected return** – it shows how big of a profit can be expected from the invested value (in this case, in one year).

Portfolio's **standard deviation** (σ) – a statistical ratio, showing how volatile were the fluctuations of the moving annual returns of the portfolio, compared to the average. Higher ratio indicates a bigger overall risk and that the portfolio might not be well diversified.

Portfolio's **Sharpe ratio** – one of the main risk measuring ratios currently used in the world, along with alpha, beta and R^2. Introduced in 1966 by an American professor William Sharpe, who also received a Nobel Prize in Economics, was praised for its simplicity. The ratio shows how much of an excess return the investor receives, if he increases the risk by one more percent. If the ratio is < 1, it means the risk taken did not generate a higher return, therefore the portfolio is ineffective. Ratio of > 1 is considered good, whereas ratio of > 3 is considered great (Sharpe, 1966).

To calculate the ratio, it's required to select the risk free rate, which, in the case of this research, is 0,0015 (the returns of 1–6 month German Treasury Bills, which is the main risk-free asset in the Bank of Lithuania's investment strategy). The formula[3] of the calculus:

$$Sharpe\ ratio\ S = \frac{E_{r\,p} - R_f}{\sigma_p}.$$

[3] In this formula R_f – risk free rate; $E_{r\,p}$ – expected return of the risky optimal portfolio; σ_p – standard deviation of the risky optimal portfolio

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The next issue is to measure the currency risk and overall risk of the portfolio. One of the most common methods of risk measurement is calculating the value at risk (**VaR**). The Basel Committee on Banking Supervision introduced such methodology in 1995, and since then the Banks of EU are using it to determine their capital requirements and managing market risks (Habibnia, 2013). VaR method also identifies currency risk, if the selected variables are the investor's balance positions in foreign currencies and foreign investments.

VaR is understood as a maximum possible loss of value that any investment can suffer during a set period of time, when the market conditions are normal. The size of VaR depends on three parameters: 1) **time** (in this case VaR will be measured for 1 year); 2) **confidence level** (usually 95 or 99 %, in this case 95 %) and 3) **currency** (in this case – euros).

The goal of this research is to construct a portfolio, whose VaR would not exceed the risk budget limit of -100 M euros per year.

There are three RiskMetrics Group methods to calculate VaR (Mina, Xiao, 2001):

1. **Historical** method. This method analyses a set of historically attained losses and "cuts off" a specific percentage of worst cases, least of which would be the value of VaR. It is considered to be the most complicated method, in terms of time consumption and calculus;
2. **Monte Carlo** simulation. This method is the most advanced mathematically. It generates a sporadic matrix of possible losses and also selects the value of VaR after "cutting off" a specific percentage of worst results;
3. **JP Morgan** variation/co-variation, also known as the **parametric** method. It is a simple and straightforward way to calculate VaR after assuming that the markets operate under normal conditions, and the data is normally dispersed around the average. Although such simplification distorts reality, this method is the most widely used in calculating VaR. The formula of the calculus is:

$$VaR\ (with\ 95\ \%\ confidence\ level) = \sigma_p \times value\ invested \times 1{,}645$$

The risk of a single security/asset can be expressed as the standard deviation of its moving annual returns. On the other hand, the risk of the whole portfolio depends on the interaction of all variables, expressed in their correlations. This dependency can be described in three main principles (Bodie, Kane, Marcus, 2011):

1. The smaller correlation (ideally negative) between two securities decreases the risk, investing in them both. The portfolio benefits *the most* if we include two least correlated parameters;
2. The perfect negative correlation of -1 is an extreme case, meaning the opportunity of a perfect hedge. The portfolio, comprised of two perfectly negatively correlated securities will have a **risk of zero**;

3. An optimal portfolio will *always* have a smaller risk than every single variable that is included in the portfolio – it's called the diversification effect.

As we examine our portfolio combinations, we can quickly notice which variables can be considered "hedges", if we analyse the correlation matrix. Every security that has a negative correlation with any other variable is called a hedging factor. Moreover, it is possible to construct a portfolio with negative weights of certain variables – it means those variables should be sold rather than bought/invested to, and that further decreases the portfolio's risk. In case of this research, it is possible to either buy or sell foreign currencies to achieve that. If the currency is positively correlated to bonds and stocks (i.e. JPY and TOPIX), it is recommended to sell it, and vice versa (i.e. euro negatively correlates to German stocks) (Cavallaro, 2012).

Finally, if the calculated VaR of our optimal portfolio will prove unsatisfactory (exceeding the limit of -100 M euros), the final step of Markowitz's method would be to diversify it with the risk-free asset, which provides the risk of zero and, naturally, decreases the expected return as well. The percentage of the amount invested risk-freely will depend on the subjective risk aversion level of the investor, which in this case will be determined by the risk budget limit requirement. The formulas[4] of the calculus are:

$$Non\ risk\ free\ part\ of\ the\ portfolio = \frac{E_{r\,p} - R_f}{A \times \sigma_p^2}$$

$$Risk\ free\ part\ of\ the\ portfolio = 1 - non\ risk\ free\ part\ of\ the\ portfolio$$

The optimized final portfolio

The portfolio we selected can be optimized via Markowitz's model, using MS Excel Solver optimizer platform. It can be done in three different ways, selecting different desired parameters:

1) Maximizing the **expected return**;
2) Minimizing the **standard deviation** (risk);
3) Maximizing the **Sharpe ratio** (the effectiveness of the investment):

Additionally, for the optimizer to work, limiting conditions must be introduced, such as: the sum of the weights of all variables must equal to 1 or 100 %.

1) The characteristics of the **maximized return** portfolio: 16,4 % expected return, which is achieved by buying and short-selling certain securities for 100 %. However, in reality, no sane investor would choose this portfolio, as it comes with a high risk – standard deviation exceeds the expected return, VaR highly exceeds the risk budget requirements of 100 M EUR, and Sharpe ratio is also lower than 1, which signifies inefficiency.

[4] In these formulas R_f – risk free rate; $E_{r\,p}$ – expected return of the risky optimal portfolio; σ_p – standard deviation of the risky optimal portfolio; A – preferred level of risk aversion

TABLE 5. **Characteristics of the optimized portfolios (asset weights, portfolio expected returns, risks, Sharpe ratios and VaR). Source: compiled by authors**

	1) Maximized return portfolio	2) Minimized risk portfolio	3) Maximized Sharpe portfolio
US Government bonds	-7,72%	45,63%	70,09%
US Quasi-government bonds	-34,66%	-7,53%	10,77%
US Corporate bonds	99,78%	-33,09%	-41,63%
Japan Government bonds	-38,00%	-82,13%	-67,72%
Japan Quasi-government bonds	-12,56%	100,00%	129,39%
Japan Corporate bonds	-99,94%	46,87%	-46,75%
UK Government bonds	95,28%	12,02%	-20,86%
UK Quasi-government bonds	50,70%	-3,15%	45,13%
UK Corporate bonds	100,00%	-2,24%	-3,95%
Euro zone Government bonds	-3,52%	-34,81%	-36,90%
Euro zone Quasi-government bonds	56,18%	-36,85%	-86,51%
Euro zone Corporate bonds	-24,32%	85,03%	141,14%
S&P500 stocks	-14,64%	-3,21%	2,06%
TOPIX stocks	-39,59%	4,33%	8,40%
FTSE350 stocks	15,29%	0,12%	-5,40%
Bloomberg EURO500 stocks	-19,60%	2,11%	3,14%
USD	22,74%	2,93%	4,57%
JPY	-7,55%	3,21%	-1,87%
GBP	-37,88%	0,78%	-3,10%
Expected return	16,37%	2,27%	4,59%
Risk	19,81%	0,94%	1,35%
Sharpe ratio	0,8186	2,2682	3,2884
VaR	1349.772.988 EUR	63.809.115,8 EUR	91.920.618,4 EUR

2) The characteristics of the **minimized risk** portfolio: standard deviation of only 0,9 %, almost 2,3 % expected return and a Sharpe ratio that is considered good. Such strategy is viable for conservative investors, but it is still not the best choice, as long as the investor gets increasing returns by taking a higher risk.
3) The characteristics of the **maximized Sharpe** ratio portfolio: 4,6 % expected return, small risk of 1,35 % and Sharpe ratio above 3, which is considered great. No other asset allocation would increase it. The unique structure of such portfolio advises to buy more than 100 % of some specific assets (in this case, Japanese Quasi-government bonds and European corporate bonds), meaning more than the initial value of the portfolio. Such combination does not need to be diversified by a risk-free asset, as its VaR meets the requirements.

This might seem like an ideal choice for the final portfolio, sadly, in reality such strategy could not be implemented. The reasons for that are elaborated further on.

The Bank of Lithuania, as an institutional investor, must restrict its investment behaviour with certain conditions, as it cannot afford to act otherwise:

- The Bank cannot take a short position in bonds and stocks (as in, sell borrowed securities that the Bank doesn't actually possess);
- The Bank cannot allow investing into a single country more than 50 %, for the possibility of that country facing recession in the future (this eliminates the possibility of investing the majority of value into USA bonds);
- The Bank cannot allow investing into Quasi-government sector bonds more than into the Government sector bonds.

Therefore, more restrictive conditions have been introduced into the MS Excel Solver optimizer, limiting the weights of bonds and stocks, and ignoring the foreign currencies. After doing that and maximizing the Sharpe ratio again, we get a new portfolio, which has an undesirable value of VaR – 111 M EUR. Introducing the risk-free asset then diversifies the portfolio and the result is the final optimal portfolio choice of this research.

TABLE 6. **Optimal portfolio(1) structure and characteristics**

OPTIMAL PORTFOLIO(1)	Weight	Sum
Risk-free asset	9,85%	407.893.381,79 EUR
US Government bonds	37,86%	1.568.142.499,65 EUR
US Quasi-government bonds	9,47%	392.035.624,91 EUR
Japan Government bonds	9,16%	379.304.171,08 EUR
Japan Quasi-government bonds	2,29%	94.826.042,77 EUR
UK Government bonds	2,67%	110.508.965,28 EUR
UK Quasi-government bonds	0,67%	27.627.241,32 EUR
Euro zone Corporate bonds	15,89%	658.115.003,29 EUR
S&P500 stocks	0,05%	2.270.146,21 EUR
TOPIX stocks	9,20%	381.158.517,54 EUR
Bloomberg EURO500 stocks	0,11%	4.388.003,29 EUR
USD	1,63%	67.563.185,82 EUR
JPY	-4,52%	-187.290.762,53 EUR
GBP	5,67%	235.023.949,51 EUR
	100,00%	4.141.565.969,93 EUR

Main characteristics (1)	
Expected return	4,07%
Standard Deviation	1,47%
VaR	99.931.955,06 EUR

Source: compiled by authors

Optimal portfolio(1): this portfolio maximizes the Sharpe ratio (effectiveness) of the investment, is diversified with the risk-free asset and meets the risk budget requirements. The risk-free asset part makes up 9,85 % of the portfolio, whereas risky part 90,15 %. With the confidence level of 95 % and risk aversion level of 182, the portfolio's VaR value does not exceed the critical annual limit of 100 M euros.

The structure of the portfolio consists only of 13 risky assets – other initial variables were discarded as not optimal choices. Biggest weights are given to USA Government bonds (37,86 % or 1,568 M euros) and EZ Corporate bonds (15,89 % or 658 M euros).

The portfolio has a return rate of 4,07 % and risk rate of 1,47 %.

Although portfolio(1) is an acceptable and recommended strategy, it is wise to consider the lower risk alternative. Therefore, the authors constructed an additional, safer portfolio(2), which diversifies risk the same way, but has an increased the weight of the risk-free asset. The goal was to decrease the desired VaR from 100 M to 75 M euros.

TABLE 7. **Optimal portfolio(2) structure and characteristics**

OPTIMAL PORTFOLIO(2)	Weight	Sum
Risk-free asset	32,76%	1.356.613.473,30 EUR
US Government bonds	28,24%	1.169.680.061,21 EUR
US Quasi-government bonds	7,06%	292.420.015,30 EUR
Japan Government bonds	6,83%	282.923.603,02 EUR
Japan Quasi-government bonds	1,71%	70.730.900,75 EUR
UK Government bonds	1,99%	82.428.818,36 EUR
UK Quasi-government bonds	0,50%	20.607.204,59 EUR
Euro zone Corporate bonds	11,85%	490.889.059,83 EUR
S&P500 stocks	0,04%	1.693.305,78 EUR
TOPIX stocks	6,86%	284.306.763,08 EUR
Bloomberg EURO500 stocks	0,08%	3.273.018,85 EUR
USD	1,22%	50.395.491,06 EUR
JPY	-3,37%	-139.700.486,81 EUR
GBP	4,23%	175.304.749,23 EUR
	100,00%	4.141.565.977,57 EUR

Main characteristics (2)	
Expected return	3,07%
Standard Deviation	1,09%
VaR	74.539.409,1 EUR

Source: compiled by authors

Optimal portfolio(2): the risk free asset makes up 32,76 % part of the portfolio, whereas risky part 67,24 %. With the confidence level of 95 % and risk aversion level of 244, the portfolio's VaR value does not exceed the dangerous annual limit of 75 M euros.

In the case of the safer portfolio, it is recommended to invest the biggest value into the risk-free asset (1356,6 M euros). The weights of the risky assets do not change drastically, although each of them decreases proportionally.

The portfolio has a return rate of 3,07 % and risk rate of 1,09 %.

Such a portfolio is recommended and easy to shift to, if the Bank expects an economic recession in the near future.

Slightly diverging from guidelines

this part of the research highlights how different is the final optimal portfolio compared to the initial guidelines, which were provided in the chapter ***Research Assumptions***.

Optimal portfolio(1) is further structurally analysed in two aspects:

1) Comparing weights of four different currencies in the portfolio with the standard structure of the SDR basket;
2) Comparing weights of different asset classes in the portfolio with the strategic asset allocation for the end of 2016, provided by the Bank of Lithuania.

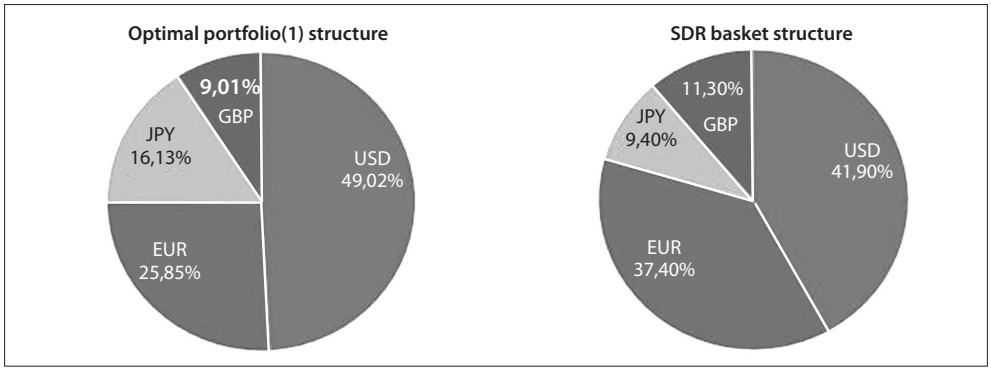

FIGURE 2. **Portfolio(1) structure by currencies**

Source: compiled by authors, using data from optimal portfolio(1) table and the current SDR basket structure (Antweiler, 2013)

It is evident that the optimal portfolio rather significantly diverges from the SDR basket's structure. The most similar proportion are the UK assets, although the biggest part of it is the recommendation to hedge by buying 5,67 % GBP.

The optimal choice is to invest more in Japanese markets, even if it is recommended to sell 4,5 % of the JPY. A significant weight is given to the TOPIX stocks, as they turn out to be an appealing hedging factor (close to perfect negative >80% correlation with the USA bonds, and also negative correlation with the Japanese and UK bonds).

It is also advised to significantly reduce investing in euros and keep it at 25,85 %. The relatively poor risks and returns arising from recent years of prolonged euro zone economic depression, reduces euro's appeal in comparison to what other countries have to offer.

Finally, USA bonds seem unambiguously the most appealing, displaying high average annual returns, so it is recommended to invest 49 % of the portfolio in dollars.

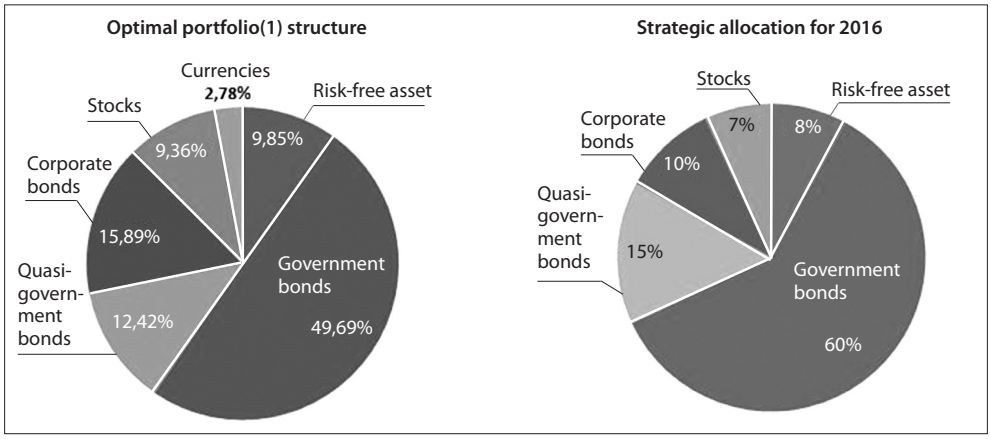

FIGURE 3. **Portfolio(1) structure by asset classes**

Source: compiled by authors, using data from optimal portfolio(1) table and the Bank of Lithuania's investment guidelines for the end of 2016.

The optimal portfolio also slightly diverges from the recommended structure. It is advised to invest 10 % less into the Government sector and 2,5% less into Quasi-government sector, also to increase the part invested into the corporate sector up to 16%. The weight of stocks, and surprisingly enough risk-free assets, should also be slightly increased, and finally, the authors introduced the addition of foreign exchange deals, which make up about 2,8 % of the portfolio.

However, the structure ultimately remains true to the guidelines – the highest weight is placed on the Government sector and the smallest – on stocks (if we ignore the currencies).

The benefits of diversification and the addition of foreign currencies

The benefits of risk diversification in the portfolio can be illustrated by drawing a H. Markowitz's efficient frontier curve. The possible optimal portfolios are represented as points on the curve, and the separate assets with their representative risk/return characteristics are always found below the efficient frontier – meaning the portfolio's risk will always be lower than its components'.

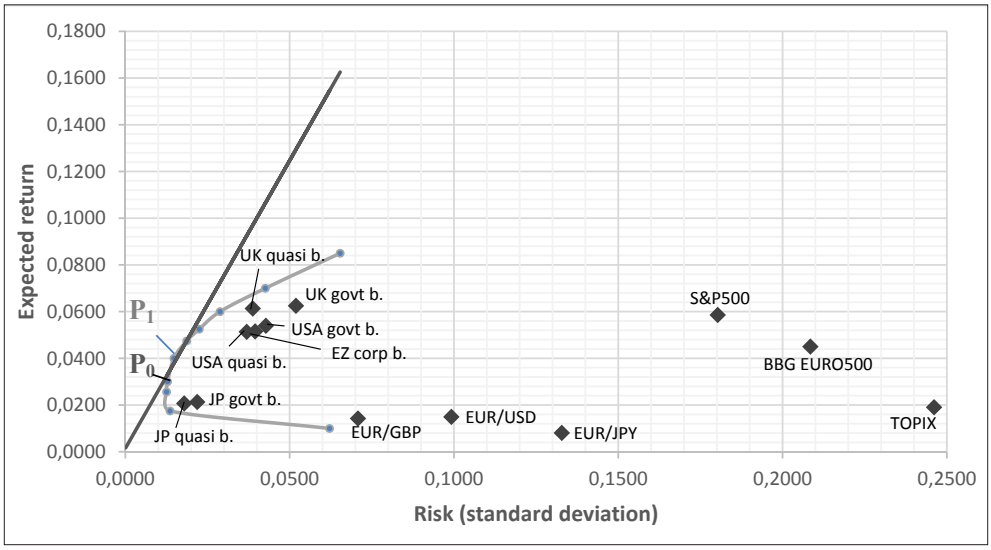

FIGURE 4. **Efficient frontier of Markowitz portfolio selection with the addition of all separate assets**

Source: compiled by authors

The graph shows the blue portfolio selection curve, representing all possible optimal portfolio combinations. The straight red CAL line has the slope equal to the Sharpe ratio, and is raised above the (0;0) by the value of the risk-free rate (0,0015). The P_0 point on the curve represents minimized risk portfolio, and is closest to the Yaxis. The

curve *above* P_0 is called the **efficient frontier**. P_1 marks the optimal portfolio(1) with the maximized Sharpe.

It is evident that in P_1 the portfolio risk is lower than the separate assets, also displayed on the chart. The stocks are obvious sources of risk, especially Japanese TOPIX index – investing full portfolio into it amounts to VaR of 1,7 bn euros, which is close to 30 % of the value of all financial assets, owned by the Bank of Lithuania. The foreign currencies are averagely risky and also the least profitable assets.

The average standard deviation of all portfolio components equals to 8,3 %, which is almost six times higher than the optimal portfolio(1) risk characteristic. The diversification effect is strongly evident.

Finally, the authors measured the benefits of adding three foreign currencies – USD, JPY and GBP into the portfolio. It was conducted by comparing the characteristics of optimal portfolio(1) and the new optimized portfolio with foreign currency weights set to zero. The goal was to measure structural changes and changes in return. The risk was set to constant.

TABLE 8. **The effects of adding foreign currencies into optimal portfolio**

	1) Portfolio without foreign currencies	2) Optimal portfolio(1)
Risky portfolio part	81,62%	**90,15%**↑
Risk-free portfolio part	18,38%	9,85%↓
Expected return	3,8%	**4,1%**↑
Standard deviation	1,47%	1,47%

Source: compiled by authors

The portfolio without foreign currencies is more true to the Bank's strategic investment allocation proposed for 2016, where addition of currencies was not considered. However, optimizing such portfolio and keeping the level of risk stable (to meet the VaR requirements) significantly increases the weight of the risk-free asset (over 18 %) and the expected return decreases by 0,3 %. This ratio equals to -12,3 M euros.

In conclusion, theoretically, the benefits of the portfolio diversification can be practically unlimited – if we consider increasing the weight of the risk-free asset almost to a 100 %, reducing VaR to less than 10 M euros. However, profitability must also be considered. In the case of this research, the diversification benefits are **risk reduction of 5,6 times**, and the benefits of hedging with foreign currencies equal to **0,3 % higher portfolio returns**, while the risk remains the same.

Discussing alternative strategies

One alternative strategy was already constructed in the chapter ***The Optimized Final Portfolio*** – a more conservative optimal portfolio(2) with an increased weight of the risk-free asset. Such a safe strategy is currently inefficient, as the main current goal is to increase the Bank's profitability. This chapter discusses and illustrates more alternative strategies.

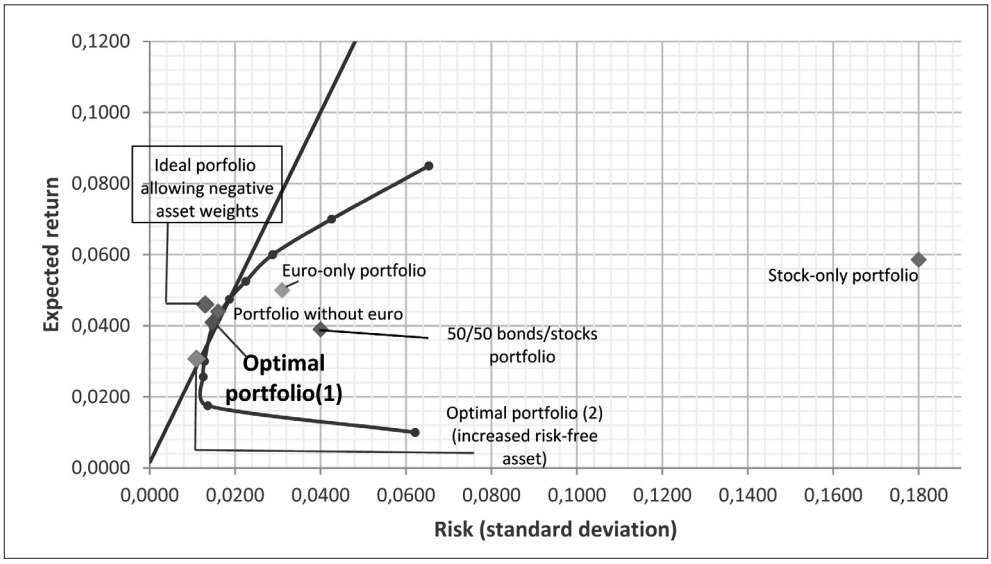

FIGURE 5. **Efficient frontier of Markowitz portfolio selection with the addition of alternate strategies portfolio examples**

Source: compiled by authors

This graph provides additional specific portfolios, constructed by the authors, with the goal to evaluate the effectiveness of such investment strategies. Closest to the Y axis lies an optimal portfolio(2) with an increased risk-free asset part and it displays the lowest risk characteristic of 1,1 %, and also the lowest return characteristic of 3,1%. Above it, is the ideal portfolio with maximized Sharpe, that allows the Bank to short-sell its assets, as discussed in the beginning of ***The Optimized Final Portfolio*** chapter. Such portfolio has a risk value of 1,3 % and yields higher returns, however, it is currently impossible to achieve, without changing the Bank's inner investment requirements. Between the ideal and the conservative portfolio lies the green marked **optimal portfolio(1)** with the risk of 1,5 % and VaR that meets the Bank's requirements.

Out of sheer curiosity, the authors also compared the characteristics of a portfolio comprised only of euro investments, and another portfolio comprised only of foreign currency investments. The results are as follows:

- **Euro-only** portfolio. In this case, the set of options are highly reduced, and the Bank can only invest to euro zone bonds and stocks. In this case, it is optimal to direct 93 % of the portfolio into Quasi-government sector bonds (if it was possible) and 7 % into EZ stocks, which partially hedges the risk. Such a portfolio has a risk of 3 % with a relatively high return, but it is not optimal. In conclusion – investing only in euros is not efficient;
- **Foreign currency** portfolio without euro. Such portfolio would be the most acceptable, non-optimal alternative. Its characteristics seem even better, more profitable than optimal portfolio(1), although it must be noted that VaR does not meet the requirements, and the Bank cannot allow itself to invest only in foreign currencies for the possible liquidity issues;

Finally, two more portfolios with an increased weight of volatile stocks were also compared, with the goal to measure how rapidly the risk would increase:

- **50/50 bonds/stocks proportion** portfolio. After giving the condition for stock weight values to be no less than the bond weights, the optimizer compiles a portfolio which has the risk of 4 % and return of 3,9 %. A relatively higher weight is given to foreign currencies, which hedge the risk in stocks, while having the lowest returns out of all assets;
- **Stock-only** portfolio (without hedging). It's only natural that this portfolio is the riskiest one – its standard deviation reaches 18 %. In this case, it is recommended to choose only American S&P500 stocks, which display the most favourable return/risk ratio, and, unfortunately, can't be hedged by other stocks.

In conclusion, the portfolios with an increased stock weight are highly inefficient, which validates the global practice of institutional investors to keep the investments to stocks to a minimum (up to 10 %), or not invest at all. Additionally, portfolios comprised only of euros are way riskier and more inefficient than foreign currency portfolios. Finally, it is evident that including a variety of different asset classes allows us to achieve better results.

Conclusions and recommendations

The results of the analysis of the Bank of Lithuania's financial assets carried out by the authors reveal these conclusions:

1) The data collection in official statistics is extremely problematic regarding the value of state assets (i.e. the lack of data past the year of 2011 and different values provided for the same year), however, it is safe to say that the value of financial assets as a percentage of total state assets doubled in the span of 10 years;
2) A strong correlation between the real GDP growth and Bank of Lithuania's financial assets/profitability implies that the effectiveness of financial assets management has a nationally wide impact. However, the Bank's profit/invested value indicator has reached a record low in 2012 – 2013, meaning the previous strategy was highly ineffective;

3) The authors have selected 19 securities/other assets as variables and constructed an optimal portfolio, which yields 4,1 percent annual return at 1,5 percent risk (annual VaR being less than -100 M euros). The portfolio consists of 9,85 percent risk-free asset (1–6 month German Treasury bills) and 90,15 percent risky part, comprised of 13 assets, the riskiest being Japanese TOPIX stock index with the standard deviation of 24,6 percent;
4) The addition of foreign currency spot exchanges provides positive results of increasing the returns by 0,3 percent (while the risk remains the same), which translates to +12,3 M euros annually;
5) Alternatively, if the Bank predicts an economic slump, it is relatively simple to tighten the risk budget and shift the investments to the risk-free asset, and as a result decreasing the returns by 1 percent;
6) On the other hand, if the Bank is willing to risk slightly more, it is possible to decrease or completely omit the euro investments and invest only in foreign currencies, which provides better returns with an additional liquidity risk.

Following that, several investment recommendations can be made:

1) Since the structure of the optimal portfolio slightly deviates from the Bank of Lithuania's strategic guidelines, it is recommended to invest less in euros and more in US dollars and Japanese yen, adding spot exchanges of foreign currencies. It is also advised to decrease safe Government bonds investment weight by 10 percent, and to increase stocks weight by 2,4 percent;
2) It is strongly not recommended to increase the stocks weight in the portfolio to more than 10 percent, as the risk rises greatly;
3) While the addition of foreign currencies is proven to be unquestionably beneficial in this case, it is advised not to ignore the additional costs it would cause (such as rebalancing costs or liquidity risks) and reach the decision whether to buy foreign currency or not, after weighing those costs against benefits;
4) If possible, it would be helpful to provide short annual investment strategy reports, which would summarize the goals and results, the actual returns' differences from expected values, and the overall effectiveness of the strategy, while keeping the exact structure of the portfolio (sensitive information) unrevealed.

Ultimately, this research was a mini-simulation of an actual investment portfolio selection process, that occurs in the Bank of Lithuania. In reality, the number of variables that comprise the model is way higher than 19, and the selection is an automatic process. The market imperfections and transaction costs the Markowitz model initially ignores are also taken into account. Moreover, the investment horizon, asset weight requirements and restrictions in the Bank of Lithuania can change annually, as the new goals come into picture and the new financial instruments get introduced to the markets; so the model assumptions in this case are neither unchangeable, nor universal.

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