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Asymptotic behaviour of mild solution of nonlinear stochastic partial functional equations driven by Rosenblatt process

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Abstract. This paper presents conditions to assure existence, uniqueness and stability for impulsive neutral stochastic integrodifferential equations with delay driven by Rosenblatt process and Poisson jumps. The Banach fixed point theorem and the theory of resolvent operator developed by Grimmer [R.C. Grimmer, Resolvent operators for integral equations in a Banach space, *Trans. Am. Math. Soc.*, 273(1):333–349, 1982] are used. An example illustrates the potential benefits of these results.

Keywords: partial functional differential equations, existence result, resolvent operator, stability, Rosenblatt process, Poison jumps.

1 Introduction

Stochastic differential equations (SDEs) arise in many areas of science and engineering, wherein, quite often the future state of such systems depends not only on present state, but also on its history leading to stochastic functional differential equations with delays rather than SDEs. However, many stochastic dynamical systems depend not only on present and past states, but also contain the derivatives with delays. Neutral stochastic differential equations with delays are often used to describe such systems (see, e.g., [19, 30]). On the other hand, the stability of impulsive differential equations has been discussed by several authors (see, e.g., [2, 10, 26, 32, 33, 38]).

Stochastic integrodifferential equations with delay are important for investigating several problems raised from natural phenomena. As far as applications are concerned,

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stochastic evolution equations have been motivated by such phenomena as wave propagation in random media [16] and turbulence [12]. Important motivations came also from biological sciences, in particular from population biology; see Dawson [27] and Fleming [7]. One has finally to mention early control theoretic application of Wang [8], Kushner [11], Bensoussan and Viot [36]. In addition, the study of neutral stochastic functional differential equations (SFDEs) driven by jumps process also have begin to gain attention and strong growth in recent years (see [15, 21, 31] and references therein).

Regarding the fractional Brownian motion (fBm), one can find results involving existence, uniqueness and stability of solutions for stochastic functional differential equations; see [3–6, 20, 22–24, 28, 34]. Let $\mathcal{H}$ be a separable Hilbert space.

In this work, we shall prove the existence, uniqueness and asymptotic behavior of mild solution for a class of impulsive stochastic neutral functional integrodifferential equation with delays driven by Rosenblatt process and Poisson jumps described in the form:

$$
\begin{aligned}
&\mathrm{d}\big[u(t) + G\big(t, u\big(t - r(t)\big)\big)\big] \\
&\quad = \Bigg[A\big[u(t) + G\big(t, u\big(t - r(t)\big)\big)\big] + \int_0^t \Theta(t-s)\big(u(s) + G\big(s, u\big(s - r(s)\big)\big)\big)\Bigg]\,\mathrm{d}t \\
&\qquad + F\big(t, x\big(t - \rho(t)\big)\big)\,\mathrm{d}t + \int_{\Lambda} \sigma\big(t, x\big(t - k(t)\big), \nu\big)\,\widetilde{N}(\mathrm{d}t, \mathrm{d}\nu) \qquad (1) \\
&\qquad + \Gamma(t)\mathrm{d}Z_H(t), \quad t \geqslant 0,\ t \neq t_k, \\
&\Delta u(t_k) = x\big(t_k^+\big) - x\big(t_k^-\big) = I_k\big(u(t_k)\big), \quad t = t_k, k = 1, 2, \dots, \\
&u(\cdot) = \zeta(\cdot) \in \mathcal{D}_{\mathcal{F}_0}\big([-\tau, 0], \mathbb{H}\big), \quad \tau > 0,
\end{aligned}
$$

where A, which is the infinitesimal generator of a strongly continuous semigroup $(S(t))_{t>0}$ on a Hilbert space $\mathbb{H}$ with domain $D(A)$, $(\Theta(t))_{t\geqslant 0}$, is a closed linear operator on $\mathbb{H}$ with domain $D(\Theta(t)) \supset D(A)$, $\Lambda \in \mathcal{B}_\sigma(\mathcal{H} - \{0\})$ ($\mathcal{B}_\sigma(\mathcal{H} - \{0\})$ is the Borel trace σ-algebra on $\mathcal{H} - \{0\}$) is a Borel set, and $\widetilde{N}$ will be defined later.

Let $\mathcal{D}_{\mathcal{F}_0} = \mathcal{D}_{\mathcal{F}_0}([-\tau, 0], \mathbb{H})$ be the space of càdlàg $\mathcal{F}_0$-measurable functions almost surely bounded from $[-\tau, 0] \times \Omega$ into $\mathbb{H}$, equipped with the supremum norm $\|\zeta\|_{\mathcal{D}} = \sup_{-\tau\leqslant\theta\leqslant 0} \|\zeta(\theta)\|_{\mathbb{H}}$, and ζ has finite second moment. Z_H is a Rosenblatt process on a real and separable Hilbert space $\mathbb{K}_0$. $r, \rho, k : [0, \infty) \to [0, \tau]$ ($\tau > 0$) are continuous functions, and $G, F : [0, +\infty) \times \mathbb{H} \to \mathbb{H}$, $G : [0, +\infty) \to \mathcal{L}_2^0(\mathbb{K}, \mathbb{H})$, $\sigma : [0, +\infty) \times \mathbb{H} \times \Lambda \to \mathbb{H}$ are appropriate functions. Here $\mathcal{L}_2^0(\mathbb{K}, \mathbb{H})$ denotes the space of all Q-Hilbert–Schmidt operators from $\mathbb{K}$ into $\mathbb{H}$. Moreover, the fixed moments of time t_k satisfy $0 < t_1 < t_2 < \cdots < t_k < \cdots$, and $\lim_{k\to\infty} t_k = \infty$, $x(t_k^-)$ and $x(t_k^+)$ represent the left and right limits of $u(s)$ at time t_k with $I_k(\cdot) : \mathbb{H} \to \mathbb{H}$ determining the size of the jump.

The analysis of (1) when $B \equiv 0$ was initiated in Ouahra et al. [28], where the authors proved the existence and stability of solutions by using a strict contraction principle. The main contribution is towards this direction by presenting conditions to assure existence, uniqueness and stability for such a class of system with the integrodifferential term. Our paper expands the usefulness of stochastic integrodifferential equations since the literature shows results for existence and exponential stability for such equations under semigroup theory.

The remaining of the paper is organized as follows. Section 2 presents notation and preliminary results. Section 3 shows the main results for existence, uniqueness of mild solutions for impulsive neutral stochastic integrodifferential equations driven by Rosenblatt process and Poisson processes and conditions for the exponential stability in mean square. Finally, Section 4 presents an example that illustrates our results.

2 Preliminaries

In this section, we provide some basic results about Poisson process, resolvent operator and Rosenblatt process.

2.1 Poisson jumps process

We denote by $\mathcal{B}_\sigma(\mathcal{H})$ the Borel σ-algebra of $\mathcal{H}$. Let $(p(t))_{t\geqslant 0}$ be an $\mathcal{H}$-valued, σ-finite stationary $\mathcal{F}_t$-adapted Poisson point process on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geqslant 0}, \mathbf{P})$. The counting random measure N defined by

$$N\big((t_1, t_2] \times \Lambda\big)(w) := \sum_{t_1 < s \leqslant t_2} \mathbf{1}_\Lambda\big(p(s)(w)\big)$$

for any $\Lambda \in \mathcal{B}_\sigma(\mathcal{H} - \{0\})$; where $0 \notin \bar{\Lambda}$ (the closure of Λ) is called the Poisson random measure associated to the Poisson point process p. The following notation is used:

$$N(t, \Lambda) = N\big((0, t] \times \Lambda\big).$$

Then it is known that there exists a σ-finite measure ν such that

$$\mathbf{E}\big(N(t, \Lambda)\big) = \nu(\Lambda)t, \qquad \mathbf{P}\big(N(t, \Lambda) = k\big) = \frac{\exp(-t\nu(\Lambda))(t\nu(\Lambda))^k}{k!}.$$

This measure ν is said the Lévy measure. Then the measure $\widetilde{N}$ is defined by

$$\widetilde{N}\big((0, t] \times \Lambda\big) = N\big((0, t] \times \Lambda\big) - t\nu(\Lambda).$$

This measure $\widetilde{N}(\mathrm{d}t, \mathrm{d}y)$ is called the compensated Poisson random measure, and $\mathrm{d}t\nu(\Lambda)$ is called the compensator (see [18]).

Definition 1. Let $\Lambda \in \mathcal{B}_\sigma(\mathcal{H} - \{0\})$. $\mathcal{P}^2([0, T] \times \Lambda; \mathbb{H})$ is the space of all predictable mappings $h : [0, T] \times \Lambda \times \Omega \to \mathbb{H}$ for which

$$\int_0^T \int_\Lambda \mathbf{E}\big\|h(t, \nu)\big\|^2 \,\mathrm{d}t\, \lambda(\mathrm{d}\nu) < \infty.$$

We may then define the $\mathbb{H}$-valued stochastic integral $\int_0^T \int_\Lambda L(t, \nu)\widetilde{N}(\mathrm{d}t, \mathrm{d}\nu)$, which is a centered square-integrable martingale [25].

2.2 Deterministic integrodifferential equations

In this subsection, we recall some knowledge on partial integrodifferential equations and the related resolvent operators. Let $\mathbb{H}$ and Y be two Banach spaces such that

$$\|y\|_Y = \|Ay\| + \|y\|, \quad y \in Y.$$

A and $\Theta(t)$ are closed linear operator on $\mathbb{H}$. Let $C([0,+\infty);Y)$, $B(Y,\mathbb{H})$ stand for the space of all continuous functions from $[0,+\infty)$ into Y, the set of all bounded linear operators from Y into $\mathbb{H}$, respectively. In what follows, we suppose the following assumptions:

(H1) A represents the infinitesimal generator of a strogly continuous semigroup on $\mathbb{H}$.

(H2) For all $t > 0$, Θ denotes a closed, continuous linear operator from $D(A)$ to $\mathbb{H}$; in addition, $\Theta(t)$ is a bounded linear operator from $(Y, \|\cdot\|_Y)$ into $(\mathbb{H}, \|\cdot\|_{\mathbb{H}})$. For any $y \in (Y, \|\cdot\|_Y)$, the map $t \to \Theta(t)y$ is bounded, differentiable, and the derivative $d\Theta(t)y/\,\mathrm{d}t$ is bounded and uniformly continuous on $[0,+\infty[$.

By Grimmer [13], under assumptions (H1) and (H2), the following Cauchy problem

$$\begin{aligned} L'(t) &= AL(t) + \int_0^t \Theta(t-s)L(s)\,\mathrm{d}s, \quad t \geqslant 0, \\ L(0) &= l_0 \in \mathbb{H}. \end{aligned} \tag{2}$$

has an associated resolvent operator of bounded linear operator valued function $\mathcal{R}(t) \in \mathcal{L}(\mathbb{H})$ for $t \geqslant 0$.

Definition 2. (See [13].) A resolvent operator for Eq. (2) is a bounded linear operator valued function $\mathcal{R}(t) \in \mathcal{L}(\mathbb{H})$, $t \geqslant 0$, satisfying the following properties:

(i) $\mathcal{R}(0) = I$ and $\|\mathcal{R}(t)\| \leqslant M\mathrm{e}^{kt}$ for some constants M and k.

(ii) For each $x \in \mathbb{H}$, $\mathcal{R}(t)x$ is strongly continuous for $t \geqslant 0$.

(iii) For $x \in Y$, $\mathcal{R}(\cdot)x \in C^1([0,+\infty[;\mathbb{H}) \cap C([0,+\infty[;Y)$ and

$$\begin{aligned} u(t)\mathcal{R}'(t)x &= A\mathcal{R}(t)x + \int_0^t \Theta(t-s)\mathcal{R}(s)x\,\mathrm{d}s \\ &= \mathcal{R}(t)Ax + \int_0^t \mathcal{R}(t-s)\Theta(s)x\,\mathrm{d}s, \quad t \geqslant 0. \end{aligned}$$

Hereafter, the resolvent operator for (2) is assumed to be continuous and exponentially stable.

(H3) The resolvent operator $R(\cdot)$ is both norm continuous and exponentially stable (the exponential stability means that there exist constants $M > 0$ and $k \geqslant 1$ such that $\|\mathcal{R}(t)\| \leqslant M\mathrm{e}^{-\kappa t}$ for all $t \geqslant 0$).

2.3 Rosenblatt process

Self-similar processes are invariant in distribution under suitable scaling. They are of considerable interest in practice since aspects of the self-similarity appear in different phenomena like telecommunications, turbulence, hydrology or economics (see, e.g., [14, 17, 24, 37]).

A self-similar processes can be defined as limits that appear in the so-called noncentral limit theorem (see [34]). We briefly recall the Rosenblatt process as well the Wiener integral with respect to it.

Let us recall the notion of Hermite rank. Denote by $H_j(x)$ the Hermite polynomial of degree j given by $H_j(x) = (-1)^j \mathrm{e}^{x^2/2}(\mathrm{d}^j/\mathrm{d}x^j)\mathrm{e}^{-x^2/2}$, and let g be a function on $\mathbb{R}$ such that $\mathbf{E}[g(\zeta_0)] = 0$ and $\mathbf{E}[(g(\zeta_0))^2] < \infty$. Assume that g has the following expansion in Hermite polynomials

$$g(x) = \sum_{j\geqslant 0} c_j H_j(x),$$

where $c_j = \mathbf{E}[g(\zeta_0 H_j(\zeta))]/j!$. The Hermite rank of g is defined by

$$k = \min\{j\colon\ c_j \neq 0\}.$$

Consider $(\zeta_n)_{n\in\mathbb{Z}}$ a stationary Gaussian sequence with mean zero and variance 1, which exhibits long range dependence in the sense that the correlation function satisfies

$$r(n) = \mathbf{E}(\zeta_0\zeta_n) = n^{(2H-2)/k}L(n),$$

with $H \in (1/2, 1)$ and L is a slowly varying function at infinity. Since $\mathbf{E}[g(\zeta_0)] = 0$, we have $k \geqslant 1$. Then the following family of stochastic processes

$$\frac{1}{n^H}\sum_{j=1}^{[nt]} g(\zeta)$$

converges as $n \to \infty$, in the sense of finite dimensional distributions, to the self-similar stochastic process with stationary increments

$$Z_H^k(t) = c(H,k) \times \int_{\mathbb{R}^k} \left(\int_0^t \prod_{j=1}^k (s-y_j)_+^{-(1/2+(1-H)/k)}\,\mathrm{d}s \right) \mathrm{d}B(y_1)\,\mathrm{d}B(y_1)\cdots\,\mathrm{d}B(y_k), \tag{3}$$

where $x_+ = max(x, 0)$. The above integral is a Wiener-Itô multiple integral of order k with respect to the standard Brownian motion $(B(y))_{y\in\mathbb{R}}$, and the constant $c(H, k)$ is a normalizing constant that ensures $\mathbf{E}(Z_H^k(1))^2 = 1$. The process $(Z_H^k(t))_{t\geqslant 0}$ is called the Hermite process. When $k = 1$, the process given by (3) is nothing else that the fractional Brownian motion (fBm) with Hurst parameter $H \in (1/2, 1)$. For $k = 2$, the process is not Gaussian. If $k = 2$, then process (3) is known as the Rosenblatt process [34]. The

Rosenblatt process is of course the most studied process in the class of Hermite processes due to its significant importance in modelling. The Rosenblatt process is, after fBm, the most well know Hermite process.

We also recall the following properties of the Rosenblatt process:

- The process Z_H^k is H-self-similar in the sense that for any $c > 0$,
$$\big(Z_H^k(ct)\big) \stackrel{\mathrm{d}}{=} \big(c^H Z_H^k(t)\big),$$
where $\stackrel{\mathrm{d}}{=}$ means equivalence of all finite dimensional distributions. It has stationary increments and all moments are finite.
- From the stationarity of increments and the self-similar, it follows that, for any $p \geqslant 1$,
$$\mathbf{E}\big\|Z_H(t) - Z_H(s)\big\|_{\mathbb{K}_0}^p \leqslant \big\|\mathbf{E}\big(Z_H(1)\big)\big\|_{\mathbb{K}_0}^p |t-s|^{pH}.$$
As a convergence, the Rosenblatt process has Hölder continuous paths of order γ with $0 < \gamma < H$.

Self-similarity and long-range dependence make this process a useful driving noise in models arising in physics, telecommunication networks, finance and other fields.

Consider a time interval $[0, T]$ with arbitrary fixed horizon T and let $\{Z_H(t),\ t \in [0, T]\}$ the on-dimensional Rosenblatt process with parameter $H \in (1/2, 1)$. By Tudor [35], it is well known that Z_H has the following integral representation:

$$Z_H(t) = \mathrm{d}(H) \int_0^t \int_0^t \left[\int_{y_1 \vee y_2}^t \frac{\partial K^{H'}}{\partial u}(u, y_1) \frac{\partial K^{H'}}{\partial u}(u, y_2)\, \mathrm{d}u \right] \mathrm{d}B(y_1)\, \mathrm{d}B(y_2), \tag{4}$$

where $B = \{B(t)\colon t \in [0, T]\}$ is a Wiener process, $H' = (H{+}1)/2$, $\mathrm{d}(H) = (1/(H{+}1)) \times \sqrt{H/(2(2H-1))}$ is a normalizing constant, and $K^H(t, s)$ is a Kernel given by

$$K^H(t, s) = c_H s^{1/2-H} \int_s^t (u-s)^{H-3/2} u^{H-3/2} \mathrm{d}u$$

for $t > s$, where $c_H = \sqrt{H(2H-1)/(\beta(2-2H, H-1/2))}$, and $\beta(\cdot,\cdot)$ denotes the beta function. We put $K^H(t, s) = 0$ if $t \leqslant s$.

The basic observation is the fact that the covariance structure of the Rosenblatt process is similar to the one of the Rosenblatt process, and this allows the use of the same classes of deterministic integrands as in the Rosenblatt process. By formula (4) we can write

$$Z_H(t) = \int_0^t \int_0^t I(\mathbf{1}_{[0,t]})(y_1, y_2)\, \mathrm{d}B(y_1)\, \mathrm{d}B(y_2),$$

where I denote the mapping on the set of functions $f : [0,T] \to \mathbb{R}$ to the set of functions $f : [0,T]^2 \to \mathbb{R}$ such that

$$I(f)(y_1, y_2) = \mathrm{d}(H) \int\limits_{y_1 \vee y_2}^{T} f(u) \frac{\partial K^{H'}}{\partial u}(u, y_1) \frac{\partial K^{H'}}{\partial u}(u, y_2)\, \mathrm{d}u.$$

Let us denote by ξ the class of elementary functions on $\mathbb{R}$ of the form

$$f(\cdot) = \sum_{j=1}^{n} a_j \mathbf{1}_{(t_j, t_{j+1}]}(\cdot), \quad 0 \leqslant t_j < t_{j+1} \leqslant T,\ a_j \in \mathbb{R},\ j = 1, \dots, n.$$

For $f \in \xi$ as above, it is natural to define its Wiener integral with respect to the Rosenblatt process Z_H by

$$\begin{aligned}\int\limits_0^T f(s)\, \mathrm{d}Z_\Gamma(s) &:= \sum_{j=1}^{n} a_j \big[Z_H(t_{j+1}) - Z_H(t_j)\big] \\ &= \int\limits_0^T \int\limits_0^T I(f)(y_1, y_2)\, \mathrm{d}B(y_1)\, \mathrm{d}B(y_2).\end{aligned}$$

Let $\mathcal{K}$ be the set of functions f such that

$$\mathcal{K} = \Biggl\{ f : [0,T] \to \mathbb{R}\colon\ \|f\|_{\mathcal{K}} := \int\limits_0^T \int\limits_0^T \bigl(I(f)(y_1, y_2)\bigr)\, \mathrm{d}B(y_1)\, \mathrm{d}B(y_2) < \infty \Biggr\}.$$

It holds that

$$\|f\|_{\mathcal{K}} = H(2H-1) \int\limits_0^T \int\limits_0^T f(u) f(v) |u - v|^{2H-2} \mathrm{d}u\, \mathrm{d}v,$$

and the mapping

$$f \to \int\limits_0^T f(u)\, \mathrm{d}Z_H(u) \tag{5}$$

provides an isometry from ξ to $L^2(\Omega)$. On the other hand, it has been proved in [29] that the set of elementary functions ξ is dense in $\mathcal{K}$. As a consequence, mapping (5) can be extended to an isometry from $\mathcal{K}$ to $L^2(\Omega)$. We call this extension as the Wiener integral of $f \in \mathcal{K}$ with respect to Z_H.

Let us consider the operator K_H^* from ξ to $\mathcal{L}^2([0,T])$ defined by

$$(K_H^* \varphi)(y_1, y_2) = \int\limits_{y_1 \vee y_2}^{T} \varphi(r) \frac{\partial K^{H'}}{\partial r}(r, y_1, y_2)\, \mathrm{d}r,$$

where $K(\cdot,\cdot,\cdot)$ is the kernel of Rosenblatt process in representation (4)

$$K(t,y_1,y_2)=\mathbf{1}_{[0,t]}(y_1)\mathbf{1}_{[0,t]}(y_2)\int\limits_{y_1\vee y_2}^{t}\frac{\partial K^{H'}}{\partial u}(u,y_1)\frac{\partial K^{H'}}{\partial u}(u,y_2)\,\mathrm{d}u.$$

We refer to [35] for the proof of the fact that K_H^* is an isometry between $\mathcal{K}$ and $\Lambda^2([0,T])$. It follows from [35] that $\mathcal{K}$ contains not only functions, but its elements could be also distributions. In order to obtain a space of functions contained in $\mathcal{K}$, we consider the linear space $|\mathcal{K}|$ generated by the measurable functions ψ such that

$$\|\psi\|_{|\mathcal{K}|}^2:=\alpha_H\int\limits_0^T\int\limits_0^T\big\|\psi(s)\big\|_{\mathbb{K}}\big\|\psi(t)\big\|_{\mathbb{K}}|s-t|^{2H-2}\,\mathrm{d}s\,\mathrm{d}t<\infty,$$

where $\alpha_H=H(2H-1)$. The space $|\mathcal{K}|$ is a Banach space with the norm $\|\psi\|_{|\mathcal{K}|}$, and we have the following inclusions (see [35]).

Lemma 1.

$$\mathcal{L}^2\big([0,T]\big)\subseteq\mathcal{L}^{1/H}\big([0,T]\big)\subseteq|\mathcal{K}|\subseteq\mathcal{K},$$

and for any $\psi\in\mathcal{L}^2([0,T])$*, we have*

$$\|\psi\|_{|\mathcal{K}|}^2\leqslant 2HT^{2H-1}\int\limits_0^T\big\|\psi(s)\big\|_{\mathbb{K}}^2\,\mathrm{d}s.$$

Let $\mathbb{H}$ and $\mathbb{K}$ be two real, separable Hilbert spaces, and let $\mathcal{L}(\mathbb{K},\mathbb{H})$ be the space of bounded linear operator from $\mathbb{K}$ to $\mathbb{H}$. For the sake of convenience, we shall use the same notation to denote the norms in $\mathbb{H}$, $\mathbb{K}$ and $\mathcal{L}(\mathbb{K},\mathbb{H})$. Let $Q\in\mathcal{L}(\mathbb{K},\mathbb{H})$ be an operator defined by $Qe_n=\lambda_n e_n$ with finite trace $\mathrm{tr}Q=\sum_{n=1}^{\infty}\lambda_n<\infty$, where $\lambda_n\geqslant 0$ $(n=1,2\dots)$ are nonnegative real numbers, and $\{e_n\}$ $(n=1,2,\dots)$ is a complete orthonormal basis of $\mathbb{K}$. We define the infinite dimensional Q-Rosenblatt process on $\mathbb{K}$ as

$$Z_H(t)=Z_Q(t)=\sum_{n=1}^{\infty}\sqrt{\lambda_n}e_n z_n(t),\tag{6}$$

where $(z_n)_{n\geqslant 0}$ is a family of real independent Rosenblatt process. Note that series (6) is convergent in $L^2(\Omega)$ for every $t\in[0,T]$ since

$$\mathbf{E}\big\|Z_Q(t)\big\|_{\mathbb{K}}^2=\sum_{n=1}^{\infty}\lambda_n\big(\mathbf{E}\big(z_n(t)\big)\big)^2=t^{2H}\sum_{n=1}^{\infty}\lambda_n<\infty.$$

Note also that Z_Q has covariance function in the sense that

$$\mathbf{E}\big\langle Z_Q(t),x\big\rangle\big\langle Z_Q(s),y\big\rangle=R(s,t)\big\langle Q(x),y\big\rangle,\quad x,y\in\mathbb{K},\ t,s\in[0,T].$$

In order to define Wiener integrals with respect to the Q-Rosenblatt process, we introduce the space $\mathcal{L}_2^0 := \mathcal{L}_2^0(\mathbb{K},\mathbb{H})$ of all Q-Hilbert–Schmidt operators $\psi : \mathbb{K} \to \mathbb{H}$. We recall that $\psi \in \mathcal{L}(\mathbb{K},\mathbb{H})$ is called a Q-Hilbert–Schmidt operator if

$$\|\psi\|_{\mathcal{L}_2^0} := \sum_{n=1}^{\infty} \|\sqrt{\lambda_n}\psi e_n\|^2 < \infty,$$

and that the space $\mathcal{L}_2^0$ equipped with the inner product $\langle \varphi, \psi\rangle_{\mathcal{L}_2^0} = \sum_{n=1}^{\infty}\langle \varphi e_n, \psi e_n\rangle$ is a separable Hilbert space.

Now, let $\phi(s)$, $s \in [0,T]$, be a function with values in $\mathcal{L}_2^0(\mathbb{K},\mathbb{H})$ such that $\sum_{n=1}^{\infty} \|K^*\phi Q^{1/2} e_n\|^2_{\mathcal{L}_2^0} < \infty$. The Wiener integral of ϕ with respect to Z_Q is defined by

$$\begin{aligned}\int_0^t \phi(s)\,\mathrm{d}Z_Q(s) &= \sum_{n=1}^{\infty} \int_0^t \sqrt{\lambda_n}\phi(s)\,\mathrm{d}z_n(s) \\ &= \sum_{n=1}^{\infty} \int_0^t \int_0^t \sqrt{\lambda_n} K_H^*(\phi e_n)(y_1,y_2)\,\mathrm{d}B(y_1)\,\mathrm{d}B(y_2).\end{aligned}$$

Now, we end this subsection by stating the following result, which is useful to prove the main result.

Lemma 2. *If $\psi : [0,T] \to \mathcal{L}_2^0(\mathbb{K},\mathbb{H})$ satisfies $\int_0^T \|\psi(s)\|^2_{\mathcal{L}_2^0}\,\mathrm{d}s < \infty$, then the above sum in* (3) *is well defined as a $\mathbb{H}$-valued random variable, and we have*

$$\mathbf{E}\left\|\int_0^t \psi(s)\,\mathrm{d}Z_H(s)\right\|^2_{\mathbb{H}} \leqslant 2Ht^{2H-1}\int_0^t \left\|\psi(s)\right\|^2_{\mathcal{L}_2^0}\,\mathrm{d}s.$$

Proof. Let $\{e_n\}$ $(n = 1,2,\dots)$ be the complete orthonormal basis of $\mathbb{K}$ introduced above. Applying Hölder inequality, we have

$$\begin{aligned}\mathbf{E}\left\|\int_0^t \psi(s)\,\mathrm{d}Z_\Gamma(s)\right\|^2_{\mathbb{H}} &= \mathbf{E}\left\|\sum_{n=1}^{\infty}\int_0^t\int_0^t \sqrt{\lambda_n}K_H^*(\psi e_n)(y_1,y_2)\,\mathrm{d}B(y_1)\,\mathrm{d}B(y_2)\right\|^2 \\ &= \sum_{n=1}^{\infty}\mathbf{E}\left\|\int_0^t\int_0^t \sqrt{\lambda_n}K_H^*(\psi e_n)(y_1,y_2)\,\mathrm{d}B(y_1)\,\mathrm{d}B(y_2)\right\|^2 \\ &\leqslant \sum_{n=1}^{\infty} 2Ht^{2H-1}\int_0^t \lambda_n\left\|\psi(s)e_n\right\|^2_{\mathcal{L}_2^0}\,\mathrm{d}s \\ &= 2Ht^{2H-1}\int_0^t \left\|\psi(s)\right\|^2_{\mathcal{L}_2^0}\,\mathrm{d}s.\end{aligned}$$

□

3 Main results

In this part, conditions guaranteeing existence, uniqueness and exponential stability of mild solution to Eq. (1) are presented. The next definition introduces the concept of *solution* for the stochastic system (1).

Definition 3. A process $\{u(t),\ t \in [-\tau, T]\}$ (with $T > 0$) that evolves on $\mathbb{H}$ is referred to as mild solution of (1) if it satisfies the next four conditions:

(i) $u(t)$ is $\mathcal{F}_t$-adapted and satisfies $\int_0^T \|u(t)\|^2\,\mathrm{d}t < +\infty$ almost surely, $t \geqslant 0$;
(ii) $u(t)$ is càdlàg, i.e., it is right continuous with left-limit paths on $[0, T]$ almost surely;
(iii) $u(t) = \zeta(t)$, $-\tau \leqslant t \leqslant 0$;
(iv) For all $t \in [0, T]$, $u(t)$ satisfies

$$\begin{aligned} &u(t) + G\big(t, u\big(t - \mathcal{R}(t)\big)\big) \\ &\quad = \mathcal{R}(t)\big(\zeta(0) + G\big(0, \zeta\big(-r(0)\big)\big)\big) \\ &\qquad + \int_0^t \mathcal{R}(t-s) F\big(s, u\big(s - \rho(s)\big)\big)\,\mathrm{d}s + \int_0^t \mathcal{R}(t-s)\varGamma(s)\,\mathrm{d}Z_H(s) \\ &\qquad + \int_0^t \int_{\varLambda} \mathcal{R}(t-s)\sigma\big(s, u\big(s - k(s)\big), \nu\big)\,\widetilde{N}(\mathrm{d}s, \mathrm{d}\nu) \\ &\qquad + \sum_{0<t_k<t} R(t - t_k) I_k\big(u(t_k)\big) \quad \mathbf{P}\text{-p.s.} \end{aligned}$$

In order to prove the main result in this section, we require the following assumptions:

(H4) The functions $F(t, \cdot)$, $G(t, \cdot)$ and σ satisfy global Lipschitz conditions, that is, there exists $K, \bar{K} > 0$ such that for any $\eta, \zeta \in \mathbb{H}$ and $t \geqslant 0$,

$$\begin{gathered} \big\|F(t,\zeta) - F(t,\eta)\big\|_{\mathbb{H}}^2 \leqslant K\|\zeta - \eta\|_{\mathbb{H}}^2, \\ \int_{\varLambda} \big\|\sigma(t,\zeta,\nu) - \sigma(t,\eta,\nu)\big\|_{\mathbb{H}}^2\,\lambda(\mathrm{d}\nu) \leqslant K\|\zeta - \eta\|_{\mathbb{H}}^2, \\ \big\|G(t,\zeta) - G(t,\zeta)\big\|_{\mathbb{H}}^2 \leqslant \bar{K}\|\zeta - \eta\|_{\mathbb{H}}^2, \\ F(t,0) = G(t,0) = \sigma(t,0,\nu) = 0, \quad t \geqslant 0,\ \nu \in \varLambda. \end{gathered}$$

(H5) The function G is continuous in the quadratic mean sense, i.e., for all $\kappa_1, \kappa_2 \in \mathcal{D}([0, T], L^2(\varOmega, \mathbb{H}))$,

$$\lim_{t \to s} \big\|G(t, \kappa_1) - G(s, \kappa_2)\big\|^2 = 0.$$

(H6) There exists a real number $\gamma > 0$ such that the function $G : [0, +\infty) \to \mathcal{L}_2^0(\mathbb{K}, \mathbb{H})$ satisfies

$$\int_0^{+\infty} \mathrm{e}^{2\gamma s} \big\|G(s)\big\|_{\mathcal{L}_2^0}^2 \, \mathrm{d}s < \infty.$$

(H7) The impulsive functions I_k ($k = 1, 2, \dots$) satisfy the following conditions: $I_k \in C(\mathbb{H}, \mathbb{H})$, and there exists some positive numbers q_k, $k = 1, 2, \dots$, such that $\|I_k(x) - I_k(x)\|_{\mathbb{H}} \leqslant q_k \|x - y\|_{\mathbb{H}}$ and $I_k(0) = 0$ for all $x, y \in \mathbb{H}$.

The main result of this paper is given in the next theorem.

Theorem 1. *Suppose that* (H1)–(H7) *hold and that*

(i) *There exists a constant $\tilde{q} > 0$ such that $q_k \leqslant \tilde{q}(t_k - t_{k-1})$, $k = 1, 2, \dots$;*
(ii) *The initial value ζ satisfies $\mathbf{E}\|\zeta(t)\|_{\mathbb{H}}^2 \leqslant M_0 \mathbf{E}\|\zeta\|_{\mathcal{D}}^2 \mathrm{e}^{-at}$, $t \in [-\tau, 0]$, for some $M_0 > 0$, $a > 0$;*
(iii) $4[\bar{K} + M^2 K \kappa^{-2} + M^2 K (2\kappa)^{-1} + M^{-2} \tilde{q} \kappa^{-2}] \leqslant 1$.

Then, for all $T > 0$, Eq. (1) *has a unique mild solution on $[-\tau, T]$ and is exponential decay to zero in mean square, i.e., there exists a pair of positive constants $a > 0$ and $\mu = \mu(\zeta, a)$ such that*

$$\mathbf{E}\big\|u(t)\big\|_{\mathbb{H}}^2 \leqslant \mu \mathrm{e}^{-at}, \quad t \geqslant 0.$$

Proof. In this proof, we let $\mathcal{M}_\zeta$ be the set all càdlàg processes $u(t, \omega) : [-\tau, T] \times \Omega \to \mathbb{H}$ satisfying the next two conditions: (i) $u(t) = \zeta(t)$, $t \in [-\tau, 0]$, and (ii) there exist some constants $\mu = \mu(\zeta, a) > 0$ and $a > 0$ such that $\mathbf{E}\|u(t)\|^2 \leqslant \mu \mathrm{e}^{-at}$ for all $t \geqslant 0$. We point out that $\mathcal{M}_\zeta$ equipped with the norm $\|\cdot\|_{\mathcal{M}_\zeta}$ is a Banach space.

To clarify the next reasoning, take the operator Φ on $\mathcal{M}_\zeta$ as

$$\Phi(u)(t) = \zeta(t), \quad t \in [-\tau, 0],$$

and for $t \in [0, T]$,

$$\begin{aligned} \Phi(u)(t) &= \mathcal{R}(t)\big(\zeta(0) + G\big(0, \zeta\big(-r(0)\big)\big)\big) - G\big(t, u\big(t - r(t)\big)\big) \\ &\quad + \int_0^t \mathcal{R}(t-s) F\big(s, u\big(s - \rho(s)\big)\big) \, \mathrm{d}s + \int_0^t \mathcal{R}(t-s) \Gamma(s) \, \mathrm{d}Z_H(s) \\ &\quad + \int_0^t \int_\Lambda \mathcal{R}(t-s) \sigma\big(s, u\big(s - k(s)\big), \nu\big) \, \widetilde{N}(\mathrm{d}s, \mathrm{d}\nu) \\ &\quad + \sum_{0 < t_k < t} \mathcal{R}(t - t_k) I_k\big(u(t_k)\big). \end{aligned}$$

Now, we prove that Φ has a fixed point in $\mathcal{M}_\zeta$.

The remaining arguments for proving the above theorem are divided into the following three main steps.

Step 1. We show that $\Phi(\mathcal{M}_\zeta) \subset \mathcal{M}_\zeta$.

Let $u(\cdot) \in \mathcal{M}_\zeta$, then we have

$$\begin{aligned}
&\big\|\mathbf{E}\Phi(u)(t)\big\|_{\mathbb{H}}^2 \\
&\quad \leqslant 6\Bigg[\mathbf{E}\big\|\mathcal{R}(t)\big(\zeta(0)+G\big(0,\zeta\big(-r(0)\big)\big)\big)\big\|_{\mathbb{H}}^2 + \mathbf{E}\big\|G\big(t,u\big(t-r(t)(t)\big)\big)\big\|_{\mathbb{H}}^2 \\
&\qquad + \mathbf{E}\Bigg\|\int_0^t \mathcal{R}(t-s)F\big(s,u\big(s-\rho(s)\big)\big)\,\mathrm{d}s\Bigg\|_{\mathbb{H}}^2 + \mathbf{E}\Bigg\|\int_0^t \mathcal{R}(t-s)\Gamma(s)\,\mathrm{d}Z_H(s)\Bigg\|_{\mathbb{H}}^2 \\
&\qquad + \mathbf{E}\Bigg\|\int_0^t\int_\Lambda \mathcal{R}(t-s)\sigma\big(s,u\big(s-k(s)\big),\nu\big)\,\widetilde{N}(\mathrm{d}s,\mathrm{d}\nu)\Bigg\|_{\mathbb{H}}^2 \\
&\qquad + \mathbf{E}\Bigg\|\sum_{0<t_k<t}\mathcal{R}(t-t_k)I_k\big(u(t_k)\big)\Bigg\|_{\mathbb{H}}^2\Bigg] \\
&\quad := 6\sum_{k=1}^6 P_k.
\end{aligned} \tag{7}$$

Without loss of generally, we may assume that $0 < a < \kappa$. Now, let us estimate the terms on right-hand side of inequality (7). Let $\mu = \mu(g,a) > 0$ and $a > 0$ such that

$$\mathbf{E}\big\|u(t)\big\|_{\mathbb{H}}^2 \leqslant \mu \mathrm{e}^{-at}, \quad t \geqslant 0.$$

Then by assumption (H3) we have

$$P_1 \leqslant M^2\mathbf{E}\big\|\zeta(0)+G\big(0,\zeta\big(-r(0)\big)\big)\big\|_{\mathbb{H}}^2\mathrm{e}^{-\kappa t} \leqslant C_1\mathrm{e}^{-\kappa t}, \tag{8}$$

where $C_1 = \mathbf{E}\|\zeta(0)+G(0,\zeta(-r(0)))\|_{\mathbb{H}}^2 < \infty$. By using assumption (H4) we obtain

$$\begin{aligned}
P_2 &\leqslant \mathbf{E}\big\|G\big(t,u\big(t-r(t)\big)\big) - G(t,0)\big\|_{\mathbb{H}}^2 \\
&\leqslant \bar{K}\mathbf{E}\big\|u\big(t-r(t)\big)\big\|_{\mathbb{H}}^2 \\
&\leqslant \bar{K}\big[\mu\mathrm{e}^{-a(t-r(t))} + \mathbf{E}\big\|\zeta\big(t-r(t)\big)\big\|_{\mathbb{H}}^2\big] \\
&\leqslant \bar{K}\big[\mu + M_0\mathbf{E}\|\zeta\|_{\mathcal{D}}^2\big]\mathrm{e}^{-a(t-r(t))} \\
&\leqslant \bar{K}\big[\mu + M_0\mathbf{E}\|\zeta\|_{\mathcal{D}}^2\big]\mathrm{e}^{-at}\mathrm{e}^{a\tau} \leqslant C_2\mathrm{e}^{-at},
\end{aligned} \tag{9}$$

where $C_2 = \bar{K}[\mu + M_0\mathbf{E}\|\zeta\|_{\mathcal{D}}^2]\mathrm{e}^{a\tau}$.

Using Hölder inequality and (H4), we have

$$\begin{aligned}
P_3 &\leqslant \mathbf{E}\bigg\| \int_0^t \mathcal{R}(t-s)F\big(s,u\big(s-\rho(s)\big)\big)\,\mathrm{d}s\bigg\|_{\mathbb{H}}^2 \\
&\leqslant \mathbf{E}\bigg(\int_0^t \big\|\mathcal{R}(t-s)F\big(s,u\big(s-\rho(s)\big)\big)\big\|_{\mathbb{H}}\,\mathrm{d}s\bigg)^2 \\
&\leqslant M^2 \int_0^t \mathrm{e}^{-\kappa(t-s)}\,\mathrm{d}s \int_0^t \mathrm{e}^{-\kappa(t-s)}\mathbf{E}\big\|F\big(s,u\big(s-\rho(s)\big)\big)\big\|_{\mathbb{H}}^2\,\mathrm{d}s \\
&\leqslant M^2 K \int_0^t \mathrm{e}^{-\kappa(t-s)}\,\mathrm{d}s \int_0^t \mathrm{e}^{-\kappa(t-s)}\mathbf{E}\big\|u\big(s-\rho(s)\big)\big\|_{\mathbb{H}}^2\,\mathrm{d}s \\
&\leqslant M^2 K\kappa^{-1}\big(\mu + M_0\mathbf{E}\|\zeta\|_{\mathcal{D}}^2\big)\mathrm{e}^{a\tau}\mathrm{e}^{-at}\int_0^t \mathrm{e}^{(a-\kappa)(t-s)}\,\mathrm{d}s \\
&\leqslant M^2 K\kappa^{-1}(\kappa-a)^{-1}\big(\mu + M_0\mathbf{E}\|\zeta\|_{\mathcal{D}}^2\big)\mathrm{e}^{a\tau}\mathrm{e}^{-at} \leqslant C_3\mathrm{e}^{-at},
\end{aligned} \tag{10}$$

where $C_3 = M^2K\kappa^{-1}(\kappa-a)^{-1}(\mu + M_0\mathbf{E}\|\zeta\|_{\mathcal{D}}^2)\mathrm{e}^{a\tau} < +\infty$.

By using Lemma 2, we get that

$$\begin{aligned}
P_4 &= \mathbf{E}\bigg\|\int_0^t \mathcal{R}(t-s)\varGamma(s)\,\mathrm{d}Z_H(s)\bigg\|_{\mathbb{H}}^2 \\
&\leqslant 2M^2Ht^{2H-1}\int_0^t \mathrm{e}^{-2\kappa(t-s)}\big\|\varGamma(s)\big\|_{\mathcal{L}_2^0}^2\,\mathrm{d}s.
\end{aligned} \tag{11}$$

If $\gamma < \kappa$, then the following estimate holds:

$$\begin{aligned}
P_4 &\leqslant 2M^2Ht^{2H-1}\int_0^t \mathrm{e}^{-2\kappa(t-s)}\mathrm{e}^{-2\gamma(t-s)}\mathrm{e}^{2\gamma(t-s)}\big\|\varGamma(s)\big\|_{\mathcal{L}_2^0}^2\,\mathrm{d}s \\
&\leqslant 2M^2Ht^{2H-1}\mathrm{e}^{-2\gamma t}\int_0^t \mathrm{e}^{-2(\kappa-\gamma)(t-s)}\mathrm{e}^{2\gamma s}\big\|\varGamma(s)\big\|_{\mathcal{L}_2^0}^2\,\mathrm{d}s \\
&\leqslant 2M^2HT^{2H-1}\mathrm{e}^{-2\gamma t}\int_0^T \mathrm{e}^{-2(\kappa-\gamma)(t-s)}\mathrm{e}^{2\gamma s}\big\|\varGamma(s)\big\|_{\mathcal{L}_2^0}^2\,\mathrm{d}s \\
&\leqslant 2M^2HT^{2H-1}\mathrm{e}^{-2\gamma t}\int_0^T \mathrm{e}^{2\gamma s}\big\|\varGamma(s)\big\|_{\mathcal{L}_2^0}^2\,\mathrm{d}s.
\end{aligned} \tag{12}$$

If $\kappa < \gamma$, then the following estimate holds:

$$P_4 \leqslant 2M^2HT^{2H-1}\mathrm{e}^{-2\kappa t}\int_0^T \mathrm{e}^{2\gamma s}\big\|\varGamma(s)\big\|^2_{\mathcal{L}_2^0}\,\mathrm{d}s. \tag{13}$$

Using (11), (12) and (13), we have

$$P_4 \leqslant C_4\mathrm{e}^{-\min(\kappa,\gamma)t}, \tag{14}$$

where $C_4 = 2M^2HT^{2H-1}\int_0^T \mathrm{e}^{2\gamma s}\|\varGamma(s)\|^2_{\mathcal{L}_2^0}\,\mathrm{d}s < +\infty$. On the other hand, by assumptions (H3) and (H4) we get

$$\begin{aligned}
P_5 &= \mathbf{E}\|\int_0^t\int_\Lambda \mathcal{R}(t-s)\sigma\big(s,u\big(s-k(s)\big),\nu\big)\,\widetilde{N}(\mathrm{d}s,\mathrm{d}\nu)\|^2_{\mathbb{H}}\\
&\leqslant M^2\mathbf{E}\int_0^t \mathrm{e}^{-2\kappa(t-s)}\int_\Lambda \big\|\sigma\big(s,u\big(s-k(s)\big),\nu\big)\big\|^2_{\mathbb{H}}\,\lambda(\mathrm{d}\nu)\,\mathrm{d}s\\
&\leqslant M^2K\int_0^t \mathrm{e}^{-2\kappa(t-s)}\mathbf{E}\big\|u\big(s-k(s)\big)\big\|^2_{\mathbb{H}}\,\mathrm{d}s\\
&\leqslant M^2K\int_0^t \mathrm{e}^{-2\kappa(t-s)}\big(\mu+M_0\mathbf{E}\|\zeta\|^2_{\mathcal{D}}\big)\mathrm{e}^{-a(s-\rho(s))}\,\mathrm{d}s\\
&\leqslant M^2K\big(\mu+M_0\mathbf{E}\|\zeta\|^2_{\mathcal{D}}\big)\mathrm{e}^{\tau a}\mathrm{e}^{-at}\int_0^t \mathrm{e}^{-2\kappa(t-s)}\mathrm{e}^{-as}\mathrm{e}^{at}\,\mathrm{d}s\\
&\leqslant M^2K\big(\mu+M_0\mathbf{E}\|\zeta\|^2_{\mathcal{D}}\big)\mathrm{e}^{\tau a}\mathrm{e}^{-at}\int_0^t \mathrm{e}^{-(2\kappa-a)(t-s)}\,\mathrm{d}s\\
&\leqslant M^2K\big(\mu+M_0\mathbf{E}\|\zeta\|^2_{\mathcal{D}}\big)(2\kappa-a)^{-1}\mathrm{e}^{\tau a}\mathrm{e}^{-at} \leqslant C_5\mathrm{e}^{-at},
\end{aligned} \tag{15}$$

where $C_5 = M^2K(\mu+M_0\mathbf{E}\|\zeta\|^2_{\mathcal{D}})(2\kappa-a)^{-1}\mathrm{e}^{\tau a} < +\infty$. Now, by combining (H7) and Cauchy–Schwartz inequality, we obtain the following estimation for the impulsive term:

$$\begin{aligned}
P_6 &\leqslant \mathbf{E}\Big[\sum_{0<t_k<t} M\mathrm{e}^{-\kappa(t-t_k)}q_k\big\|u(t_k)\big\|_{\mathbb{H}}\Big]^2\\
&\leqslant M^2\mathbf{E}\Big[\sum_{0<t_k<t} \mathrm{e}^{-\kappa(t-t_k)}\tilde{q}(t_k-t_{k-1})\big\|u(t_k)\big\|_{\mathbb{H}}\Big]^2\\
&\leqslant M^2\tilde{q}^2\mathbf{E}\bigg(\int_0^t \mathrm{e}^{-\kappa(t-s)}\|u(s)\|_{\mathbb{H}}\,\mathrm{d}s\bigg)^2
\end{aligned}$$

$$\leqslant M^2\tilde{q}^2\mathbf{E}\int_0^t \mathrm{e}^{-\kappa(t-s)}\,\mathrm{d}s\int_0^t \mathrm{e}^{-\kappa(t-s)}\big\|u(s)\big\|_{\mathbb{H}}^2\,\mathrm{d}s$$
$$\leqslant M^2\tilde{q}^2\kappa^{-1}\mu\mathrm{e}^{-at}\int_0^t \mathrm{e}^{-(\kappa-a)(t-s)}\,\mathrm{d}s$$
$$\leqslant M^2\tilde{q}^2\kappa^{-1}\mu(\kappa-a)^{-1}\mathrm{e}^{-at}, \tag{16}$$

where $C_6 = M^2\tilde{q}^2\kappa^{-1}\mu(\kappa-a)^{-1} < +\infty$. Inequality (8), (9), (10), (14), (15) and (16) together imply that $\mathbf{E}\|\Phi(u)(t)\|_{\mathbb{H}}^2 \leqslant \bar{M}\mathrm{e}^{-\bar{a}t}$ for some $\bar{M} > 0$ and $\bar{a} > 0$.

Step 2. Next, we show that $\Phi(u)(t)$ is càdlàg process on $\mathcal{M}_\zeta$. Let $0 < t < T$ and $h > 0$ be sufficiently small. Then, for any fixed $u(t) \in \mathcal{M}_\zeta$, we have

$$\begin{aligned}
&\mathbf{E}\big\|\Phi(x)(t+h) - \Phi(u)(t)\big\|_{\mathbb{H}}^2\\
&\quad\leqslant 6\Bigg[\mathbf{E}\big\|\big(\mathcal{R}(t+h)-\mathcal{R}(t)\big)\big(G(0)+\zeta\big(0,G\big(-r(0)\big)\big)\big)\big\|_{\mathbb{H}}^2\\
&\qquad+\mathbf{E}\big\|G\big(t+h,u\big(t+h-r(t+h)\big)\big)-G\big(t,u\big(t-r(t)\big)\big)\big\|_{\mathbb{H}}^2\\
&\qquad+\mathbf{E}\Bigg\|\int_0^{t+h}\mathcal{R}(t+h-s)F\big(s,u\big(s-\rho(s)\big)\big)\,\mathrm{d}s\\
&\qquad-\int_0^{t}\mathcal{R}(t-s)F\big(s,u\big(s-\rho(s)\big)\big)\,\mathrm{d}s\Bigg\|_{\mathbb{H}}^2\\
&\qquad+\mathbf{E}\Bigg\|\int_0^{t+h}\mathcal{R}(t+h-s)\Gamma(s)\,\mathrm{d}Z_H(s)-\int_0^{t}\mathcal{R}(t-s)\Gamma(s)\,\mathrm{d}Z_H(s)\Bigg\|_{\mathbb{H}}^2\\
&\qquad+\mathbf{E}\Bigg\|\int_0^{t+h}\int_\Lambda\mathcal{R}(t+h-s)\sigma\big(s,u\big(s-k(s)\big),\nu\big)\,\widetilde{N}(\mathrm{d}s,\mathrm{d}\nu)\\
&\qquad-\int_0^{t}\int_\Lambda\mathcal{R}(t-s)\sigma\big(s,u\big(s-k(s)\big),\nu\big)\,\widetilde{N}(\mathrm{d}s,\mathrm{d}\nu)\Bigg\|_{\mathbb{H}}^2\\
&\qquad+\mathbf{E}\Bigg\|\sum_{0<t_k<t+h}\mathcal{R}(t+h-t_k)I_k\big(u(t_k)\big)\\
&\qquad-\sum_{0<t_k<t}\mathcal{R}(t-t_k)I_k\big(u(t_k)\big)\Bigg\|_{\mathbb{H}}^2\Bigg]\\
&\quad:= 6\sum_{k=1}^{6}\mathbf{E}\big\|P_k(t+h)-P_k(t)\big\|_{\mathbb{H}}^2,
\end{aligned}$$

$$\begin{aligned}
&\mathbf{E}\big\|P_3(t+h)-P_3(t)\big\|_{\mathbb{H}}^2\\
&\quad\leqslant \mathbf{E}\Bigg\|\int_0^t\big[\mathcal{R}(t+h-s)-\mathcal{R}(t-s)\big]F\big(s,u\big(s-\rho(s)\big)\big)\,\mathrm{d}s\Bigg\|_{\mathbb{H}}^2\\
&\qquad+\mathbf{E}\Bigg\|\int_t^{t+h}\mathcal{R}(t+h-s)F\big(s,u\big(s-\rho(s)\big)\big)\,\mathrm{d}s\Bigg\|_{\mathbb{H}}^2\\
&\quad\leqslant \int_0^t\big\|\mathcal{R}(s+h)-\mathcal{R}(s)\big\|^2\,\mathrm{d}s\mathbf{E}\int_0^t\big\|F\big(s,u\big(s-\rho(s)\big)\big)\big\|_{\mathbb{H}}^2\,\mathrm{d}s\\
&\qquad+M^2T\mathbf{E}\int_t^{t+h}\mathrm{e}^{-2\kappa(t+h-s)}\big\|F\big(s,u\big(s-\rho(s)\big)\big)\big\|_{\mathbb{H}}^2\,\mathrm{d}s
\end{aligned}\tag{17}$$

and

$$\begin{aligned}
&\mathbf{E}\int_0^t\big\|F\big(s,u\big(s-\rho(s)\big)\big)\big\|_{\mathbb{H}}^2\,\mathrm{d}s\\
&\quad\leqslant K\int_0^t\mathbf{E}\big\|u\big(s-\rho(s)\big)\big\|_{\mathbb{H}}^2\,\mathrm{d}s\leqslant K\int_0^t\big(\mu+M_0\mathbf{E}\|\zeta\|_{\mathcal{D}}^2\big)\mathrm{e}^{-a(s-\rho(s))}\,\mathrm{d}s\\
&\quad\leqslant K(\mu+M_0\mathbf{E}\|\zeta\|_{\mathcal{D}}^2)\mathrm{e}^{\tau t}\int_0^t\mathrm{e}^{-as}\,\mathrm{d}s\leqslant K(\mu+M_0\mathbf{E}\|\zeta\|_{\mathcal{D}}^2)\mathrm{e}^{\tau t}\mathrm{e}^{-at}\int_0^t\mathrm{e}^{a(t-s)}\,\mathrm{d}s\\
&\quad\leqslant K(\mu+M_0\mathbf{E}\|\zeta\|_{\mathcal{D}}^2)\mathrm{e}^{\tau t}a^{-1}\mathrm{e}^{-at}.
\end{aligned}\tag{18}$$

From inequality (18) it follows that there exist a constant $K^*>0$ such that

$$\mathbf{E}\int_0^t\mathrm{e}^{-2\kappa(t-s)}\big\|F\big(s,u\big(s-\rho(s)\big)\big)\big\|_{\mathbb{H}}^2\,\mathrm{d}s\leqslant K^*.\tag{19}$$

By using the norm continuity of the resolvent operator, inequalities (17), (19) and Lebesgue's dominated convergence theorem, it follows that $\mathbf{E}\|P_3(t+h)-P_3(t)\|_{\mathbb{H}}^2\to 0$ as $h\to 0$.

Similarly, we can verify that $\mathbf{E}\|P_k(t+h)-P_k(t)\|_{\mathbb{H}}^2\to 0$ as $h\to 0$, $k=1,2,4,5,6$. The above arguments show that $t\to\Phi(u)(t)$ is càdlàg process. Then we conclude that $\Phi(\mathcal{M}_\zeta)\subset\mathcal{M}_\zeta$.

Step 3. In this part, we show that $\Phi:\mathcal{M}_\zeta\to\mathcal{M}_\zeta$ is a contraction mapping. Now, fix $x,y\in\mathcal{M}_\zeta$, and we have

$$\mathbf{E}\big\|\Phi(x)(t)-\Phi(y)(t)\big\|_{\mathbb{H}}^2\leqslant 4\Big[\mathbf{E}\big\|G\big(t,x\big(t-r(t)\big)\big)-G\big(t,y\big(t-r(t)\big)\big)\big\|_{\mathbb{H}}^2$$

$$
\begin{aligned}
&+ \mathbf{E}\Bigg\| \int_0^t \mathcal{R}(t-s)\big[F\big(s,x\big(s-\rho(s)\big)\big) - F\big(s,y\big(s-\rho(s)\big)\big)\big]\,\mathrm{d}s\Bigg\|_{\mathbb{H}}^2 \\
&+ \mathbf{E}\Bigg\| \int_0^t \int_\Lambda \mathcal{R}(t-s)\big[\sigma\big(s,x\big(s-k(s)\big),\nu\big) - \sigma\big(s,y\big(s-k(s)\big),\nu\big)\big]\,\widetilde{N}(\mathrm{d}s,\mathrm{d}\nu)\Bigg\|_{\mathbb{H}}^2 \\
&+ \mathbf{E}\Bigg\| \sum_{0<t_k<t} \mathcal{R}(t-t_k)\big[I_k\big(x(t_k)\big) - I_k\big(y(t_k)\big)\big]\Bigg\|_{\mathbb{H}}^2\Bigg] \\
&:= 4\sum_{k=1}^4 J_k. \qquad (20)
\end{aligned}
$$

Using assumption (H4), the first term of (20) imply

$$
J_1 \leqslant E\big\|G\big(t,x\big(t-r(t)\big)\big) - \varphi\big(t,y\big(t-r(t)\big)\big)\|_{\mathbb{H}}^2,
$$

$$
\begin{aligned}
u\big(s-k(s)\big) &\leqslant \bar{K}\mathbf{E}\big\|x\big(s-r(s)\big) - y\big(t-\mathcal{R}(t-s)\big)\big\|_{\mathbb{H}}^2 \\
&\leqslant \bar{K}\sup_{t\geqslant 0}\mathbf{E}\big\|u(t)-y(t)\big\|_{\mathbb{H}}^2. \qquad (21)
\end{aligned}
$$

Using Hölder inequality and assumption (H4), we have

$$
\begin{aligned}
J_2 &\leqslant \mathbf{E}\Bigg\| \int_0^t \mathcal{R}(t-s)\big[F\big(s,x\big(s-\rho(s)\big)\big) - F\big(s,y\big(s-\rho(s)\big)\big)\big]\,\mathrm{d}s\Bigg\|_{\mathbb{H}}^2 \\
&\leqslant M^2K\int_0^t \mathrm{e}^{-\kappa(t-s)}\,\mathrm{d}s \int_0^t \mathrm{e}^{-\kappa(t-s)}\mathbf{E}\big\|x\big(s-\rho(s)\big) - y\big(s-\rho(s)\big)\big\|_{\mathbb{H}}^2\,\mathrm{d}s \\
&\leqslant M^2K\Bigg(\int_0^t \mathrm{e}^{-\kappa(t-s)}\,\mathrm{d}s\Bigg)^2 \sup_{t\geqslant 0}\mathbf{E}\big\|x(t)-y(t)\big\|_{\mathbb{H}}^2 \\
&\leqslant M^2K\kappa^{-2}\sup_{t\geqslant 0}\mathbf{E}\big\|x(t)-y(t)\big\|_{\mathbb{H}}^2, \qquad (22)
\end{aligned}
$$

$$
\begin{aligned}
J_3 &\leqslant \mathbf{E}\Bigg\| \int_0^t \int_\Lambda \mathcal{R}(t-s)\big[\sigma\big(s,x\big(s-k(s)\big),\nu\big) - \sigma\big(s,y\big(s-k(s)\big),\nu\big)\big]\,\widetilde{N}(\mathrm{d}s,\mathrm{d}\nu)\Bigg\|_{\mathbb{H}}^2 \\
&\leqslant \mathbf{E}\int_0^t \int_\Lambda \big\|\mathcal{R}(t-s)\big[\sigma\big(s,x\big(s-k(s)\big),\nu\big) - \sigma\big(s,y\big(s-k(s)\big),\nu\big)\big]\|_{\mathbb{H}}^2\,\lambda(\mathrm{d}\nu)\,\mathrm{d}s
\end{aligned}
$$

$$
\begin{aligned}
&\leqslant M^2\mathbf{E}\int_0^t \mathrm{e}^{-2\kappa(t-s)}\int_\Lambda \big\|\sigma\big(s,x\big(s-k(s)\big),\nu\big)-\sigma\big(s,y\big(s-k(s)\big),\nu\big)\big]\big\|_{\mathbb{H}}^2\,\lambda(\mathrm{d}\nu)\,\mathrm{d}s\\
&\leqslant M^2K\int_0^t \mathrm{e}^{-2\kappa(t-s)}\sup_{t\geqslant 0}\mathbf{E}\big\|x(t)-y(t)\big\|_{\mathbb{H}}^2\,\mathrm{d}s\\
&\leqslant M^2K(2\kappa)^{-1}\Big(\sup_{t\geqslant 0}\mathbf{E}\big\|u(t)-y(t)\big\|_{\mathbb{H}}^2\Big).
\end{aligned}
\tag{23}
$$

For the last term, we have

$$
\begin{aligned}
J_4 &\leqslant \mathbf{E}\bigg\|\sum_{0<t_k<t}\mathcal{R}(t-t_k)\big[I_k\big(x(t_k)\big)-I_k\big(y(t_k)\big)\big]\bigg\|_{\mathbb{H}}^2\\
&\leqslant M^2\mathbf{E}\bigg[\sum_{0<t_k<t}\mathrm{e}^{-\kappa(t-t_k)}\tilde{q}(t_k-t_{k-1})\big\|x(t_k)-y(t_k)\big\|_{\mathbb{H}}\big]^2\\
&\leqslant M^2\tilde{q}^2\mathbf{E}\bigg(\int_0^t \mathrm{e}^{-\kappa(t-s)}\big\|x(s)-y(s)\big\|_{\mathbb{H}}\,\mathrm{d}s\bigg)^2\\
&\leqslant M^2\tilde{q}^2\mathbf{E}\int_0^t \mathrm{e}^{-\kappa(t-s)}\,\mathrm{d}s\int_0^t \mathrm{e}^{-\kappa(t-s)}\big\|x(s)-y(s)\big\|_{\mathbb{H}}^2\,\mathrm{d}s\\
&\leqslant M^2\tilde{q}^2\kappa^{-2}(\sup_{t\geqslant 0}\mathbf{E}\big\|x(t)-y(t)\big\|_{\mathbb{H}}^2).
\end{aligned}
\tag{24}
$$

Thus inequalities (21), (22), (23) and (24) together imply

$$
\begin{aligned}
\mathbf{E}\big\|\Phi(x)(t)-\Phi(y)(t)\big\|_{\mathbb{H}}^2 &\leqslant 4\big[\bar{K}+M^2K\kappa^{-2}+M^2K(2\kappa)^{-1}+M^2\tilde{q}^2\kappa^{-2}\big]\\
&\quad\times\Big(\sup_{t\geqslant 0}\mathbf{E}\big\|x(t)-y(t)\big\|_{\mathbb{H}}^2\Big).
\end{aligned}
$$

Therefore, by inequality (iii) of Theorem 1 it follows that Φ is a contractive mapping. Then the fixed point theorem implies that system (1) possesses a unique mild solution and this solution is exponentially stable in mean square. □

4 Application

In this section, an example is presented to illustrate the obtained theory, which can be used, for example, as models for population dynamics like in [1], where the author use model 1 with $F\equiv 0$, $A\equiv 0$, $\Theta\equiv 0$ and $\sigma\equiv 0$. We consider the following impulsive neutral stochastic integrodifferential equations:

$$\begin{aligned}
&\frac{\partial}{\partial t}\big[\beta(t,\xi)+u_1\big(t,\beta\big(t-\rho(t),\xi\big)\big)\big]\\
&\quad=\frac{\partial^2}{\partial\xi^2}\big[\beta(t,\xi)+u_1\big(t,\beta\big(t-\rho(t),\xi\big)\big)\big]\\
&\qquad+\int_0^t \tilde{b}(t-s)\frac{\partial^2}{\partial\xi^2}\big[\beta(t,\xi)+u_1\big(t,\beta\big(t-\rho(t),\xi\big)\big)\big]\,\mathrm{d}s\\
&\qquad+f_1\big(t,\beta\big(t-\tau(t),\xi\big)\big)\,\mathrm{d}t+\mathrm{e}^{-t}\,\mathrm{d}Z_H(t)\\
&\qquad+\int_{\varLambda} h_1\big(t,\beta\big(t-k(t),\xi\big),\nu\big)\widetilde{N}(\mathrm{d}t,\mathrm{d}\nu),\quad t\neq t_k,\ t\geqslant 0,\\
&\Delta\beta(t_k,\xi)=\beta\big(t_k^+,\xi\big)-\beta\big(t_k^-,\xi\big)=I_k\big(\beta(t_k,\xi)\big),\quad t=t_k,\ k=1,2,\dots,\\
&\beta(t,0)+u_1\big(t,\beta(t-\rho(t),0)\big)=0,\quad t\geqslant 0,\\
&\beta(t,\pi)+u_1\big(t,\beta(t-\rho(t),\pi)\big)=0,\quad t\geqslant 0,\\
&\beta(\theta,\xi)=\beta_0(\theta,\xi),\quad \theta\in[-\tau,0],\ \xi\in[0,\pi],
\end{aligned}$$

where $r>0$. Let $\mathbb{H}=L^2(0,\pi)$ with the norm $\|\cdot\|$, $e_n:=\sqrt{2/\pi}\sin(nx)$ $(n=1,2,\dots)$ denote the completed orthonormal basis in $\mathbb{H}$, and Z_H is a Rosenblatt process.

Define $A: D(A)\subset\mathbb{H}\to\mathbb{H}$ by $A=\partial^2/\partial z^2$ with $D(A)=H^2(0,\pi)\cap H_0^1(0,\pi)$. Then

$$A\hat{h}=-\sum_{n=1}^{\infty}n^2<\hat{h},\quad e_n>e_n,\ \hat{h}\in D(A),$$

where $(e_n)_{n\in\mathbb{N}}$ is the orthonormal set of eigenvectors of operator A. It is easy to prove that A is the infinitesimal generator of a strongly continuous semigroup $(S(t))_{t\geqslant 0}$; thus, (H1) is true and $\|S(t)\|\leqslant 1/(\mathrm{e}^t)\leqslant 1$.

We denote by $\varTheta(t):\mathcal{D}(A)\subset\mathbb{H}\to\mathbb{H}$ the operator defined by $\varTheta(t)z=\tilde{b}(t)Az$ for $t\geqslant 0$ and $z\in\mathcal{D}(A)$.

Let $\mathcal{H}=\mathbb{R}$, and let $\varLambda=\{z\in\mathcal{H}\colon 0<|z|_{\mathcal{H}}\leqslant c,\ c>0\}$. We suppose that

(i) For $t\geqslant 0$, $\nu\in\varLambda$, $u_1(t,0)=f_1(t,0)=h(t,0,\nu)=0$.

(ii) There exist a positive constant $\bar{k}$ such that for all $t\geqslant 0$, $\xi_1,\xi_2\in\mathbb{R}$,
$$\big\|u_1(t,\xi_1)-u_1(t,\xi_2)\big\|_{\mathbb{H}}^2\leqslant\bar{k}\|\xi_1-\xi_2\|_{\mathbb{H}}^2.$$

(iii) There exist a positive constant k_1 such that for all $t\geqslant 0$, $\xi_1,\xi_2\in\mathbb{R}$,
$$\int_{\varLambda}\big\|h_1(t,\xi_1)-h_1(t,\xi_2)\big\|_{\mathbb{H}}^2\,\lambda(\mathrm{d}\nu)\vee\big\|f_1(t,\xi_1)-f_1(t,\xi_2)\big\|_{\mathbb{H}}^2\leqslant k_1\|\xi_1-\xi_2\|_{\mathbb{H}}^2.$$

(iv) There exist a positive constant q_k, $k=1,2,\dots$, such that, for $k=1,2,\dots$ and $\xi_1,\xi_2\in\mathbb{H}$,
$$\big\|I_k(\xi_1)-I_k(\xi_2)\big\|_{\mathbb{H}}\leqslant q_k\|\xi_1-\xi_2\|_{\mathbb{H}}.$$

For $t \geqslant 0$, $\xi \in [0,\pi]$ and ζ a $\mathcal{D}$-valued function, define the functions $G : \mathbb{R}^+ \times \mathcal{D} \to \mathbb{H}$, $\psi : \mathbb{R}^+ \times \mathcal{D} \to \mathcal{L}^0(\mathbb{H},\mathbb{H})$ $F : \mathbb{R}^+ \times \mathcal{D} \to \mathbb{H}$ and $\sigma : \mathbb{R}^2 \times \mathcal{H}_c \times \to \mathbb{H}$ for the finite delay as follows:

$$\begin{aligned}
G(t,\varsigma)(\xi) &= u_1\bigl(t, \varsigma\bigl(t-\rho(t),\xi\bigr)\bigr),\\
F(t,\varsigma)(\xi) &= f_1\bigl(t, \varsigma\bigl(t-\tau(t),\xi\bigr)\bigr),\\
\Gamma(t)(\xi) &= \mathrm{e}^{-t},\\
\sigma(t,\varsigma,z)(\xi) &= h_1\bigl(t, \varsigma\bigl(t-k(t,\xi)\bigr),\nu\bigr).
\end{aligned}$$

For $\xi \in [0,\pi]$, we put

$$u(t) = \beta(t,\xi), \quad t \geqslant 0, \qquad \zeta(\theta)(\xi) = \beta_0(\theta,\xi), \quad \theta \in [-\tau,0],$$

then Eq. (1) takes the following form:

$$\begin{aligned}
&d\bigl[u(t) + G(t,x_t)\bigr]\\
&\quad = \Biggl(A\bigl[u(t) + G(t,x_t)\bigr] + \int_0^t \Theta(t-s)\bigl[u(s) + G(s,x_s)\bigr]\,\mathrm{d}s + f(t,x_t)\Biggr)\mathrm{d}t\\
&\qquad + \Gamma(t)\,\mathrm{d}Z_H(t) + \int_\Lambda \phi\bigl(t,u(t),\nu\bigr)\widetilde{N}(\mathrm{d}t,\mathrm{d}\nu), \quad t \geqslant 0,\ t \neq t_k,\\
&\Delta u(t_k) = x(t_k^+) - x(t_k^-) = I_k u(t_k), \quad t = t_k,\ k = 1,2,\dots,\\
&u_0(\cdot) = \zeta(\cdot).
\end{aligned}$$

Moreover, if $\tilde{b}$ is bounded and a C^1 function such that its derivative $\tilde{b}'$ is bounded and uniformly continuous, then (H1) and (H2) are satisfied, and hence, by Theorem 2.1 in [9], Eq. (2) has a resolvent operator $(r(t))_{t\geqslant 0}$ on $\mathbb{H}$.

By assumptions (i)–(iv) we have

$$\bigl\|u(t,\phi_1) - u(t,\phi_2)\bigr\|_{\mathbb{H}}^2 \leqslant \bar{K}\|\phi_1 - \phi_2\|_{\mathbb{H}}^2,$$

$$\int_\Lambda \bigl\|h_1(t,\phi_1) - h_1(t,\phi_2)\bigr\|_{\mathbb{H}}^2 \lambda(\mathrm{d}\nu) \vee \bigl\|f_1(t,\phi_1) - f_1(t,\phi_2)\bigr\|_{\mathbb{H}}^2 \leqslant K\|\phi_1 - \phi_2\|_{\mathbb{H}}^2$$

and

$$\bigl\|I_k(\zeta) - I_k(\eta)\bigr\|_{\mathbb{H}} \leqslant q_k\|\zeta - \eta\|_{\mathbb{H}}$$

for $k = 1,2,\dots$ and $\eta, \in \mathbb{H}$. Then all assumptions (H1)–(H6) are fulfilled, and hence, there exists a mild solution for (1).

We assume moreover that there exists $\beta > a_1 > 1$ and $\tilde{b}(t) < \mathrm{e}^{-\beta t}/a_1$ for all $t \geqslant 0$. Thanks to Lemma 5.2 in [9], we have the following estimates: $\|\mathcal{R}(t)\| \leqslant \mathrm{e}^{-\lambda t}$, where $\lambda = 1 - 1/a$. Consequently, all the hypotheses of Theorem 1 are fulfilled. Therefore, Eq. (1) possesses a unique mild solution, which is exponentially stable, provided that $\bar{K} + M^2K\kappa^{-2} + M^2K(2\kappa)^{-1} + M^{-2}\tilde{q}\kappa^{-2} \leqslant 1/4$, and there exist a constant $\tilde{q} \leqslant q_k(t_k - t_{k-1})$, $k = 1,2,\dots$.

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