Nonlinear Analysis: Modelling and Control, 2018, Vol. 23, No. 3, 380–400 ISSN 1392-5113
https://doi.org/10.15388/NA.2018.3.6

μ-stability criteria for nonlinear differential systems with additive leakage and transmission time-varying delays

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Received: August 21, 2017 / **Revised:** March 16, 2018 / **Published online:** April 20, 2018

Abstract. In this paper, the μ-stability analysis issue of nonlinear differential systems along with two kinds of delay components, namely leakage delay and transmission delay, is investigated. By constructing a suitable Lyapunov–Krasovskii's functional and utilizing Finsler's lemma, some novel μ-stability criteria for the concerned nonlinear system are obtained. These criteria are expressed in the framework of linear matrix inequalities (LMIs), which can be verified easily by means of standard software. Finally, two examples are presented to exhibit the advantage and effectiveness of the proposed theoretical results.

Keywords: μ-stability, leakage delay, linear matrix inequality (LMI).

1 Introduction or the first section

It is obvious that time delay often occur in many industrial and engineering systems, such as chemical engineering systems, long transmission lines in pneumatic systems, population dynamic models, network control systems, etc. It cannot be avoided in modeling such practical systems. A delay may induce instability, oscillation, and thereby the system suffers poor performance [9, 16]. That is why the problem on stability analysis of delayed systems has attracted interests of many researchers for past several decades. In general, the methods for dealing with delay conditions are Lyapunov–Razumikhin method [28,29], Lyapunov–Krasovskii functional [24] and linear matrix inequality (LMI) approach. Until recent times, a large number of publications has come out in dedication to time delay systems, regarding which some important and interesting results can be seen in [21, 22]. However, majority of the published results have focused only on bounded time delays. In contrary, time delays may be unbounded in many practical systems [2, 8, 15], and there by considering this fact, the results accomplished so far in [21, 22] seem to be insufficient and restrictive.

An obvious question is therefore placed as how to analyze the stability of delayed systems in case of unbounded time delays. Recently, a new rescuing concept has been introduced, namely μ-stability, which answers the aforementioned question. In this connection, few papers have been devoted in establishing some novel μ-stability criteria for delayed systems with or without uncertainties [3, 20, 26, 30]. In [20], the authors have analyzed the μ-stability of impulsive neural networks with unbounded time-varying delays and continuously distributed delays by constructing a suitable Lyapunov–Krasovskii functional. In [26], authors discussed the μ-stability of impulsive differential systems with unbounded time-varying delays and nonlinear perturbations and obtained some μ-stability criteria that depend on the range of distributed delay and the decay rate of discrete delay.

Alongside, time delay in leakage term will also highly affect the stability of dynamical networks. This may be due to some theoretical as well as technical difficulties, as can be seen in [7]. Inspite of this fact, we cannot see much existing work on time delays in leakage term or forgetting term, see, for example, [1, 7, 12, 13, 23]. Particularly, in [1], leakage delay in the leakage term has been used to destabilize the neuron states, where an appropriate Lyapunov–Krasovskii functional with a triple integral term has been constructed to improvise the stability criterion of neural network systems. The state estimation problem for neural networks with leakage delay and time-varying delay has been studied in [23], where a new Lyapunov–Krasovskii functional and matrix inequality techniques have been employed. Some authors have done extensive works on stability of neural networks with constant leakage delays, see, for example, [18, 19]. It is clear that all results regarding existence and uniqueness of equilibrium point are independent of both time delays and initial conditions. This shows that leakage term with time delay does not affect the existence and uniqueness of equilibrium point.

Recently, some authors have studied the dynamics of impulsive functional differential equations with leakage time delay and impulsive effects [19] by using some analytical methods. In the existing literature, leakage delay, which endures as a constant in negative feedback terms, has been taken into account, and results have been published. But, still considerations on the stability analysis of dynamical systems with leakage time-varying delay are rare in the literature. More recently, authors in [17] have discussed the stability issues of nonlinear differential systems with leakage time-varying delays.

In [5, 6], authors have reported that in networked control systems, signals transmitted from one device to another via some components of systems would originate a way to many delay components to occur with different physical properties. For example, time delays $d_1(t)$ and $d_2(t)$ in the dynamical model $\dot{x}(t) = Ax(t) + BKx(t - d_1(t) - d_2(t))$ has been induced from sensor to controller and from controller to actuator, respectively. Consequently, stability analysis on such systems has been carried out in [5, 6, 27] by introducing two additive time-varying delay components $d_1(t)$ and $d_2(t)$ with $d_1(t) + d_2(t) = d(t)$. It is worth pointing out that information on the additive delays is not suitably taken into account in the aforementioned works, and the introduction of many slack variables inevitably increases the computational burden, which motivates the study of this paper. Inspired by this idea, in this paper, μ-stability of nonlinear differential systems with two kinds of additive time-varying delays, namely leakage time-varying delay and transmission time-varying delay, is investigated. However, to the optimum of our knowledge, very few works in the existing literature that consider μ-stability problem of nonlinear differential systems with two delay components.

By applying the Finsler's Lemma and constructing appropriate Lyapunov–Krasovskii functional, several delay-dependent μ-stability conditions of the addressed system are derived in the form of linear matrix inequalities (LMIs). These conditions can be easily tested with any of the available numerical packages. As the present conditions involve no free-weighting matrices, the computational burden is largely reduced. The μ-stability criteria subject to the equality constraints are derived based on a Lyapunov–Krasovskii functional, and then less conservative μ-stability criteria are obtained via LMIs by applying the Finsler's lemma.

Notations. Let $\mathbb{R}^n$ denote the n-dimensional real spaces equipped with the Euclidean norm $|\cdot|$, and let $\mathbb{R}^{n\times m}$ denote the $n\times m$-dimensional real spaces. $X>0$ means that the matrix X is a real symmetric positive definite matrix, whereas $X\geqslant 0$ indicates a positive semi-definite matrix. The notations $X^\top$ and X^{-1} denote the transpose and the inverse of X, respectively. Denote the largest eigenvalue and the smallest eigenvalue of matrix X by $\lambda_{\max}(X)$ and $\lambda_{\min}(X)$, respectively. I denotes the identity matrix with proper dimensions. The notion $\mathcal{C}(A,B)$ denotes the space of all continuous functions from A to B. $B^\perp$ denotes a matrix whose columns form the bases of the right null space of B. In addition, the notation $*$ denotes the symmetric terms in a symmetric matrix.

2 Problem description and preliminaries

In this section, we introduce our system and some notations and lemmas to facilitate the presentation of our main results in the following sections.

Consider the following nonlinear differential system with additive leakage and transmission time-varying delay components:

$$\begin{aligned}\dot{x}(t) &= -\mathcal{A}x\bigl(t-\xi_1(t)-\xi_2(t)\bigr)+f\bigl(t,x(t)\bigr)\\&\quad+f\bigl(t,x\bigl(t-\varrho_1(t)-\varrho_2(t)\bigr)\bigr),\quad t>0,\\x(t)&=\varphi(t),\quad t\in(-\infty,0],\end{aligned}\tag{1}$$

where $x(t)\in\mathbb{R}^n$ denotes the state vector, $\mathcal{A}\in\mathbb{R}^{n\times n}$ denotes a constant matrix, and $\varphi\in\mathcal{C}:=\mathcal{C}((-\infty,0],\mathbb{R}^n)$. For any $\varphi\in\mathcal{C}$, define $\|\varphi\|_\infty=\max_{s\in(-\infty,0]}\{|\varphi(s)|,|\dot{\varphi}(s)|\}$.

Till the end of this paper, it is assumed that

($\mathcal{B}_1$) $\xi_1(t),\xi_2(t)$ and $\varrho_1(t),\varrho_2(t)$ are the nonnegative and continuously differentiable additive time-varying leakage delays and additive time-varying transmission delays, respectively, and satisfy $|\dot{\xi}_1(t)|\leqslant\omega_1<1$, $|\dot{\xi}_2(t)|\leqslant\omega_2<1$, $\dot{\varrho}_1(t)\leqslant\eta_1<1$, $\dot{\varrho}_2(t)\leqslant\eta_2<1$, where ω_1,ω_2,η_1, and η_2 are some real constants.

($\mathcal{B}_2$) $f\in\mathcal{C}([0,\infty)\times\mathbb{R}^n,\mathbb{R}^n)$ and satisfies $l_j^-\leqslant(f_j(t,s_1)-f_j(t,s_2))/(s_1-s_2)\leqslant l_j^+$ for any $t\in[0,\infty)$, $s_1\neq s_2$, $j=1,2,\dots,n$, where $f=(f_1,f_2,\dots,f_n)^\top$ and l_i^-, l_i^+ are some real constants, and they may be positive, zero or negative.

For representation convenience, the following notations are introduced:

$$L_1 = \mathrm{diag}\{l_1^- l_1^+, l_2^- l_2^+, \ldots, l_n^- l_n^+\},$$
$$L_2 = \mathrm{diag}\left\{\frac{l_1^- + l_1^+}{2}, \frac{l_2^- + l_2^+}{2}, \ldots, \frac{l_n^- + l_n^+}{2}\right\}.$$

For completeness, we first give the following definition and lemmas.

Definition 1. (See [3].) Assume $\mu(t)$ to be a nonnegative continuous function, which satisfies $\mu(t) \to \infty$ as $t \to \infty$. If there exists a scalar $K > 0$ such that

$$\left|x(t)\right| \leqslant \frac{K}{\mu(t)}, \quad t \geqslant 0,$$

then system (1) is said to be μ-stable.

Clearly, Definition 1 includes the global asymptotical stability and the global exponential stability.

Lemma 1. (*See* [8].) *For any constant matrix $M \in \mathbb{R}^{n\times n}$, $M = M^\top > 0$, a scalar $d > 0$, and a vector function $g(\cdot) : [0, d] \to \mathbb{R}^n$ such that the integrations concerned are well defined,*

$$\left[\int_0^d g(s)\,\mathrm{d}s\right]^\top Q \left[\int_0^d g(s)\,\mathrm{d}s\right] \leqslant d \int_0^d g^\top(s) Q g(s)\,\mathrm{d}s.$$

Lemma 2 [Finsler's lemma]. (*See* [4].) *Let $\Omega \in \mathbb{R}^n$, $\Gamma = \Gamma^\top \in \mathbb{R}^{n\times n}$, and $B \in \mathbb{R}^{m\times n}$ such that* $\mathrm{rank}(B) < n$. *The following statements are equivalent*:

(i) $\Omega^\top \Gamma \Omega < 0$ *for all* $B\Omega = 0$, $\Omega \neq 0$;
(ii) $(B^\perp)^\top \Gamma (B^\perp) < 0$;
(iii) *there exists* $X \in \mathbb{R}^{n\times m}$ *such that* $\Gamma + XB + B^\top X^\top < 0$.

3 Main results

Before deriving our main results, the notations of several vectors are defined for simplicity:

$$\Xi_1 = [0,\ 0,\ 0,\ -\mathcal{A},\ \underbrace{0,\ 0,\ \ldots,\ 0,\ 0,}_{22} -I,\ I,\ I],$$
$$\Xi_2 = [0,\ 0,\ 0,\ -\mathcal{A},\ \underbrace{0,\ 0,\ \ldots,\ 0,\ 0,}_{13} -I,\ I,\ I].$$

3.1 Leakage and transmission delays are bounded

In the following theorem, we consider both leakage and transmission time-varying delays to be bounded ($\xi_1(t) \leqslant \gamma_1$, $\xi_2(t) \leqslant \gamma_2$, $\varrho_1(t) \leqslant \rho_1$, $\varrho_2(t) \leqslant \rho_2$), and the criterion for μ-stability in terms of LMI for system (1) has been derived.

Theorem 1. *Assume that* $(\mathcal{B}_1)$ *and* $(\mathcal{B}_2)$ *hold. Then the equilibrium solution of system* (1) *is* μ*-stable if there exist constants* $\alpha_i \geqslant 0$ $(i = 1, 2, \dots, 31)$, $\widehat{T} > 0$, *positive definite matrices* P, Q_j, R_j $(j = 1, 2, \dots, 6)$, M_l, N_l, S_l, T_l, U_l, W_l $(l = 1, 2, 3)$, *any matrix* H *with proper dimension, and a continuous differential function* $\mu(t) \geqslant 0$, *which is defined on* $[0, \infty)$ *such that, for* $t \geqslant \widehat{T}$,

$$\frac{\dot{\mu}(t)}{\mu(t)} \leqslant \alpha_1, \qquad \frac{\mu(t-\xi_1(t))}{\mu(t)} \geqslant \alpha_2, \qquad \frac{\mu(t-\xi_2(t))}{\mu(t)} \geqslant \alpha_3,$$
$$\frac{\mu(t-\xi_1(t)-\xi_2(t))}{\mu(t)} \geqslant \alpha_4, \qquad \frac{\mu(t-\gamma_1)}{\mu(t)} \geqslant \alpha_5, \qquad \frac{\mu(t-\gamma_2)}{\mu(t)} \geqslant \alpha_6,$$
$$\frac{\mu(t-\gamma_1-\gamma_2)}{\mu(t)} \geqslant \alpha_7, \qquad \frac{\mu(t-\varrho_1(t))}{\mu(t)} \geqslant \alpha_8, \qquad \frac{\mu(t-\varrho_2(t))}{\mu(t)} \geqslant \alpha_9,$$
$$\frac{\mu(t-\varrho_1(t)-\varrho_2(t))}{\mu(t)} \geqslant \alpha_{10}, \qquad \frac{\mu(t-\rho_1)}{\mu(t)} \geqslant \alpha_{11}, \qquad \frac{\mu(t-\rho_2)}{\mu(t)} \geqslant \alpha_{12},$$
$$\frac{\mu(t-\rho_1-\rho_2)}{\mu(t)} \geqslant \alpha_{13}, \qquad \frac{\int_{-\gamma_1}^0 \mu(t-s)\,\mathrm{d}s}{\mu(t)} \leqslant \alpha_{14}, \qquad \frac{\int_{-\gamma_2}^0 \mu(t-s)\,\mathrm{d}s}{\mu(t)} \leqslant \alpha_{15},$$
$$\frac{\int_{-\gamma_1-\gamma_2}^0 \mu(t-s)\,\mathrm{d}s}{\mu(t)} \leqslant \alpha_{16}, \qquad \frac{\int_{-\rho_1}^0 \mu(t-s)\,\mathrm{d}s}{\mu(t)} \leqslant \alpha_{17}, \qquad \frac{\int_{-\rho_2}^0 \mu(t-s)\,\mathrm{d}s}{\mu(t)} \leqslant \alpha_{18},$$
$$\frac{\int_{-\rho_1-\rho_2}^0 \mu(t-s)\,\mathrm{d}s}{\mu(t)} \leqslant \alpha_{19}, \qquad \frac{\int_{-\gamma_1}^0 \int_\theta^0 \mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta}{\mu(t)} \leqslant \alpha_{20},$$
$$\frac{\int_{-\gamma_2}^0 \int_\theta^0 \mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta}{\mu(t)} \leqslant \alpha_{21}, \qquad \frac{\int_{-\gamma_1-\gamma_2}^0 \int_\theta^0 \mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta}{\mu(t)} \leqslant \alpha_{22},$$
$$\frac{\int_{-\rho_1}^0 \int_\theta^0 \mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta}{\mu(t)} \leqslant \alpha_{23}, \qquad \frac{\int_{-\rho_2}^0 \int_\theta^0 \mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta}{\mu(t)} \leqslant \alpha_{24},$$
$$\frac{\int_{-\rho_1-\rho_2}^0 \int_\theta^0 \mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta}{\mu(t)} \leqslant \alpha_{25}, \qquad \frac{\int_{-\gamma_1}^0 \int_\lambda^0 \int_\theta^0 \mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\lambda}{\mu(t)} \leqslant \alpha_{26},$$
$$\frac{\int_{-\gamma_2}^0 \int_\lambda^0 \int_\theta^0 \mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\lambda}{\mu(t)} \leqslant \alpha_{27}, \qquad \frac{\int_{-\gamma_1-\gamma_2}^0 \int_\lambda^0 \int_\theta^0 \mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\lambda}{\mu(t)} \leqslant \alpha_{28},$$
$$\frac{\int_{-\rho_1}^0 \int_\lambda^0 \int_\theta^0 \mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\lambda}{\mu(t)} \leqslant \alpha_{29}, \qquad \frac{\int_{-\rho_2}^0 \int_\lambda^0 \int_\theta^0 \mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\lambda}{\mu(t)} \leqslant \alpha_{30},$$
$$\frac{\int_{-\rho_1-\rho_2}^0 \int_\lambda^0 \int_\theta^0 \mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\lambda}{\mu(t)} \leqslant \alpha_{31},$$

and the following inequality holds:

$$\left(\Xi_1^\perp\right)^\top \Phi\left(\Xi_1^\perp\right) < 0,$$

where $\Xi_1^\perp$ *is the orthogonal complement of the matrix* Ξ_1 *and* Φ *is defined in Appendix A.*

Proof. Consider the Lyapunov–Krasovskii functional $V(t,y)=\sum_{j=1}^{9}V_j(t,y)$, where

$$V_1(t,y)=\mu(t)\left[y(t)-\mathcal{A}\int_{t-\xi_1(t)-\xi_2(t)}^{t}y(s)\,\mathrm{d}s\right]^\top P\left[y(t)-\mathcal{A}\int_{t-\xi_1(t)-\xi_2(t)}^{t}y(s)\,\mathrm{d}s\right],$$

$$\begin{aligned}V_2(t,y)={}&\int_{t-\xi_1(t)}^{t}\mu(s)y^\top(s)Q_1y(s)\,\mathrm{d}s+\int_{t-\xi_2(t)}^{t}\mu(s)y^\top(s)Q_2y(s)\,\mathrm{d}s\\&+\int_{t-\xi_1(t)-\xi_2(t)}^{t}\mu(s)y^\top(s)Q_3y(s)\,\mathrm{d}s+\int_{t-\gamma_1}^{t}\mu(s)y^\top(s)Q_4y(s)\,\mathrm{d}s\\&+\int_{t-\gamma_2}^{t}\mu(s)y^\top(s)Q_5y(s)\,\mathrm{d}s+\int_{t-\gamma_1-\gamma_2}^{t}\mu(s)y^\top(s)Q_6y(s)\,\mathrm{d}s,\end{aligned}$$

$$\begin{aligned}V_3(t,y)={}&\int_{t-\varrho_1(t)}^{t}\mu(s)y^\top(s)R_1y(s)\,\mathrm{d}s+\int_{t-\varrho_2(t)}^{t}\mu(s)y^\top(s)R_2y(s)\,\mathrm{d}s\\&+\int_{t-\varrho_1(t)-\varrho_2(t)}^{t}\mu(s)y^\top(s)R_3y(s)\,\mathrm{d}s+\int_{t-\rho_1}^{t}\mu(s)y^\top(s)R_4y(s)\,\mathrm{d}s\\&+\int_{t-\rho_2}^{t}\mu(s)y^\top(s)R_5y(s)\,\mathrm{d}s+\int_{t-\rho_1-\rho_2}^{t}\mu(s)y^\top(s)R_6y(s)\,\mathrm{d}s,\end{aligned}$$

$$\begin{aligned}V_4(t,y)={}&\gamma_1\int_{-\gamma_1}^{0}\int_{t+s}^{t}\mu(u-s)y^\top(u)S_1y(u)\,\mathrm{d}u\,\mathrm{d}s\\&\gamma_2\int_{-\gamma_2}^{0}\int_{t+s}^{t}\mu(u-s)y^\top(u)S_2y(u)\,\mathrm{d}u\,\mathrm{d}s\\&+(\gamma_1+\gamma_2)\int_{-\gamma_1-\gamma_2}^{0}\int_{t+s}^{t}\mu(u-s)y^\top(u)S_3y(u)\,\mathrm{d}u\,\mathrm{d}s,\end{aligned}$$

$$\begin{aligned}V_5(t,y)={}&\rho_1\int_{-\rho_1}^{0}\int_{t+s}^{t}\mu(u-s)y^\top(u)T_1y(u)\,\mathrm{d}u\,\mathrm{d}s\\&+(\rho_1+\rho_2)\int_{-\rho_1-\rho_2}^{0}\int_{t+s}^{t}\mu(u-s)y^\top(u)T_3y(u)\,\mathrm{d}u\,\mathrm{d}s,\end{aligned}$$

$$V_6(t,y) = \frac{\gamma_1^2}{2}\int_{-\gamma_1}^{0}\int_{\theta}^{0}\int_{t+s}^{t}\mu(u-s)\dot{y}^\top(u)U_1\dot{y}(u)\,\mathrm{d}u\,\mathrm{d}s\,\mathrm{d}\theta$$
$$+\frac{\gamma_2^2}{2}\int_{-\gamma_2}^{0}\int_{\theta}^{0}\int_{t+s}^{t}\mu(u-s)\dot{y}^\top(u)U_2\dot{y}(u)\,\mathrm{d}u\,\mathrm{d}s\,\mathrm{d}\theta$$
$$+\frac{(\gamma_1+\gamma_2)^2}{2}\int_{-\gamma_1-\gamma_2}^{0}\int_{\theta}^{0}\int_{t+s}^{t}\mu(u-s)\dot{y}^\top(u)U_3\dot{y}(u)\,\mathrm{d}u\,\mathrm{d}s\,\mathrm{d}\theta,$$

$$V_7(t,y) = \frac{\rho_1^2}{2}\int_{-\rho_1}^{0}\int_{\theta}^{0}\int_{t+s}^{t}\mu(u-s)\dot{y}^\top(u)W_1\dot{y}(u)\,\mathrm{d}u\,\mathrm{d}s\,\mathrm{d}\theta$$
$$+\frac{\rho_2^2}{2}\int_{-\rho_2}^{0}\int_{\theta}^{0}\int_{t+s}^{t}\mu(u-s)\dot{y}^\top(u)W_2\dot{y}(u)\,\mathrm{d}u\,\mathrm{d}s\,\mathrm{d}s\theta$$
$$+\frac{(\rho_1+\rho_2)^2}{2}\int_{-\rho_1-\rho_2}^{0}\int_{\theta}^{0}\int_{t+s}^{t}\mu(u-s)\dot{y}^\top(u)W_3\dot{y}(u)\,\mathrm{d}u\,\mathrm{d}s\,\mathrm{d}\theta,$$

$$V_8(t,y) = \frac{\gamma_1^3}{6}\int_{-\gamma_1}^{0}\int_{\lambda}^{0}\int_{\theta}^{0}\int_{t+s}^{t}\mu(u-s)\dot{y}^\top(u)M_1\dot{y}(u)\,\mathrm{d}u\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\lambda$$
$$+\frac{\gamma_2^3}{6}\int_{-\gamma_2}^{0}\int_{\lambda}^{0}\int_{\theta}^{0}\int_{t+s}^{t}\mu(u-s)\dot{y}^\top(u)M_2\dot{y}(u)\,\mathrm{d}u\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\lambda$$
$$+\frac{(\gamma_1+\gamma_2)^3}{6}\int_{-\gamma_1-\gamma_2}^{0}\int_{\lambda}^{0}\int_{\theta}^{0}\int_{t+s}^{t}\mu(u-s)\dot{y}^\top(u)M_3\dot{y}(u)\,\mathrm{d}u\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\lambda,$$

$$V_9(t,y) = \frac{\rho_1^3}{6}\int_{-\rho_1}^{0}\int_{\lambda}^{0}\int_{\theta}^{0}\int_{t+s}^{t}\mu(u-s)\dot{y}^\top(u)N_1\dot{y}(u)\,\mathrm{d}u\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\lambda$$
$$+\frac{\rho_2^3}{6}\int_{-\rho_2}^{0}\int_{\lambda}^{0}\int_{\theta}^{0}\int_{t+s}^{t}\mu(u-s)\dot{y}^\top(u)N_2\dot{y}(u)\,\mathrm{d}u\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\lambda$$
$$+\frac{(\rho_1+\rho_2)^3}{6}\int_{-\rho_1-\rho_2}^{0}\int_{\lambda}^{0}\int_{\theta}^{0}\int_{t+s}^{t}\mu(u-s)\dot{y}^\top(u)N_3\dot{y}(u)\,\mathrm{d}u\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\lambda.$$

The derivatives of $V_j(t,y)$ ($j=1,2,\dots,9$) with respect to time along trajectory (1) are obtained as follows:

$$\begin{aligned}
\dot{V}_1(t,y) \leqslant{}& \dot{\mu}(t)\Biggl[y^\top(t)Py(t) - y^\top(t)P\mathcal{A}\Biggl(\int\limits_{t-\xi_1(t)-\xi_2(t)}^{t} y(s)\,\mathrm{d}s\Biggr) \\
&- \Biggl(\int\limits_{t-\xi_1(t)-\xi_2(t)}^{t} y^\top(s)\,\mathrm{d}s\Biggr)\mathcal{A}^\top Py(t) + \Biggl(\int\limits_{t-\xi_1(t)-\xi_2(t)}^{t} y^\top(s)\,\mathrm{d}s\Biggr)\mathcal{A}^\top P\mathcal{A} \\
&\times \Biggl(\int\limits_{t-\xi_1(t)-\xi_2(t)}^{t} y(s)\,\mathrm{d}s\Biggr)\Biggr] + 2\mu(t)\Biggl[y^\top(t)P\dot{y}(t) - y^\top(t)P\mathcal{A}y(t) \\
&+ y^\top(t)P\mathcal{A}(1-\omega_1-\omega_2)y\bigl(t-\xi_1(t)-\xi_2(t)\bigr) - \Biggl(\int\limits_{t-\xi_1(t)-\xi_2(t)}^{t} y^\top(s)\,\mathrm{d}s\Biggr) \\
&\times \mathcal{A}^\top P\dot{y}(t) + \Biggl(\int\limits_{t-\xi_1(t)-\xi_2(t)}^{t} y^\top(s)\,\mathrm{d}s\Biggr)\mathcal{A}^\top P\mathcal{A}y(t) \\
&- \Biggl(\int\limits_{t-\xi_1(t)-\xi_2(t)}^{t} y^\top(s)\,\mathrm{d}s\Biggr)\mathcal{A}^\top P\mathcal{A}(1-\omega_1-\omega_2)y\bigl(t-\xi_1(t)-\xi_2(t)\bigr)\Biggr],
\end{aligned}$$

$$\begin{aligned}
\dot{V}_2(t,y) \leqslant{}& \mu(t)y^\top(t)[Q_1+Q_2+Q_3+Q_4+Q_5+Q_6]y(t) \\
&- \mu\bigl(t-\xi_1(t)\bigr)\bigl[y^\top\bigl(t-\xi_1(t)\bigr)Q_1y\bigl(t-\xi_1(t)\bigr)\bigr](1-\omega_1) \\
&- \mu\bigl(t-\xi_2(t)\bigr)\bigl[y^\top\bigl(t-\xi_2(t)\bigr)Q_2y\bigl(t-\xi_2(t)\bigr)\bigr](1-\omega_2) \\
&- \mu\bigl(t-\xi_1(t)-\xi_2(t)\bigr)\bigl[y^\top\bigl(t-\xi_1(t)-\xi_2(t)\bigr)Q_3y\bigl(t-\xi_1(t)-\xi_2(t)\bigr)\bigr] \\
&\times (1-\omega_1-\omega_2) - \mu(t-\gamma_1)\bigl[y^\top(t-\gamma_1)Q_4y(t-\gamma_1)\bigr] \\
&- \mu(t-\gamma_1-\gamma_2)\bigl[y^\top(t-\gamma_1-\gamma_2)Q_6y(t-\gamma_1-\gamma_2)\bigr] \\
&- \mu(t-\gamma_2)\bigl[y^\top(t-\gamma_2)Q_5y(t-\gamma_2)\bigr],
\end{aligned}$$

$$\begin{aligned}
\dot{V}_3(t,y) \leqslant{}& \mu(t)y^\top(t)[R_1+R_2+R_3+R_4+R_5+R_6]y(t) \\
&- \mu\bigl(t-\varrho_1(t)\bigr)\bigl[y^\top\bigl(t-\varrho_1(t)\bigr)R_1y\bigl(t-\varrho_1(t)\bigr)\bigr](1-\eta_1) \\
&- \mu\bigl(t-\varrho_2(t)\bigr)\bigl[y^\top\bigl(t-\varrho_2(t)\bigr)R_2y\bigl(t-\varrho_2(t)\bigr)\bigr](1-\eta_2) \\
&- \mu\bigl(t-\varrho_1(t)-\varrho_2(t)\bigr)\bigl[y^\top\bigl(t-\varrho_1(t)-\varrho_2(t)\bigr)R_3y\bigl(t-\varrho_1(t)-\varrho_2(t)\bigr)\bigr] \\
&\times (1-\eta_1-\eta_2) - \mu(t-\rho_1)\bigl[y^\top(t-\rho_1)Q_4y(t-\rho_1)\bigr] \\
&- \mu(t-\rho_1-\rho_2)\bigl[y^\top(t-\rho_1-\rho_2)Q_6y(t-\rho_1-\rho_2)\bigr] \\
&- \mu(t-\rho_2)\bigl[y^\top(t-\rho_2)Q_5y(t-\rho_2)\bigr],
\end{aligned}$$

$$
\begin{aligned}
\dot{V}_4(t,y) \leqslant{}& \left(\int_{-\gamma_1}^{0} \mu(t-s)\,\mathrm{d}s\right)\gamma_1 y^\top(t) S_1 y(t) - \left(\int_{t-\gamma_1}^{t} y^\top(u)\,\mathrm{d}u\right)\mu(t) S_1 \\
&\times \left(\int_{t-\gamma_1}^{t} y(u)\,\mathrm{d}u\right) + \left(\int_{-\gamma_2}^{0} \mu(t-s)\,\mathrm{d}s\right)\gamma_2 y^\top(t) S_2 y(t) \\
&- \left(\int_{t-\gamma_2}^{t} y^\top(u)\,\mathrm{d}u\right)\mu(t) S_2 \left(\int_{t-\gamma_2}^{t} y(u)\,\mathrm{d}u\right) + \left(\int_{-\gamma_1-\gamma_2}^{0} \mu(t-s)\,\mathrm{d}s\right) \\
&\times (\gamma_1+\gamma_2) y^\top(t) S_3 y(t) - \left(\int_{t-\gamma_1-\gamma_2}^{t} y^\top(u)\,\mathrm{d}u\right)\mu(t) S_3 \left(\int_{t-\gamma_1-\gamma_2}^{t} y(u)\,\mathrm{d}u\right), \\
\dot{V}_5(t,y) \leqslant{}& \left(\int_{-\rho_1}^{0} \mu(t-s)\,\mathrm{d}s\right)\rho_1 y^\top(t) T_1 y(t) - \left(\int_{t-\rho_1}^{t} y^\top(u)\,\mathrm{d}u\right)\mu(t) T_1 \\
&\times \left(\int_{t-\rho_1}^{t} y(u)\,\mathrm{d}u\right) + \left(\int_{-\rho_2}^{0} \mu(t-s)\,\mathrm{d}s\right)\rho_2 y^\top(t) T_2 y(t) \\
&- \left(\int_{t-\rho_2}^{t} y^\top(u)\,\mathrm{d}u\right)\mu(t) T_2 \left(\int_{t-\rho_2}^{t} y(u)\,\mathrm{d}u\right) + \left(\int_{-\rho_1-\rho_2}^{0} \mu(t-s)\,\mathrm{d}s\right) \\
&\times (\rho_1+\rho_2) y^\top(t) T_3 y(t) - \left(\int_{t-\rho_1-\rho_2}^{t} y^\top(u)\,\mathrm{d}u\right)\mu(t) T_3 \left(\int_{t-\rho_1-\rho_2}^{t} y(u)\,\mathrm{d}u\right), \\
\dot{V}_6(t,y) \leqslant{}& \left(\int_{-\gamma_1}^{0}\int_{\theta}^{0} \mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta\right)\frac{\gamma_1^2}{2}\dot{y}^\top(t) U_1 \dot{y}(t) + \left(\int_{-\gamma_2}^{0}\int_{\theta}^{0} \mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta\right) \\
&\times \frac{\gamma_2^2}{2}\dot{y}^\top(t) U_2 \dot{y}(t) + \left(\int_{-\gamma_1-\gamma_2}^{0}\int_{\theta}^{0} \mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta\right)\frac{(\gamma_1+\gamma_2)^2}{2} \\
&\times \dot{y}^\top(t) U_3 \dot{y}(t) - \mu(t)\big[y^\top(t)\big[\gamma_1^2 U_1 + \gamma_2^2 U_2 + (\gamma_1+\gamma_2)^2 U_3\big] y(t) \\
&- y^\top(t)\gamma_1 U_1 \left(\int_{t-\gamma_1}^{t} y(u)\,\mathrm{d}u\right) - \left(\int_{t-\gamma_1}^{t} y^\top(u)\,\mathrm{d}u\right)\gamma_1 U_1 y(t) \\
&+ \left(\int_{t-\gamma_1}^{t} y^\top(u)\,\mathrm{d}u\right) U_1 \left(\int_{t-\gamma_1}^{t} y(u)\,\mathrm{d}u\right) - y^\top(t)\gamma_2 U_2 \left(\int_{t-\gamma_2}^{t} y(u)\,\mathrm{d}u\right)
\end{aligned}
$$

$$
\begin{aligned}
&- \left(\int_{t-\gamma_2}^{t} y^\top(u)\,\mathrm{d}u\right)\gamma_2 U_2 y(t) + \left(\int_{t-\gamma_2}^{t} y^\top(u)\,\mathrm{d}u\right)U_2\left(\int_{t-\gamma_2}^{t} y(u)\,\mathrm{d}u\right) \\
&- y^\top(t)(\gamma_1+\gamma_2)U_3\left(\int_{t-\gamma_1-\gamma_2}^{t} y(u)\,\mathrm{d}u\right) - \left(\int_{t-\gamma_1-\gamma_2}^{t} y^\top(u)\,\mathrm{d}u\right) \\
&\times (\gamma_1+\gamma_2)U_3 y(t) + \left(\int_{t-\gamma_1-\gamma_2}^{t} y^\top(u)\,\mathrm{d}u\right)U_3\left(\int_{t-\gamma_1-\gamma_2}^{t} y(u)\,\mathrm{d}u\right)\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
\dot V_7(t,y) \leqslant{}& \left(\int_{-\rho_1}^{0}\int_{\theta}^{0}\mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta\right)\frac{\rho_1^2}{2}\dot y^\top(t)W_1\dot y(t) + \left(\int_{-\rho_2}^{0}\int_{\theta}^{0}\mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta\right) \\
&\times \frac{\rho_2^2}{2}\dot y^\top(t)W_2\dot y(t) + \left(\int_{-\rho_1-\rho_2}^{0}\int_{\theta}^{0}\mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta\right)\frac{(\rho_1+\rho_2)^2}{2}\dot y^\top(t)W_3\dot y(t) \\
&- \mu(t)\Bigg[y^\top(t)\big[\rho_1^2W_1 + \rho_2^2W_2 + (\rho_1+\rho_2)^2W_3\big]y(t) - y^\top(t)\rho_1W_1 \\
&\times \left(\int_{t-\rho_1}^{t} y(u)\,\mathrm{d}u\right) - \left(\int_{t-\rho_1}^{t} y^\top(u)\,\mathrm{d}u\right)\rho_1W_1y(t) + \left(\int_{t-\rho_1}^{t} y^\top(u)\,\mathrm{d}u\right) \\
&\times W_1\left(\int_{t-\rho_1}^{t} y(u)\,\mathrm{d}u\right) - y^\top(t)\rho_2W_2\left(\int_{t-\rho_2}^{t} y(u)\,\mathrm{d}u\right) - \left(\int_{t-\rho_2}^{t} y^\top(u)\,\mathrm{d}u\right) \\
&\times \rho_2W_2y(t) + \left(\int_{t-\rho_2}^{t} y^\top(u)\,\mathrm{d}u\right)W_2\left(\int_{t-\rho_2}^{t} y(u)\,\mathrm{d}u\right) - y^\top(t)(\rho_1+\rho_2) \\
&\times W_3\left(\int_{t-\rho_1-\rho_2}^{t} y(u)\,\mathrm{d}u\right) - \left(\int_{t-\rho_1-\rho_2}^{t} y^\top(u)\,\mathrm{d}u\right) \\
&\times (\rho_1+\rho_2)W_3y(t) + \left(\int_{t-\rho_1-\rho_2}^{t} y^\top(u)\,\mathrm{d}u\right)W_3\left(\int_{t-\rho_1-\rho_2}^{t} y(u)\,\mathrm{d}u\right)\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
\dot V_8(t,y) \leqslant{}& \left(\int_{-\gamma_1}^{0}\int_{\lambda}^{0}\int_{\theta}^{0}\mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\lambda\right)\frac{\gamma_1^3}{6}\dot y^\top(t)M_1\dot y(t) \\
&+ \left(\int_{-\gamma_2}^{0}\int_{\lambda}^{0}\int_{\theta}^{0}\mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\lambda\right)\frac{\gamma_2^3}{6}\dot y^\top(t)M_2\dot y(t)
\end{aligned}
$$

$$
\begin{aligned}
&+\left(\int_{-\gamma_1-\gamma_2}^{0}\int_{\lambda}^{0}\int_{\theta}^{0}\mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\lambda\right)\frac{(\gamma_1+\gamma_2)^3}{6}\dot{y}^\top(t)M_3\dot{y}(t)\\
&-\mu(t)\Bigg[y^\top(t)\left(\frac{\gamma_1^4}{4}M_1+\frac{\gamma_2^4}{4}M_2+\frac{(\gamma_1+\gamma_2)^4}{4}M_3\right)y(t)\\
&-\left(\int_{-\gamma_1}^{0}\int_{t+\lambda}^{t}y^\top(u)\,\mathrm{d}u\,\mathrm{d}\lambda\right)\frac{\gamma_1^2}{2}M_1y(t)-y^\top(t)\frac{\gamma_1^2}{2}M_1\left(\int_{-\gamma_1}^{0}\int_{t+\lambda}^{t}y(u)\,\mathrm{d}u\,\mathrm{d}\lambda\right)\\
&+\left(\int_{-\gamma_1}^{0}\int_{t+\lambda}^{t}y^\top(u)\,\mathrm{d}u\,\mathrm{d}\lambda\right)M_1\left(\int_{-\gamma_1}^{0}\int_{t+\lambda}^{t}y(u)\,\mathrm{d}u\,\mathrm{d}\lambda\right)\\
&-y^\top(t)\frac{\gamma_2^2}{2}M_2\left(\int_{-\gamma_2}^{0}\int_{t+\lambda}^{t}y(u)\,\mathrm{d}u\,\mathrm{d}\lambda\right)-\left(\int_{-\gamma_2}^{0}\int_{t+\lambda}^{t}y^\top(u)\,\mathrm{d}u\,\mathrm{d}\lambda\right)\frac{\gamma_2^2}{2}M_2y(t)\\
&+\left(\int_{-\gamma_2}^{0}\int_{t+\lambda}^{t}y^\top(u)\,\mathrm{d}u\,\mathrm{d}\lambda\right)M_2\left(\int_{-\gamma_2}^{0}\int_{t+\lambda}^{t}y(u)\,\mathrm{d}u\,\mathrm{d}\lambda\right)\\
&-y^\top(t)\frac{(\gamma_1+\gamma_2)^2}{2}M_3\left(\int_{-\gamma_1-\gamma_2}^{0}\int_{t+\lambda}^{t}y(u)\,\mathrm{d}u\,\mathrm{d}\lambda\right)\\
&+\left(\int_{-\gamma_1-\gamma_2}^{0}\int_{t+\lambda}^{t}y^\top(u)\,\mathrm{d}u\,\mathrm{d}\lambda\right)M_3\left(\int_{-\gamma_1-\gamma_2}^{0}\int_{t+\lambda}^{t}y(u)\,\mathrm{d}u\,\mathrm{d}\lambda\right)\Bigg]\\
&-\left(\int_{-\gamma_1-\gamma_2}^{0}\int_{t+\lambda}^{t}y^\top(u)\,\mathrm{d}u\,\mathrm{d}\lambda\right)\frac{(\gamma_1+\gamma_2)^2}{2}M_3y(t),
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_9(t,y)\leqslant{}&\left(\int_{-\rho_1}^{0}\int_{\lambda}^{0}\int_{\theta}^{0}\mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\lambda\right)\frac{\rho_1^3}{6}\dot{y}^\top(t)N_1\dot{y}(t)\\
&+\left(\int_{-\rho_2}^{0}\int_{\lambda}^{0}\int_{\theta}^{0}\mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\lambda\right)\frac{\rho_2^3}{6}\dot{y}^\top(t)N_2\dot{y}(t)\\
&+\left(\int_{-\rho_1-\rho_2}^{0}\int_{\lambda}^{0}\int_{\theta}^{0}\mu(t-s)\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\lambda\right)\frac{(\rho_1+\rho_2)^3}{6}\dot{y}^\top(t)N_3\dot{y}(t)\\
&-\mu(t)\Bigg[y^\top(t)\left(\frac{\rho_1^4}{4}N_1+\frac{\rho_2^4}{4}N_2+\frac{(\rho_1+\rho_2)^4}{4}N_3\right)y(t)
\end{aligned}
$$

$$
\begin{aligned}
&- \left(\int_{-\rho_1}^{0} \int_{t+\lambda}^{t} y^\top(u)\,\mathrm{d}u\,\mathrm{d}\lambda \right) \frac{\rho_1^2}{2} N_1 y(t) - y^\top(t) \frac{\rho_1^2}{2} N_1 \left(\int_{-\rho_1}^{0} \int_{t+\lambda}^{t} y(u)\,\mathrm{d}u\,\mathrm{d}\lambda \right) \\
&+ \left(\int_{-\rho_1}^{0} \int_{t+\lambda}^{t} y^\top(u)\,\mathrm{d}u\,\mathrm{d}\lambda \right) N_1 \left(\int_{-\rho_1}^{0} \int_{t+\lambda}^{t} y(u)\,\mathrm{d}u\,\mathrm{d}\lambda \right) \\
&- y^\top(t) \frac{\rho_2^2}{2} N_2 \left(\int_{-\rho_2}^{0} \int_{t+\lambda}^{t} y(u)\,\mathrm{d}u\,\mathrm{d}\lambda \right) - \left(\int_{-\rho_2}^{0} \int_{t+\lambda}^{t} y^\top(u)\,\mathrm{d}u\,\mathrm{d}\lambda \right) \frac{\rho_2^2}{2} N_2 y(t) \\
&+ \left(\int_{-\rho_2}^{0} \int_{t+\lambda}^{t} y^\top(u)\,\mathrm{d}u\,\mathrm{d}\lambda \right) N_2 \left(\int_{-\rho_2}^{0} \int_{t+\lambda}^{t} y(u)\,\mathrm{d}u\,\mathrm{d}\lambda \right) \\
&- y^\top(t) \frac{(\rho_1+\rho_2)^2}{2} N_3 \left(\int_{-\rho_1-\rho_2}^{0} \int_{t+\lambda}^{t} y(u)\,\mathrm{d}u\,\mathrm{d}\lambda \right) \\
&- \left(\int_{-\rho_1-\rho_2}^{0} \int_{t+\lambda}^{t} y^\top(u)\,\mathrm{d}u\,\mathrm{d}\lambda \right) \frac{(\rho_1+\rho_2)^2}{2} N_3 y(t) \\
&+ \left(\int_{-\rho_1-\rho_2}^{0} \int_{t+\lambda}^{t} y^\top(u)\,\mathrm{d}u\,\mathrm{d}\lambda \right) N_3 \left(\int_{-\rho_1-\rho_2}^{0} \int_{t+\lambda}^{t} y(u)\,\mathrm{d}u\,\mathrm{d}\lambda \right) \Bigg].
\end{aligned}
$$

Furthermore, for any matrix H with appropriate dimension, we get

$$
\begin{aligned}
&2\mu(t)\dot{y}^\top(t)H\big[-\mathcal{A}y\big(t-\xi_1(t)-\xi_2(t)\big)+\mathcal{F}\big(t,y(t)\big) \\
&\quad + \mathcal{F}\big(t,y\big(t-\varrho_1(t)-\varrho_2(t)\big)\big)-\dot{y}(t)\big]=0.
\end{aligned}
$$

From our hypothesis $(\mathcal{B}_2)$ the following inequalities hold for any diagonal matrices $X_1>0$ and $X_2>0$:

$$
\mu(t)\begin{bmatrix} y^\top(t) \\ \mathcal{F}^\top(t,y(t)) \end{bmatrix} \begin{bmatrix} L_1X_1 & -L_2X_2 \\ * & X_1 \end{bmatrix} \begin{bmatrix} y(t) \\ \mathcal{F}(t,y(t)) \end{bmatrix} \leqslant 0,
$$

$$
\begin{aligned}
&\mu(t)\begin{bmatrix} y^\top(t-\varrho_1(t)-\varrho_2(t)) \\ \mathcal{F}^\top(t,y(t-\varrho_1(t)-\varrho_2(t))) \end{bmatrix} \begin{bmatrix} L_1X_2 & -L_2X_2 \\ * & X_2 \end{bmatrix} \\
&\quad \times \begin{bmatrix} y(t-\varrho_1(t)-\varrho_2(t)) \\ \mathcal{F}(t,y(t-\varrho_1(t)-\varrho_2(t))) \end{bmatrix} \leqslant 0.
\end{aligned}
$$

It then can be deduced that

$$
\dot{V}(t,y) \leqslant \mu(t)\zeta_1^\top(t)\Phi\zeta_1(t) \tag{2}
$$

with

$$\zeta_1^\top(t) = \Big[y^\top(t),\ y^\top\big(t-\xi_1(t)\big),\ y^\top\big(t-\xi_2(t)\big),\ y^\top\big(t-\xi_1(t)-\xi_2(t)\big),\ y^\top(t-\gamma_1),$$
$$y^\top(t-\gamma_2),\ y^\top(t-\gamma_1-\gamma_2,),\ y^\top\big(t-\varrho_1(t)\big),\ y^\top\big(t-\varrho_2(t)\big),$$
$$y^\top\big(t-\varrho_1(t)-\varrho_2(t)\big),\ y^\top(t-\rho_1),\ y^\top(t-\rho_2),\ y^\top(t-\rho_1-\rho_2),$$
$$\int\limits_{t-\xi_1(t)-\xi_2(t)}^{t} y^\top(s)\,\mathrm{d}s,\ \int\limits_{t-\gamma_1}^{t} y^\top(s)\,\mathrm{d}s,\ \int\limits_{t-\gamma_2}^{t} y^\top(s)\,\mathrm{d}s,$$
$$\int\limits_{t-\gamma_1-\gamma_2}^{t} y^\top(s)\,\mathrm{d}s,\ \int\limits_{t-\rho_1}^{t} y^\top(s)\,\mathrm{d}s,\ \int\limits_{t-\rho_2}^{t} y^\top(s)\,\mathrm{d}s,\ \int\limits_{t-\rho_1-\rho_2}^{t} y^\top(s)\,\mathrm{d}s,$$
$$\int\limits_{-\gamma_1}^{0}\int\limits_{t+\lambda}^{t} y^\top(s)\,\mathrm{d}s\,\mathrm{d}\lambda,\ \int\limits_{-\gamma_2}^{0}\int\limits_{t+\lambda}^{t} y^\top(s)\,\mathrm{d}s\,\mathrm{d}\lambda,\ \int\limits_{-\gamma_1-\gamma_2}^{0}\int\limits_{t+\lambda}^{t} y^\top(s)\,\mathrm{d}s\,\mathrm{d}\lambda,$$
$$\int\limits_{-\rho_1}^{0}\int\limits_{t+\lambda}^{t} y^\top(s)\,\mathrm{d}s\,\mathrm{d}\lambda,\ \int\limits_{-\rho_2}^{0}\int\limits_{t+\lambda}^{t} y^\top(s)\,\mathrm{d}s\,\mathrm{d}\lambda,\ \int\limits_{-\rho_1-\rho_2}^{0}\int\limits_{t+\lambda}^{t} y^\top(s)\,\mathrm{d}s\,\mathrm{d}\lambda,$$
$$\dot{y}^\top(t),\ \mathcal{F}^\top\big(t,y(t)\big),\ \mathcal{F}^\top\big(t,y\big(t-\varrho_1(t)-\varrho_2(t)\big)\big)\Big].$$

Finally, by use of Lemma 2, condition (2) with $\Xi_1\zeta_1(t) = 0$ is equivalent to the following condition:

$$\mu(t)\big(\Xi_1^\perp\big)^\top \Phi\big(\Xi_1^\perp\big) < 0.$$

From this relation we obtain that

$$\dot{V}(t,y) \leqslant 0, \quad t \geqslant \widehat{T}. \tag{3}$$

Then it follows from (3) and the generalized Ito's formula that

$$V(t,y) - V(0,y) = \int\limits_0^t \dot{V}(s,y)\,\mathrm{d}s \leqslant 0, \quad t \geqslant \widehat{T}. \tag{4}$$

From (3) and (4) we have

$$V(t,y) \leqslant V(0,y), \quad t \geqslant \widehat{T}.$$

By using Lemma 1, we obtain

$$\mu(t)\left|\mathcal{A}\int\limits_{t-\xi_1(t)-\xi_2(t)}^{t} y(s)\,\mathrm{d}s\right|^2 \leqslant \frac{\lambda_{\max}(\mathcal{A}^2)}{\lambda_{\min}(S_3)}(\gamma_1+\gamma_2)V(0,y), \quad t \geqslant 0.$$

Similarly, by considering the definition of $V_1(t,y)$, we have

$$\mu(t)\left|y(t)-\mathcal{A}\int\limits_{t-\xi_1(t)-\xi_2(t)}^{t} y(s)\,\mathrm{d}s\right|^2 \leqslant \frac{V(0,y)}{\lambda_{\min}(P)}.$$

Therefore, it follows that

$$\mu(t)\left|y(t)\right|^2 \leqslant 2\frac{\lambda_{\max}(\mathcal{A}^2)}{\lambda_{\min}(S_3)}(\gamma_1+\gamma_2)V(0,y)+2\frac{V(0,y)}{\lambda_{\min}(P)}<\infty, \quad t\geqslant 0. \tag{5}$$

From the definition of $V(t,y)$, we have

$$\begin{aligned}
V(0,y) \leqslant \big[&\lambda_{\max}(P)\big[1+(\gamma_1+\gamma_2)\lambda_{\max}(\mathcal{A})\big]^2+\gamma_1\lambda_{\max}(Q_1)+\gamma_2\lambda_{\max}(Q_2)\\
&+(\gamma_1+\gamma_2)\lambda_{\max}(Q_3)+\gamma_1\lambda_{\max}(Q_4)+\gamma_2\lambda_{\max}(Q_5)+(\gamma_1+\gamma_2)\lambda_{\max}(Q_6)\\
&+\lambda_{\max}(R_1)+\lambda_{\max}(R_2)+\lambda_{\max}(R_3)+\lambda_{\max}(R_4)+\lambda_{\max}(R_5)+\lambda_{\max}(R_6)\\
&+\gamma_1^3\lambda_{\max}(S_1)+\gamma_2^3\lambda_{\max}(S_2)+(\gamma_1+\gamma_2)^3\lambda_{\max}(S_3)+\rho_1\lambda_{\max}(T_1)\\
&+\rho_2\lambda_{\max}(T_2)+(\rho_1+\rho_2)\lambda_{\max}(T_3)+\gamma_1^5\lambda_{\max}(U_1)+\gamma_2^5\lambda_{\max}(U_2)\\
&+(\gamma_1+\gamma_2)^5\lambda_{\max}(U_3)+\rho_1^2\lambda_{\max}(W_1)+\rho_2^2\lambda_{\max}(W_2)\\
&+(\rho_1+\rho_2)^2\lambda_{\max}(W_3)+\gamma_1^7\lambda_{\max}(M_1)+\gamma_2^7\lambda_{\max}(M_2)\\
&+(\gamma_1+\gamma_2)^7\lambda_{\max}(M_3)+\rho_1^3\lambda_{\max}(N_1)+\rho_2^3\lambda_{\max}(N_2)\\
&+(\rho_1+\rho_2)^3\lambda_{\max}(N_3)\big]\|\varphi\|_\infty^2=K.
\end{aligned}$$

From (5), we get the following:

$$\left|y(t)\right|^2 \leqslant \frac{1}{\mu(t)}\left[2\frac{\lambda_{\max}(\mathcal{A}^2)}{\lambda_{\min}(S_3)}(\gamma_1+\gamma_2)+\frac{2}{\lambda_{\min}(P)}\right]K<\infty.$$

By Definition 1, it can be concluded that system (1) is μ-stable. □

3.2 Leakage delay is bounded and transmission delay is unbounded

In the following theorem, sufficient conditions for μ-stability of system (1) with bounded leakage time-varying delays and unbounded transmission time-varying delays ($\xi_1(t)\leqslant\gamma_1$, $\xi_2(t)\leqslant\gamma_2$) have been derived in the form of LMIs.

Theorem 2. *Assume that* $(\mathcal{B}_1)$ *and* $(\mathcal{B}_2)$ *hold. Then the equilibrium solution of system* (1) *is* μ*-stable if there exist constants* $\alpha_i\geqslant 0$ ($i=1,2,\dots,10,14,15,16,20,21,22,26,27,28$)*, which are defined as in Theorem* 1*,* $\widehat{T}>0$*, positive definite matrices* P*,* Q_j ($j=1,2,\dots,6$)*,* R_l*,* M_l*,* S_l*,* U_l ($l=1,2,3$)*, any matrix* H *with proper dimension, and a continuous differential function* $\mu(t)\geqslant 0$*, which is defined on* $[0,\infty)$*, such that, for* $t\geqslant\widehat{T}$*, the following inequality holds*:

$$\left(\Xi_2^\perp\right)^\top\Psi\left(\Xi_2^\perp\right)<0,$$

where $\Xi_2^\perp$ *is the orthogonal complement of the matrix* Ξ_2 *and* Ψ *is defined in Appendix* B.

Proof. Choose the Lyapunov–Krasovskii functional candidate as follows:

$$\widetilde{V}(t,y) = V_1(t,y) + V_2(t,y) + \widehat{V}_3(t,y) + V_4(t,y) + V_6(t,y) + V_8(t,y), \tag{6}$$

where

$$\widehat{V}_3(t,y) = \int_{t-\varrho_1(t)}^{t} \mu(s)y^\top(s)R_1 y(s)\,\mathrm{d}s + \int_{t-\varrho_2(t)}^{t} \mu(s)y^\top(s)R_2 y(s)\,\mathrm{d}s + \int_{t-\varrho_1(t)-\varrho_2(t)}^{t} \mu(s)y^\top(s)R_3 y(s)\,\mathrm{d}s,$$

and the remaining terms are defined as in Theorem 1. Then applying the similar discussion as the proof of Theorem 1, we get

$$\dot{\widetilde{V}}(t,y) \leqslant \mu(t)\zeta_2^\top(t)\Psi\zeta_2(t) \tag{7}$$

with

$$\begin{aligned}\zeta_2^\top(t) = \Bigg[& y^\top(t),\ y^\top\big(t-\xi_1(t)\big),\ y^\top\big(t-\xi_2(t)\big),\ y^\top\big(t-\xi_1(t)-\xi_2(t)\big),\\ & y^\top(t-\gamma_1),\ y^\top(t-\gamma_2),\ y^\top(t-\gamma_1-\gamma_2),\ y^\top\big(t-\varrho_1(t)\big),\\ & y^\top\big(t-\varrho_2(t)\big),\ y^\top\big(t-\varrho_1(t)-\varrho_2(t)\big),\\ & \int_{t-\xi_1(t)-\xi_2(t)}^{t} y^\top(s)\,\mathrm{d}s,\ \int_{t-\gamma_1}^{t} y^\top(s)\,\mathrm{d}s,\ \int_{t-\gamma_2}^{t} y^\top(s)\,\mathrm{d}s,\ \int_{t-\gamma_1-\gamma_2}^{t} y^\top(s)\,\mathrm{d}s,\\ & \int_{-\gamma_1}^{0}\int_{t+\lambda}^{t} y^\top(s)\,\mathrm{d}s\,\mathrm{d}\lambda,\ \int_{-\gamma_2}^{0}\int_{t+\lambda}^{t} y^\top(s)\,\mathrm{d}s\,\mathrm{d}\lambda,\ \int_{-\gamma_1-\gamma_2}^{0}\int_{t+\lambda}^{t} y^\top(s)\,\mathrm{d}s\,\mathrm{d}\lambda,\\ & \dot{y}^\top(t),\ \mathcal{F}^\top\big(t,y(t)\big),\ \mathcal{F}^\top\big(t,y\big(t-\varrho_1(t)-\varrho_2(t)\big)\big)\Bigg].\end{aligned}$$

Finally, by use of Lemma 2, condition (7) with $\Xi_2\zeta_2(t) = 0$ is equivalent to the following condition:

$$\mu(t)\big(\Xi_2^\perp\big)^\top\Psi\big(\Xi_2^\perp\big) < 0.$$

From this relation we obtain that

$$\dot{\widetilde{V}}(t,y) \leqslant 0, \quad t \geqslant \widehat{T}. \tag{8}$$

Then it follows from (6) and the generalized Ito's formula that

$$\widetilde{V}(t,y) - \widetilde{V}(0,y) = \int_0^t \dot{\widetilde{V}}(s,y)\,\mathrm{d}s \leqslant 0, \quad t \geqslant \widehat{T}. \tag{9}$$

From (8) and (9) we have

$$\widetilde{V}(t,y) \leqslant \widetilde{V}(0,y), \quad t \geqslant \widehat{T}.$$

Similar as in Theorem 1, by using Lemma 1, we obtain

$$\mu(t)\left|\mathcal{A}\int_{t-\xi_1(t)-\xi_2(t)}^{t} y(s)\,\mathrm{d}s\right|^2 \leqslant \frac{\lambda_{\max}(\mathcal{A}^2)}{\lambda_{\min}(S_3)}(\gamma_1+\gamma_2)\widetilde{V}(0,y), \quad t \geqslant 0.$$

Similarly, by considering the definition of $V_1(t,y)$, we have

$$\mu(t)\left|y(t)-\mathcal{A}\int_{t-\xi_1(t)-\xi_2(t)}^{t} y(s)\,\mathrm{d}s\right|^2 \leqslant \frac{\widetilde{V}(0,y)}{\lambda_{\min}(P)}.$$

Therefore, it follows that

$$\mu(t)\bigl|y(t)\bigr|^2 \leqslant 2\frac{\lambda_{\max}(\mathcal{A}^2)}{\lambda_{\min}(S_3)}(\gamma_1+\gamma_2)\widetilde{V}(0,y) + 2\frac{\widetilde{V}(0,y)}{\lambda_{\min}(P)} < \infty, \quad t \geqslant 0. \tag{10}$$

From the definition of $\widetilde{V}(t,y)$ we have

$$\begin{aligned}
\widetilde{V}(0,y) \leqslant{}& \bigl[\lambda_{\max}(P)\bigl[1+(\gamma_1+\gamma_2)\lambda_{\max}(\mathcal{A})\bigr]^2 + \gamma_1\lambda_{\max}(Q_1) + \gamma_2\lambda_{\max}(Q_2) \\
&+ (\gamma_1+\gamma_2)\lambda_{\max}(Q_3) + \gamma_1\lambda_{\max}(Q_4) + \gamma_2\lambda_{\max}(Q_5) + (\gamma_1+\gamma_2)\lambda_{\max}(Q_6) \\
&+ \lambda_{\max}(R_1) + \lambda_{\max}(R_2) + \lambda_{\max}(R_3) + \gamma_1^3\lambda_{\max}(S_1) + \gamma_2^3\lambda_{\max}(S_2) \\
&+ (\gamma_1+\gamma_2)^3\lambda_{\max}(S_3) + \gamma_1^5\lambda_{\max}(U_1) + \gamma_2^5\lambda_{\max}(U_2) + (\gamma_1+\gamma_2)^5\lambda_{\max}(U_3) \\
&+ \gamma_1^7\lambda_{\max}(M_1) + \gamma_2^7\lambda_{\max}(M_2) + (\gamma_1+\gamma_2)^7\lambda_{\max}(M_3)\bigr]\|\varphi\|_\infty^2 \\
={}& \widetilde{K}.
\end{aligned}$$

From (10) we get the following:

$$\bigl|y(t)\bigr|^2 \leqslant \frac{1}{\mu(t)}\left[2\frac{\lambda_{\max}(\mathcal{A}^2)}{\lambda_{\min}(S_3)}(\gamma_1+\gamma_2) + \frac{2}{\lambda_{\min}(P)}\right]\widetilde{K} < \infty.$$

By Definition 1 we conclude that system (1) is μ-stable. □

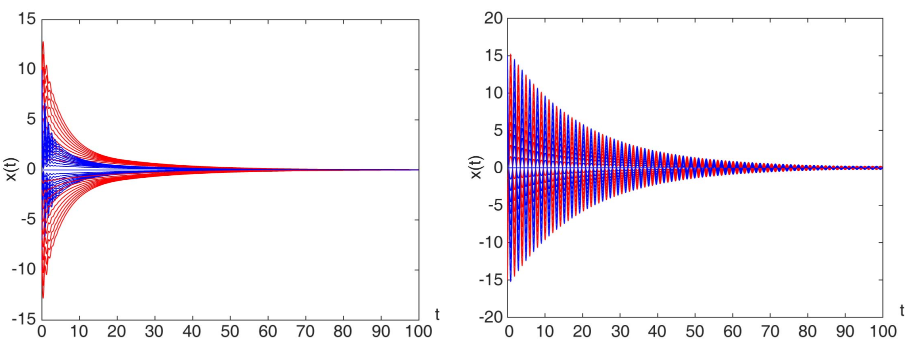
Remark 1. In the field of delay-dependent stability analysis, one of major concerns is to get maximum delay bounds with fewer decision variables [10, 11, 14]. By utilization of Finsler's lemma one can eliminate free variables, which were used in zero equalities in the works [11, 14]. From Lemma 2 one can check that the $(B^\perp)^\top \Gamma(B^\perp) < 0$ is equivalent to the existence of X such that $\Gamma + XB + B^\top X^\top < 0$ holds. Insertion of such an additional matrix X does not play a role to reduce the conservatism of $(B^\perp)^\top \Gamma(B^\perp) < 0$. It only increases the number of decision variables. Therefore, our proposed μ-stability criteria are derived in the form of (ii) in Lemma 2.

Remark 2. So far many researchers have examined the stability of time-varying delay systems via a lot of methods, for example, see [25, 31]. Unluckily, we cannot implement all those methods to the time-varying delay systems together with leakage time-varying delay owing to the occurrence of the term $\dot{\xi}(t)$ in the corresponding systems. In addition, very few researchers have investigated the stability problem of nonlinear differential systems with leakage time-varying delay [12,17]. So far, no results in the literature studies the issue of μ-stability of nonlinear differential systems with additive time-varying leakage delays. To fulfill this idea, in this paper, we have examined the μ-stability of nonlinear differential systems with bounded and unbounded additive time-varying leakage delays.

4 Numerical examples

Example 1. Consider the nonlinear system with additive leakage and transmission time-varying delays (1) with the following parameters: $\mathcal{A} = \left[\begin{smallmatrix} 1 & 2 \\ 2 & 5 \end{smallmatrix}\right]$, $\gamma_1 = 0.1$, $\gamma_2 = 0.3$, $\rho_1 = 0.2$, $\rho_2 = 0.5$, $\omega_1 = 0.2$, $\omega_2 = 0.3$, $\eta_1 = 0.4$, $\eta_2 = 0.5$, $\mathcal{F}_1(s) = \mathcal{F}_2(s) = 0.015 \tanh(s)$, $L_1 = 0$, $L_2 = 0.03I$. Let $\mu(t) = t$ and choose $\alpha_4 = 2.7$, $\alpha_{10} = 3.3$, $\alpha_{20} = 0.5$, $\alpha_{21} = 0.6$, $\alpha_{22} = 0.7$, $\alpha_{23} = 0.9$, $\alpha_{24} = 0.9$, $\alpha_{25} = 1.1$, $\alpha_{26} = 1.2$, $\alpha_{27} = 1.3$, $\alpha_{28} = 1.4$, $\alpha_{29} = 1.5$, $\alpha_{30} = 1.5$, $\alpha_{31} = 1.6$, then the LMI in Theorem 1 have the feasible solution via MATLAB LMI toolbox. By Theorem 1, system (1) with bounded leakage and transmission time-varying delays is μ-stable as shown in Fig. 1(a).

Example 2. Consider the nonlinear system with additive bounded leakage and unbounded transmission time-varying delays (1) with the following parameters: $\mathcal{A} = \left[\begin{smallmatrix} 2 & -1 \\ -2 & 1 \end{smallmatrix}\right]$, $\gamma_1 = 0.2$, $\gamma_2 = 0.4$, $\omega_1 = 0.1$, $\omega_2 = 0.3$, $\eta_1 = 0.3$, $\eta_2 = 0.5$. Choose $\mathcal{F}_1(s) = \mathcal{F}_2(s) = 0.005 \tanh(s)$, then $L_1 = 0$ and $L_2 = 0.01I$. Let $\mu(t) = t$ and take the parameters as $\alpha_4 = 1.3$, $\alpha_{10} = 1.5$, $\alpha_{20} = 0.2$, $\alpha_{21} = 0.4$, $\alpha_{22} = 0.6$, $\alpha_{26} = 1.5$, $\alpha_{27} = 1.8$, $\alpha_{28} = 1.9$, then the LMI in Theorem 2 have the feasible solution via MATLAB LMI toolbox. By Theorem 2, system 1 with bounded leakage and unbounded transmission time-varying delays is μ-stable, as shown in Fig. 1(b).



(a) Example 1 (b) Example 2

Figure 1. State trajectories of system (1).

5 Conclusion

In this paper, we have studied the problem of μ-stability for a class of nonlinear differential systems with two kinds of additive time-varying delay components, namely leakage delay and transmission delay. Based on the Finsler's lemma and Lyapunov stability theory, some new delay-dependent μ-stability criteria are derived in terms of LMIs that can be easily solved by various convex optimization algorithms. Two numerical examples have been given to show the effectiveness and usefulness of the presented criteria.

Acknowledgment. The authors would like to thank the referee for his/her very important comments that improved the results and the quality of the paper.

Appendix A

The elements of $\Phi = (\phi_{i,j})_{29\times 29}$ are defined by

$$\begin{aligned}
\phi_{1,1} &= \alpha_1 P - PA - A^\top P + \sum_{i=1}^{6}(Q_i + R_i) + \alpha_{14}\gamma_1 S_1 + \alpha_{15}\gamma_2 S_2 + \alpha_{16}(\gamma_1+\gamma_2)S_3 \\
&\quad + \alpha_{17}\rho_1 T_1 + \alpha_{18}\rho_2 T_2 + \alpha_{19}(\rho_1+\rho_2)T_3 - \gamma_1^2 U_1 - \gamma_2^2 U_2 - (\gamma_1+\gamma_2)^2 U_3 \\
&\quad - \rho_1^2 W_1 - \rho_2^2 W_2 - (\rho_1+\rho_2)^2 W_3 - \frac{\gamma_1^4}{4}M_1 - \frac{\gamma_2^4}{4}M_2 - \frac{(\gamma_1+\gamma_2)^4}{4}M_3 \\
&\quad - \frac{\rho_1^4}{4}N_1 - \frac{\rho_2^4}{4}N_2 - \frac{(\rho_1+\rho_2)^4}{4}N_3 - L_1 X_1, \\
\phi_{1,4} &= PA(1-\omega_1-\omega_2), \qquad \phi_{1,14} = -\alpha_1 PA - \alpha_1 A^\top P + A^\top PA, \\
\phi_{1,15} &= \gamma_1 U_1, \qquad \phi_{1,16} = \gamma_2 U_2, \qquad \phi_{1,17} = (\gamma_1+\gamma_2)U_3, \qquad \phi_{1,18} = \rho_1 W_1, \\
\phi_{1,19} &= \rho_2 W_2, \qquad \phi_{1,20} = (\rho_1+\rho_2)W_3, \qquad \phi_{1,21} = \frac{\gamma_1^2}{2}M_1, \qquad \phi_{1,22} = \frac{\gamma_2^2}{2}M_2, \\
\phi_{1,23} &= \frac{(\gamma_1+\gamma_2)^2}{2}M_3, \quad \phi_{1,24} = \frac{\rho_1^2}{2}N_1, \quad \phi_{1,25} = \frac{\rho_2^2}{2}N_2, \qquad \phi_{1,26} = \frac{(\rho_1+\rho_2)^2}{2}N_3, \\
\phi_{1,27} &= P, \qquad \phi_{1,28} = -L_2 X_1, \qquad \phi_{2,2} = -\alpha_2 Q_1(1-\omega_1), \qquad \phi_{3,3} = -\alpha_3 Q_2(1-\omega_2), \\
\phi_{4,4} &= -\alpha_4 Q_3(1-\omega_1-\omega_2), \qquad \phi_{4,14} = -A^\top PA(1-\omega_1-\omega_2), \qquad \phi_{4,27} = -HA, \\
\phi_{5,5} &= -\alpha_5 Q_4, \qquad \phi_{6,6} = -\alpha_6 Q_5, \qquad \phi_{7,7} = -\alpha_7 Q_6, \qquad \phi_{8,8} = -\alpha_8 R_1(1-\eta_1), \\
\phi_{9,9} &= -\alpha_9 R_2(1-\eta_2), \qquad \phi_{10,10} = -\alpha_{10} R_3(1-\eta_1-\eta_2) - L_1 X_2, \\
\phi_{10,29} &= -L_2 X_2, \qquad \phi_{11,11} = -\alpha_{11} R_4, \qquad \phi_{12,12} = -\alpha_{12} R_5, \qquad \phi_{13,13} = -\alpha_{13} R_6, \\
\phi_{14,14} &= \alpha_1 A^\top PA - S_3, \qquad \phi_{14,27} = -A^\top P, \qquad \phi_{15,15} = -S_1 - U_1, \\
\phi_{16,16} &= -S_2 - U_2, \qquad \phi_{17,17} = -U_3, \qquad \phi_{18,18} = -T_1 - W_1, \quad \phi_{19,19} = -T_2 - W_2, \\
\phi_{20,20} &= -T_3 - W_3, \qquad \phi_{21,21} = -M_1, \quad \phi_{22,22} = -M_2, \qquad \phi_{23,23} = -M_3, \\
\phi_{24,24} &= -N_1, \qquad \phi_{25,25} = -N_2, \qquad \phi_{26,26} = -N_3, \\
\phi_{27,27} &= \alpha_{20}\frac{\gamma_1^2}{2}U_1 + \alpha_{21}\frac{\gamma_2^2}{2}U_2 + \alpha_{22}\frac{(\gamma_1+\gamma_2)^2}{2}U_3 + \alpha_{23}\frac{\rho_1^2}{2}W_1 + \alpha_{24}\frac{\rho_2^2}{2}W_2
\end{aligned}$$

$$+\,\alpha_{25}\frac{(\rho_1+\rho_2)^2}{2}W_3+\alpha_{26}\frac{\gamma_1^3}{6}M_1+\alpha_{27}\frac{\gamma_2^3}{6}M_2+\alpha_{28}\frac{(\gamma_1+\gamma_2)^3}{6}M_3$$
$$+\,\alpha_{29}\frac{\rho_1^3}{6}N_1+\alpha_{30}\frac{\rho_2^3}{6}N_2+\alpha_{31}\frac{(\rho_1+\rho_2)^3}{6}N_3-H-H^\top,$$
$$\phi_{27,28}=H,\qquad \phi_{27,29}=H,\qquad \phi_{28,28}=X_1,\qquad \phi_{29,29}=X_2.$$

Appendix B

The elements of $\Psi=(\psi_{i,j})_{20\times 20}$ are defined by

$$\psi_{1,1}=\alpha_1P-PA-A^\top P+\sum_{i=1}^{6}Q_i+\sum_{i=1}^{3}R_i+\alpha_{14}\gamma_1S_1+\alpha_{15}\gamma_2S_2+\alpha_{16}(\gamma_1+\gamma_2)S_3$$
$$-\gamma_1^2U_1-\gamma_2^2U_2-(\gamma_1+\gamma_2)^2U_3-\frac{\gamma_1^4}{4}M_1-\frac{\gamma_2^4}{4}M_2-\frac{(\gamma_1+\gamma_2)^4}{4}M_3-L_1X_1,$$
$$\psi_{1,4}=PA(1-\omega_1-\omega_2),\qquad \psi_{1,11}=-\alpha_1PA-\alpha_1A^\top P+A^\top PA,\qquad \psi_{1,12}=\gamma_1U_1,$$
$$\psi_{1,13}=\gamma_2U_2,\qquad \psi_{1,14}=(\gamma_1+\gamma_2)U_3,\qquad \psi_{1,15}=\frac{\gamma_1^2}{2}M_1,\qquad \psi_{1,16}=\frac{\gamma_2^2}{2}M_2,$$
$$\psi_{1,17}=\frac{(\gamma_1+\gamma_2)^2}{2}M_3,\qquad \psi_{1,18}=P,\qquad \psi_{1,19}=-L_2X_1,\qquad \psi_{2,2}=-\alpha_2Q_1(1-\omega_1),$$
$$\psi_{3,3}=-\alpha_3Q_2(1-\omega_2),\qquad \psi_{4,4}=-\alpha_4Q_3(1-\omega_1-\omega_2),$$
$$\psi_{4,11}=-A^\top PA(1-\omega_1-\omega_2),\qquad \psi_{4,18}=-HA,\qquad \psi_{5,5}=-\alpha_5Q_4,$$
$$\psi_{6,6}=-\alpha_6Q_5,\psi_{6,6}=-\alpha_6Q_5,\qquad \psi_{7,7}=-\alpha_7Q_6,\qquad \psi_{8,8}=-\alpha_8R_1(1-\eta_1),$$
$$\psi_{9,9}=-\alpha_9R_2(1-\eta_2),\qquad \psi_{10,10}=-\alpha_{10}R_3(1-\eta_1-\eta_2)-L_1X_2,$$
$$\psi_{10,20}=-L_2X_2,\qquad \psi_{11,11}=\alpha_1A^\top PA-S_3,\qquad \psi_{11,18}=-A^\top P,$$
$$\psi_{12,12}=-S_1-U_1,\qquad \psi_{13,13}=-S_2-U_2,\qquad \psi_{14,14}=-U_3,$$
$$\psi_{15,15}=-M_1,\qquad \psi_{16,16}=-M_2,\qquad \psi_{17,17}=-M_3,$$
$$\psi_{18,18}=\alpha_{20}\frac{\gamma_1^2}{2}U_1+\alpha_{21}\frac{\gamma_2^2}{2}U_2+\alpha_{22}\frac{(\gamma_1+\gamma_2)^2}{2}U_3+\alpha_{26}\frac{\gamma_1^3}{6}M_1+\alpha_{27}\frac{\gamma_2^3}{6}M_2$$
$$+\,\alpha_{28}\frac{(\gamma_1+\gamma_2)^3}{6}M_3-H-H^\top,$$
$$\psi_{18,19}=H,\qquad \psi_{18,20}=H,\qquad \psi_{19,19}=X_1,\qquad \psi_{20,20}=X_2.$$

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