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Non-fragile mixed H_∞ and passivity control for neural networks with successive time-varying delay components*

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Abstract. This paper is concerned with the problem of nonfragile mixed H_∞ and passivity control for neural networks with successive time-varying delay components. We construct a suitable Lyapunov–Krasovskii function with triple and quadruple integral terms then utilizing Jensen's lemma and Wirtinger-type inequality technique. Some sufficient conditions are presented for the existence of nonfragile mixed H_∞ and passivity performance criterions. The expressions for the nonfragile controller can be obtained by solving a set of linear matrix inequality. Finally, two numerical examples are presented to demonstrate the effectiveness of our proposed method.

Keywords: nonfragile, H_∞ and passivity control, successive time-varying delay components, Lyapunov–Krasovskii function.

1 Introduction

Neural networks are generally recognized as one of the simplified models of neural processing in the human brain [5]. Due to its strong capability of information processing, neural networks have been applied in many areas such as signal and image processing, fault diagnosis, pattern recognition, fixed-point computations, associative memories,

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optimization and other scientific areas [7, 39, 50, 52]. In such applications, it is important to know the stability properties of the designed neural network, which makes the analysis of dynamical behavior of neural networks one of the key factors in the design and applications of neural networks. Therefore, many interesting and important results for different types of neural networks have been reported (see, e.g., [1, 13, 22, 24, 37, 42, 51] and the references therein).

In addition, the study on time-delay system has become a topic of great theoretically and practically importance since time-delay is inherent features of more physical process and may lead to instability or significantly affect performances of the corresponding time-delay system. It should be pointed out that the finite neurons communicate with speed and time-delay of the switching amplifiers when the interaction between neurons and induce time-delay neural networks are implemented by very large-scale integrated electronic circuits. Time-delay phenomena are often appeared in most physical systems such as AIDS epidemic, aircraft stabilization, chemical engineering systems, control of epidemics, distributed networks, inferred grinding model, manual control, microwave oscillator, models of lasers, neural network, nuclear reactor, population dynamic model, rolling mill, ship stabilization and systems with lossless transmission lines. Therefore, the study on stability analysis of time-delay system has been widely investigated in [19–21, 38, 56]. Some sufficient conditions were given out to ascertain the exponential stability for delayed complex-valued memristor-based neural networks in [38]. Recently, a new type neural network model with successive time-varying delay components was proposed in [57]. This model has a strong application background in remote control and networked control. For example, in network control systems, signals transmitted from one point to another may experience a few segments of networks, which can possibly induce successive time-delays with different properties due to the variable network transmission conditions [9]. Therefore, the problem of stability criteria for neural networks with successive time-varying delay components have been rarely investigated (see, e.g., [25, 28, 35, 41, 46, 55]).

In addition, the problem of H_∞ control plays a major role in performance constrained control for industrial plants. So it is important to design a valid control law to eliminate the effect of approximation errors and external disturbances to achieve the desired performance. It is the aim of this theory to design the controller such that the closed-loop system is internally stable and its H_∞ norm of the transfer function between the controlled output and the disturbances will not exceed a given H_∞ performance level γ. Hence, there has been increasing interest in the problem of H_∞ control of dynamical systems because of their useful applications in robust control, image processing, especially in classification of patterns, associative memories and other areas [4, 8, 14, 33, 36, 40, 47, 58]. Therefore, in general, it is important both theoretically and practically to studied the stability criteria of the dynamical systems. In [26], the authors presented a multiple delayed state feedback control design for H_∞ problem of a class of neural networks with multiple time-varying delays. The problem of H_∞ design for a class of neural networks with delay-dependent time-varying delays was addressed in [43].

On the other hand, stability issues are often bound up with theory of passive systems in various dynamic problems. Passive systems mainly mean that the energy supplied from external source is more than the one dissipated inside a dynamic system. Based on the concept of energy, the passivity is the property of dynamic systems and describes the energy flowing through the systems. It relates the input and output with the storage function and thus defines a set of useful input-output properties. The main concept of passivity theory is that the passive properties of a system can keep the system internal stability. The passivity theory is originated from circuit analysis [2] and since then has found successful applications in diverse areas such as stability, signal processing, complexity, chaos control and synchronization, fuzzy control, power control, group coordination, flow control and energy management [3, 6, 32, 44, 48, 53]. Therefore, the problem of passivity analysis for neural networks with time-varying delays have received a great deal of attention and a great many of related literatures have been published [17, 30, 31, 45, 49, 54]. The passivity analysis for switched neural networks with parametric uncertainties have been investigated by Lyapunov theorey and some analysis techniques [11, 18].

In recent years, the nonfragile control problem has been an attractive topic in theory analysis and practical implementation. The main topic of the nonfragile control scheme is how to design a feedback control that will be insensitive to some error or gains variation in feedback loop. Therefore, the nonfragile control problem has attracted the interest of many researchers. For example, the problem of nonfragile passivity control for dynamical systems with time-varying delay has been investigated in [10, 12, 16, 23, 27, 29]. The nonfragile passivity and passification problems for a class of nonlinear singular networked control systems with network-induced time-varying delay has been proposed in [16]. In [29], the problem of nonfragile H_∞ control has been discussed for memristor-based neural networks using passivity theory. The problem of nonfragile observer-based passive control for a class of Markovian jumping systems subjected to uncertainties and time-delays are investigated in [10]. In [23], the authors studied the state estimation problem of H_∞ and passive for memristive neural networks with random gain fluctuations. The nonfragile mixed H_∞ and passive asynchronous state estimation problem for uncertain Markov jump neural networks with time-varying delay is presented in [12]. Very recently, the finite-time nonfragile passivity control problem for neural networks with time-varying delay has been studied in [27]. However, to the best of authors knowledge, so far, no results on the nonfragile mixed H_∞ and passivity control for neural networks with successive time-varying delay components. This motivates our present research.

Motivated by the above statement, in this paper, we consider the problem of nonfragile mixed H_∞ and passivity control neural networks with successive time-varying delay components. The main contributions of this paper are summarised as follows:

- The nonfragile mixed H_∞ and passivity control neural networks with successive time-varying delay components are proposed for the first time.
- The required results are derived by using a suitable Lyapunov–Krasovskii function and using linear matrix inequality approach together with Jensen's lemma and Wirtinger-type inequality technique.
- Further, the sufficient conditions for the existence of nonfragile state feedback control gain is obtained by using the mixed H_∞ and passivity analysis.
- The conditions in our main results can be converted into linear matrix inequalities easily, which can be solved by using *Matlab LMI* toolbox.

The contributions of the above techniques are demonstrated through two numerical examples.

Notations. Throughout this paper, the superscripts T and -1 denote transpose of a matrix and matrix inverse, respectively; $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the n-dimensional Euclidean space and set of all $n \times n$ real matrices; for symmetric matrices A and B, the notation $A > B$ (respectively $A \geqslant B$) means that the matrix $A - B$ is positive definite (respectively nonnegative); symmetric terms in a symmetric matrix are denoted by $*$; I is an appropriately dimensioned identity matrix.

2 Problem formulation and preliminaries

Consider the following neural networks with discrete and distributed time-varying delays:

$$\begin{aligned}\dot{z}(t) &= -Az(t) + W_0 f\bigl(z(t)\bigr) + W_1 f\bigl(z\bigl(t-\sigma(t)\bigr)\bigr) + W_2 \int\limits_{t-\rho(t)}^{t} f\bigl(z(s)\bigr)\,\mathrm{d}s \\ &\quad + Bu(t) + B_\omega \omega(t), \\ y(t) &= Cz(t), \quad z(t) = \phi(t), \quad t \in [-\beta, 0],\ \beta = \max[\sigma_{12}, \sigma_{22}, \rho],\end{aligned} \tag{1}$$

where $z(t) = [z_1(t), z_2(t) \dots, z_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$ is the state vector associated with the neurons, $f(z(t)) = [f_1(z(t)), f_2(z(t)), \dots, f_n(z(t))]^{\mathrm{T}} \in \mathbb{R}^n$ is the neuron activation function; $u(t) \in \mathbb{R}^m$ is the control input; $y(t) \in \mathbb{R}^q$ is the system output; $\omega(t) \in \mathbb{R}^p$ is the deterministic disturbance input, which belongs to $\mathcal{L}_2[0, \infty]$; A is a diagonal matrix; W_0 is the connection weight matrix; W_1 is the discrete delayed connection weight matrix; W_2 is the distributed delayed connection weight matrix; B, B_ω and C are known real constant matrices with appropriate dimensions; The time-varying delays $\sigma(t)$ and $\rho(t)$ satisfied the following conditions:

$$\begin{gathered} 0 \leqslant \sigma_{11} \leqslant \sigma_1(t) \leqslant \sigma_{12}, \qquad \dot{\sigma}_1(t) \leqslant \mu_1 < 1, \\ 0 \leqslant \sigma_{21} \leqslant \sigma_2(t) \leqslant \sigma_{22}, \qquad \dot{\sigma}_2(t) \leqslant \mu_2 < 1, \\ 0 \leqslant \rho(t) \leqslant \rho, \qquad \dot{\rho}(t) \leqslant \eta < 1, \end{gathered}$$

where $\sigma_{12} \geqslant \sigma_{11}$, $\sigma_{22} \geqslant \sigma_{21}$ and μ_1, μ_2 are constants. Here, let us denote

$$\sigma(t) = \sigma_1(t) + \sigma_2(t), \qquad \mu = \mu_1 + \mu_2, \qquad h_1 = \sigma_{12} - \sigma_{11}, \qquad h_2 = \sigma_{22} - \sigma_{21}.$$

(A1) For any $j = 1, 2, \dots, n$, there exist constants F_j^- and F_j^+ such that

$$F_j^- \leqslant \frac{f_j(k_1) - f_j(k_2)}{k_1 - k_2} \leqslant F_j^+, \tag{2}$$

where $f_j(0) = 0$, $k_1, k_2 \in \mathbb{R}$, $k_1 \neq k_2$.

For presentation convenience, in the following, we denote $F_1 = \mathrm{diag}\{F_1^- F_1^+, \dots, F_n^- F_n^+\}$, $F_2 = \mathrm{diag}\{(F_1^- + F_1^+)/2, \dots, (F_n^- + F_n^+)/2\}$.

We consider the following nonfragile state feedback controller:

$$u(t) = K(t)z(t),$$

where $K(t) = K + \Delta K(t)$ and K is the controller gain, ΔK is perturbed matrix, which is assumed to be

$$\Delta K(t) = H_a F(t) E_a,$$

where H_a and E_a are known real constant matrices with appropriate dimensions, the time-varying matrix $F(t)$ satisfying $F^{\mathrm{T}}(t)F(t) \leqslant I$.

Definition 1. (See [45].) The neural network (1) is said to be asymptotically stable with a mixed H_∞ and passivity performance γ if, under zero initial condition, there exists a scaler $\gamma > 0$ such that

$$\int_0^{t^*} \left[-\gamma^{-1}\theta y^{\mathrm{T}}(\alpha)y(\alpha) + 2(1-\theta)y^{\mathrm{T}}(\alpha)\omega(\alpha)\right] \mathrm{d}\alpha \geqslant -\gamma \int_0^{t^*} \omega^{\mathrm{T}}(\alpha)\omega(\alpha)\,\mathrm{d}\alpha \qquad (3)$$

for all $t^* > 0$ and any nonzero $\omega(t) \in \mathcal{L}_2[0,\infty]$, where $\theta \in [0,1]$ represents a weighting parameter that defines the trade-off between mixed H_∞ and passivity performance.

Lemma 1. (*See* [15].) *For a positive matrix N and scalars $b > a > 0$ such that the following integrations are well defined, it holds that*

$$-(b-a)\int_{t-b}^{t-a} y^{\mathrm{T}}(s)Ny(s)\,\mathrm{d}s \leqslant -\left(\int_{t-b}^{t-a} y(s)\,\mathrm{d}s\right)^{\mathrm{T}} N \int_{t-b}^{t-a} y(s)\,\mathrm{d}s,$$

$$-\frac{b^2-a^2}{2}\int_{t-b}^{t-a}\int_s^t y^{\mathrm{T}}(u)Ny(u)\,\mathrm{d}u\,\mathrm{d}s \leqslant -\left(\int_{t-b}^{t-a}\int_s^t y(u)\,\mathrm{d}u\,\mathrm{d}s\right)^{\mathrm{T}} \times N \int_{t-b}^{t-a}\int_s^t y(u)\,\mathrm{d}u\,\mathrm{d}s,$$

$$-\frac{b^3-a^3}{6}\int_{t-b}^{t-a}\int_s^t\int_u^t y^{\mathrm{T}}(v)Ny(v)\,\mathrm{d}v\,\mathrm{d}u\,\mathrm{d}s$$

$$\leqslant -\left(\int_{t-b}^{t-a}\int_s^t\int_u^t y(v)\,\mathrm{d}v\,\mathrm{d}u\,\mathrm{d}s\right)^{\mathrm{T}} \times N \int_{t-b}^{t-a}\int_s^t\int_u^t y(v)\,\mathrm{d}v\,\mathrm{d}u\,\mathrm{d}s.$$

Lemma 2. (*See* [34].) *For given symmetric positive definite matrix $N > 0$ and for any differentiable function $\omega(\cdot) \in [a,b] \to \mathbb{R}^n$, the following inequality holds*:

$$\int_a^b \dot{\omega}^{\mathrm{T}}(s)N\dot{\omega}(s)\,\mathrm{d}s \geqslant \frac{1}{b-a}\begin{bmatrix}\omega(b)\\ \omega(a)\\ \upsilon\end{bmatrix}^{\mathrm{T}} R_2(N)\begin{bmatrix}\omega(b)\\ \omega(a)\\ \upsilon\end{bmatrix},$$

where

$$\upsilon = \frac{1}{b-a}\int_a^b \omega(s)\,\mathrm{d}s, \qquad R_2(N) = \begin{bmatrix} N & -N & 0 \\ * & N & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{\pi^2}{4}\begin{bmatrix} N & N & -2N \\ * & N & -2N \\ * & * & 4N \end{bmatrix}.$$

Lemma 3. (*See* [47].) *Let H, E and $F(t)$ be real matrices of appropriate dimensions with $F(t)$ satisfying $F^{\mathrm{T}}(t)F(t) \leqslant I$. Then, for any scalar $\varepsilon > 0$,*

$$HF(t)E + \big(HF(t)E\big)^{\mathrm{T}} \leqslant \varepsilon^{-1}HH^{\mathrm{T}} + \varepsilon E^{\mathrm{T}}E.$$

3 Main results

In this section, we will propose a sufficient condition of the mixed H_∞ and passivity control for neural networks with successive time-varying delay components and nonfragile controller designs.

Theorem 1. *Under assumption* (A1)*, for given scalers σ_{11}, σ_{12}, σ_{21}, σ_{22}, ρ, μ_1, μ_2 and η, the neural network* (1) *is asymptotically stable with a mixed H_∞ and passivity performance γ if there exists positive definite matrices P, Q_i $(i = 1, 2, \dots, 13)$, R_1, R_2, S_1, S_2, T_1, T_2, U_1, U_2, V_1, V_2,*

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ * & P_{22} & P_{23} \\ * & * & P_{33} \end{bmatrix},$$

and positive diagonal matrices H_1, H_2, H_3 such that the following LMIs hold:

$$\bar{\Phi} = \begin{bmatrix} \bar{\Omega} & \sqrt{\theta}\bar{C}_1 & \bar{\Gamma}_1 & \epsilon\bar{\Gamma}_2 \\ * & -\gamma I & 0 & 0 \\ * & * & -\epsilon I & * \\ * & * & * & -\epsilon I \end{bmatrix} < 0, \tag{4}$$

where $\bar{\Omega} = (\bar{\Omega}_{i,j})_{20\times 20}$ with

$$\begin{aligned} \bar{\Omega}_{1,1} &= \bar{Q}_1 + \bar{Q}_4 + \bar{Q}_7 + \bar{Q}_{10} + \bar{Q}_{11} + \bar{Q}_{12} + \bar{Q}_{13} + h_1\bar{R}_1 + h_2\bar{R}_2 - \bar{T}_1 - \bar{T}_1^{\mathrm{T}} - \bar{T}_2 \\ &\quad - \bar{T}_2^{\mathrm{T}} - \frac{3}{2}h_1\bar{U}_1 - \frac{3}{2}h_2\bar{U}_2 - AX^{\mathrm{T}} - XA + BF + F^{\mathrm{T}}B^{\mathrm{T}} - F_1\bar{H}_1 - F_1\bar{H}_3, \end{aligned}$$

$$\bar{\Omega}_{1,2} = F_1\bar{H}_3, \qquad \bar{\Omega}_{1,5} = \bar{P}_{12}, \qquad \bar{\Omega}_{1,6} = -\bar{P}_{12}, \qquad \bar{\Omega}_{1,7} = \bar{P}_{13}, \qquad \bar{\Omega}_{1,8} = -\bar{P}_{13},$$

$$\bar{\Omega}_{1,9} = \bar{P}_{11} - X^{\mathrm{T}} - AX^{\mathrm{T}} + BF, \qquad \bar{\Omega}_{1,10} = \bar{Q}_2 + \bar{Q}_5 + \bar{Q}_8 + W_0X^{\mathrm{T}} + F_2\bar{H}_1 + F_2\bar{H}_3,$$

$$\bar{\Omega}_{1,11} = W_1X^{\mathrm{T}} - F_2\bar{H}_3, \qquad \bar{\Omega}_{1,15} = \frac{2}{h_1}\bar{T}_1, \qquad \bar{\Omega}_{1,16} = \frac{2}{h_2}\bar{T}_2, \qquad \bar{\Omega}_{1,17} = W_2X^{\mathrm{T}},$$

$$\bar{\Omega}_{1,18} = \frac{3}{h_1}\bar{U}_1, \qquad \bar{\Omega}_{1,19} = \frac{3}{h_2}\bar{U}_2, \qquad \bar{\Omega}_{1,20} = B_\omega X^{\mathrm{T}} - (1-\theta)C^{\mathrm{T}},$$

$$\bar{\Omega}_{2,2} = -(1-\mu)\bar{Q}_1 - F_1\bar{H}_2 - F_1\bar{H}_3, \qquad \bar{\Omega}_{2,10} = -\bar{H}_3^{\mathrm{T}}F_2^{\mathrm{T}},$$

$$\bar{\Omega}_{2,11} = -(1-\mu)\bar{Q}_2 + F_2\bar{H}_2 + F_2\bar{H}_3, \qquad \bar{\Omega}_{3,3} = -(1-\mu_1)\bar{Q}_4,$$

$$\bar{\Omega}_{3,12} = -(1-\mu_1)\bar{Q}_5, \qquad \bar{\Omega}_{4,4} = -(1-\mu_2)\bar{Q}_7, \qquad \bar{\Omega}_{4,13} = -(1-\mu_2)\bar{Q}_8,$$
$$\bar{\Omega}_{5,5} = -\bar{Q}_{10} - \frac{1}{h_1}\left(\bar{S}_1 + \frac{\pi^2}{4}\bar{S}_1\right), \qquad \bar{\Omega}_{5,6} = -\frac{1}{h_1}\left(-\bar{S}_1 + \frac{\pi^2}{4}\bar{S}_1\right),$$
$$\bar{\Omega}_{5,15} = \bar{P}_{22}^{\mathrm{T}} + \frac{1}{h_1^2}\frac{\pi^2}{2}\bar{S}_1, \qquad \bar{\Omega}_{6,6} = -\bar{Q}_{11} - \frac{1}{h_1}\left(\bar{S}_1 + \frac{\pi^2}{4}\bar{S}_1\right),$$
$$\bar{\Omega}_{6,15} = -\bar{P}_{22}^{\mathrm{T}} + \frac{1}{h_1^2}\frac{\pi^2}{2}\bar{S}_1, \qquad \bar{\Omega}_{7,7} = -\bar{Q}_{12} - \frac{1}{h_2}\left(\bar{S}_2 + \frac{\pi^2}{4}\bar{S}_2\right),$$
$$\bar{\Omega}_{7,8} = -\frac{1}{h_2}\left(-\bar{S}_2 + \frac{\pi^2}{4}\bar{S}_2\right), \qquad \bar{\Omega}_{7,15} = \bar{P}_{23}^{\mathrm{T}}, \qquad \bar{\Omega}_{7,16} = \bar{P}_{33}^{\mathrm{T}} + \frac{1}{h_2^2}\frac{\pi^2}{2}\bar{S}_2,$$
$$\bar{\Omega}_{8,8} = -\bar{Q}_{13} - \frac{1}{h_2}\left(\bar{S}_2 + \frac{\pi^2}{4}\bar{S}_2\right), \qquad \bar{\Omega}_{8,15} = -\bar{P}_{23}^{\mathrm{T}}, \qquad \bar{\Omega}_{8,16} = -\bar{P}_{33}^{\mathrm{T}} + \frac{1}{h_2^2}\frac{\pi^2}{2}\bar{S}_2,$$
$$\bar{\Omega}_{9,9} = h_1\bar{S}_1 + h_2\bar{S}_2 + \frac{h_1^2}{2}\bar{T}_1 + \frac{h_2^2}{2}\bar{T}_2 + \frac{h_1^3}{6}\bar{U}_1 + \frac{h_2^3}{6}\bar{U}_2 - X^{\mathrm{T}} - X,$$
$$\bar{\Omega}_{9,10} = W_0X^{\mathrm{T}}, \qquad \bar{\Omega}_{9,11} = W_1X^{\mathrm{T}}, \qquad \bar{\Omega}_{9,17} = W_2X^{\mathrm{T}}, \qquad \bar{\Omega}_{9,20} = B_\omega X^{\mathrm{T}},$$
$$\bar{\Omega}_{10,10} = \bar{Q}_3 + \bar{Q}_6 + \bar{Q}_9 + \bar{V}_1 + \rho\bar{V}_2 - \bar{H}_1 - \bar{H}_3, \qquad \bar{\Omega}_{10,11} = \bar{H}_3,$$
$$\bar{\Omega}_{11,11} = -(1-\mu)\bar{Q}_3 - \bar{H}_2 - \bar{H}_3, \qquad \bar{\Omega}_{12,12} = -(1-\mu_1)\bar{Q}_6,$$
$$\bar{\Omega}_{13,13} = -(1-\mu_2)\bar{Q}_9, \bar{\Omega}_{14,14} = -\bar{V}_1, \qquad \bar{\Omega}_{15,15} = -\frac{1}{h_1}\bar{R}_1 - \frac{\pi^2}{h_1^3}\bar{S}_1 - \frac{2}{h_1^2}\bar{T}_1,$$
$$\bar{\Omega}_{16,16} = -\frac{1}{h_2}\bar{R}_2 - \frac{\pi^2}{h_2^3}\bar{S}_2 - \frac{2}{h_2^2}\bar{T}_2, \qquad \bar{\Omega}_{17,17} = -\frac{1}{\rho}\bar{V}_2, \qquad \bar{\Omega}_{18,18} = -\frac{6}{h_1^3}\bar{U}_1,$$
$$\bar{\Omega}_{19,19} = -\frac{6}{h_2^3}\bar{U}_2, \qquad \bar{\Omega}_{20,20} = -\gamma I, \qquad \bar{C}_1 = [C\overbrace{0\cdots0}^{19\ times}],$$
$$\Gamma_1^{\mathrm{T}} = [XB^{\mathrm{T}}\overbrace{0\cdots0}^{7\ times}XB^{\mathrm{T}}\overbrace{0\cdots0}^{11\ times}]^{\mathrm{T}}H_a, \qquad \Gamma_2^{\mathrm{T}} = [E_a\overbrace{0\cdots0}^{19\ times}]^{\mathrm{T}}.$$

Proof. We construct the following Lyapunov–Krasovskii function:

$$V(t) = \sum_{i=1}^{7} V_i(t), \tag{5}$$

where

$$V_1(t) = \eta^{\mathrm{T}}(t)P\eta(t), \quad \eta^{\mathrm{T}}(t) = \left[z^{\mathrm{T}}(t)\int_{t-\sigma_{12}}^{t-\sigma_{11}} z^{\mathrm{T}}(s)\,\mathrm{d}s \int_{t-\sigma_{22}}^{t-\sigma_{21}} z^{\mathrm{T}}(s)\,\mathrm{d}s\right],$$

$$V_2(t) = \int_{t-\sigma(t)}^{t} \begin{bmatrix} z(s) \\ f(z(s)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} \begin{bmatrix} z(s) \\ f(z(s)) \end{bmatrix} \mathrm{d}s$$
$$+ \int_{t-\sigma_1(t)}^{t} \begin{bmatrix} z(s) \\ f(z(s)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q_4 & Q_5 \\ * & Q_6 \end{bmatrix} \begin{bmatrix} z(s) \\ f(z(s)) \end{bmatrix} \mathrm{d}s$$

$$+ \int_{t-\sigma_2(t)}^{t} \begin{bmatrix} z(s) \\ f(z(s)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q_7 & Q_8 \\ * & Q_9 \end{bmatrix} \begin{bmatrix} z(s) \\ f(z(s)) \end{bmatrix} \mathrm{d}s + \int_{t-\sigma_{11}}^{t} z^{\mathrm{T}}(s) Q_{10} z(s)\,\mathrm{d}s$$

$$+ \int_{t-\sigma_{12}}^{t} z^{\mathrm{T}}(s) Q_{11} z(s)\,\mathrm{d}s + \int_{t-\sigma_{21}}^{t} z^{\mathrm{T}}(s) Q_{12} z(s)\,\mathrm{d}s + \int_{t-\sigma_{22}}^{t} z^{\mathrm{T}}(s) Q_{13} z(s)\,\mathrm{d}s,$$

$$V_3(t) = \int_{-\sigma_{12}}^{-\sigma_{11}} \int_{t+\xi}^{t} z^{\mathrm{T}}(s) R_1 z(s)\,\mathrm{d}s\,\mathrm{d}\xi + \int_{-\sigma_{22}}^{-\sigma_{21}} \int_{t+\xi}^{t} z^{\mathrm{T}}(s) R_2 z(s)\,\mathrm{d}s\,\mathrm{d}\xi,$$

$$V_4(t) = \int_{-\sigma_{12}}^{-\sigma_{11}} \int_{t+\xi}^{t} \dot{z}^{\mathrm{T}}(s) S_1 \dot{z}(s)\,\mathrm{d}s\,\mathrm{d}\xi + \int_{-\sigma_{22}}^{-\sigma_{21}} \int_{t+\xi}^{t} \dot{z}^{\mathrm{T}}(s) S_2 \dot{z}(s)\,\mathrm{d}s\,\mathrm{d}\xi,$$

$$V_5(t) = \int_{-\sigma_{12}}^{-\sigma_{11}} \int_{\theta}^{0} \int_{t+\xi}^{t} \dot{z}^{\mathrm{T}}(s) T_1 \dot{z}(s)\,\mathrm{d}s\,\mathrm{d}\xi\,\mathrm{d}\theta + \int_{-\sigma_{22}}^{-\sigma_{21}} \int_{\theta}^{0} \int_{t+\xi}^{t} \dot{z}^{\mathrm{T}}(s) T_2 \dot{z}(s)\,\mathrm{d}s\,\mathrm{d}\xi\,\mathrm{d}\theta,$$

$$V_6(t) = \int_{-\sigma_{12}}^{-\sigma_{11}} \int_{\alpha}^{0} \int_{\theta}^{0} \int_{t+\xi}^{t} \dot{z}^{\mathrm{T}}(s) U_1 \dot{z}(s)\,\mathrm{d}s\,\mathrm{d}\xi\,\mathrm{d}\theta\,\mathrm{d}\alpha$$

$$+ \int_{-\sigma_{22}}^{-\sigma_{21}} \int_{\lambda}^{0} \int_{\theta}^{0} \int_{t+\beta}^{t} \dot{z}^{\mathrm{T}}(s) U_2 \dot{z}(s)\,\mathrm{d}s\,\mathrm{d}\xi\,\mathrm{d}\theta\,\mathrm{d}\alpha,$$

$$V_7(t) = \int_{t-\rho(t)}^{t} f^{\mathrm{T}}\big(z(s)\big) V_1 f\big(z(s)\big)\,\mathrm{d}s + \int_{-\rho(t)}^{0} \int_{t+\xi}^{t} f^{\mathrm{T}}\big(z(s)\big) V_2 f\big(z(s)\big)\,\mathrm{d}s\,\mathrm{d}\xi.$$

Now, taking the time-derivative of $V(t)$ along the solutions of neural network (1), we have that

$$\dot{V}(t) = \sum_{i=1}^{7} \dot{V}_i(t),$$

where

$$\dot{V}_1(t) = 2 \begin{bmatrix} z^{\mathrm{T}}(t) \\ \int_{t-\sigma_{12}}^{t-\sigma_{11}} z^{\mathrm{T}}(s)\,\mathrm{d}s \\ \int_{t-\sigma_{22}}^{t-\sigma_{21}} z^{\mathrm{T}}(s)\,\mathrm{d}s \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ * & P_{22} & P_{23} \\ * & * & P_{33} \end{bmatrix} \begin{bmatrix} \dot{z}(t) \\ z(t-\sigma_{11}) - z(t-\sigma_{12}) \\ z(t-\sigma_{21}) - z(t-\sigma_{22}) \end{bmatrix},$$

$$= 2z^{\mathrm{T}}(t) P_{11} \dot{z}(t) + 2z^{\mathrm{T}}(t) P_{12} z(t-\sigma_{11}) - 2z^{\mathrm{T}}(t) P_{12} z(t-\sigma_{12})$$

$$+ 2z^{\mathrm{T}}(t) P_{13} z(t-\sigma_{21}) - 2z^{\mathrm{T}}(t) P_{13} z(t-\sigma_{22})$$

$$+ 2z^{\mathrm{T}}(t-\sigma_{11}) P_{22}^{\mathrm{T}} \int_{t-\sigma_{12}}^{t-\sigma_{11}} z(s)\,\mathrm{d}s - 2z^{\mathrm{T}}(t-\sigma_{12}) P_{22}^{\mathrm{T}} \int_{t-\sigma_{12}}^{t-\sigma_{11}} z(s)\,\mathrm{d}s$$

$$
\begin{aligned}
&+ 2z^{\mathrm{T}}(t-\sigma_{21})P_{23}^{\mathrm{T}}\int_{t-\sigma_{12}}^{t-\sigma_{11}} z(s)\,\mathrm{d}s - 2z^{\mathrm{T}}(t-\sigma_{22})P_{23}^{\mathrm{T}}\int_{t-\sigma_{12}}^{t-\sigma_{11}} z(s)\,\mathrm{d}s \\
&+ 2z^{\mathrm{T}}(t-\sigma_{21})P_{33}^{\mathrm{T}}\int_{t-\sigma_{22}}^{t-\sigma_{21}} z(s)\,\mathrm{d}s - 2z^{\mathrm{T}}(t-\sigma_{22})P_{33}^{\mathrm{T}}\int_{t-\sigma_{22}}^{t-\sigma_{21}} z(s)\,\mathrm{d}s, \qquad (6)
\end{aligned}
$$

$$
\begin{aligned}
\dot V_2(t) = {}& \begin{bmatrix} z(t) \\ f(z(t)) \end{bmatrix}^{\mathrm{T}} \left\{ \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} + \begin{bmatrix} Q_4 & Q_5 \\ * & Q_6 \end{bmatrix} + \begin{bmatrix} Q_7 & Q_8 \\ * & Q_9 \end{bmatrix} \right\} \begin{bmatrix} z(t) \\ f(z(t)) \end{bmatrix} \\
&- (1-\mu) \begin{bmatrix} z(t-\sigma(t)) \\ f(z(t-\sigma(t))) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} \begin{bmatrix} z(t-\sigma(t)) \\ f(z(t-\sigma(t))) \end{bmatrix} \\
&- (1-\mu_1) \begin{bmatrix} z(t-\sigma_1(t)) \\ f(z(t-\sigma_1(t))) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q_4 & Q_5 \\ * & Q_6 \end{bmatrix} \begin{bmatrix} z(t-\sigma_1(t)) \\ f(z(t-\sigma_1(t))) \end{bmatrix} \\
&- (1-\mu_2) \begin{bmatrix} z(t-\sigma_2(t)) \\ f(z(t-\sigma_2(t))) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q_7 & Q_8 \\ * & Q_9 \end{bmatrix} \begin{bmatrix} z(t-\sigma_2(t)) \\ f(z(t-\sigma_2(t))) \end{bmatrix} \\
&+ z^{\mathrm{T}}(t)[Q_{10}+Q_{11}+Q_{12}+Q_{13}]z(t) - z^{\mathrm{T}}(t-\sigma_{11})Q_{10}z(t-\sigma_{11}) \\
&- z^{\mathrm{T}}(t-\sigma_{12})Q_{11}z(t-\sigma_{12}) - z^{\mathrm{T}}(t-\sigma_{21})Q_{12}z(t-\sigma_{21}) \\
&- z^{\mathrm{T}}(t-\sigma_{22})Q_{13}z(t-\sigma_{22}), \qquad (7)
\end{aligned}
$$

$$
\dot V_3(t) = z^{\mathrm{T}}(t)[h_1R_1 + h_2R_2]z(t) - \int_{t-\sigma_{12}}^{t-\sigma_{11}} z^{\mathrm{T}}(s)R_1z(s)\,\mathrm{d}s - \int_{t-\sigma_{22}}^{t-\sigma_{21}} z^{\mathrm{T}}(s)R_2z(s)\,\mathrm{d}s. \qquad (8)
$$

Now, applying the Lemma 1, we have

$$
-\int_{t-\sigma_{12}}^{t-\sigma_{11}} z^{\mathrm{T}}(s)R_1z(s)\,\mathrm{d}s \leqslant -\frac{1}{h_1}\left(\int_{t-\sigma_{12}}^{t-\sigma_{11}} z(s)\,\mathrm{d}s\right)^{\mathrm{T}} R_1 \int_{t-\sigma_{12}}^{t-\sigma_{11}} z(s)\,\mathrm{d}s, \qquad (9)
$$

$$
-\int_{t-\sigma_{22}}^{t-\sigma_{21}} z^{\mathrm{T}}(s)R_2z(s)\,\mathrm{d}s \leqslant -\frac{1}{h_2}\left(\int_{t-\sigma_{22}}^{t-\sigma_{21}} z(s)\,\mathrm{d}s\right)^{\mathrm{T}} R_2 \int_{t-\sigma_{22}}^{t-\sigma_{21}} z(s)\,\mathrm{d}s, \qquad (10)
$$

$$
\dot V_4(t) = \dot z^{\mathrm{T}}(t)[h_1S_1 + h_2S_2]\dot z(t) - \int_{t-\sigma_{12}}^{t-\sigma_{11}} \dot z^{\mathrm{T}}(s)S_1\dot z(s)\,\mathrm{d}s - \int_{t-\sigma_{22}}^{t-\sigma_{21}} \dot z^{\mathrm{T}}(s)S_2\dot z(s)\,\mathrm{d}s. \qquad (11)
$$

By using Lemma 2 we can get

$$
-\int_{t-\sigma_{12}}^{t-\sigma_{11}} \dot z^{\mathrm{T}}(s)S_1\dot z(s)\,\mathrm{d}s \leqslant -\frac{1}{h_1} \begin{bmatrix} z(t-\sigma_{11}) \\ z(t-\sigma_{12}) \\ \upsilon_1 \end{bmatrix}^{\mathrm{T}} M_2(S_1) \begin{bmatrix} z(t-\sigma_{11}) \\ z(t-\sigma_{12}) \\ \upsilon_1 \end{bmatrix}, \qquad (12)
$$

where $\upsilon_1 = {}^{1/h_1}\int_{t-\sigma_{12}}^{t-\sigma_{11}} z(s)\,\mathrm{d}s$,

$$M_2(S_1) = M_0(S_1) + \frac{\pi^2}{4}\begin{bmatrix} S_1 & S_1 & -2S_1 \\ * & S_1 & -2S_1 \\ * & * & 4S_1 \end{bmatrix}, \qquad M_0(S_1) = \begin{bmatrix} S_1 & -S_1 & 0 \\ * & S_1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\begin{aligned}
&-\int_{t-\sigma_{12}}^{t-\sigma_{11}} \dot{z}^{\mathrm{T}}(s) S_1 \dot{z}(s)\,\mathrm{d}s \\
&\quad\leqslant -\frac{1}{h_1}\Bigg[z^{\mathrm{T}}(t-\sigma_{11})\left(S_1 + \frac{\pi^2}{4}S_1\right)z(t-\sigma_{11}) + 2x^{\mathrm{T}}(t-\sigma_{11})\left(-S_1 + \frac{\pi^2}{4}S_1\right) \\
&\qquad \times z(t-\sigma_{12}) + z^{\mathrm{T}}(t-\sigma_{12})\left(S_1 + \frac{\pi^2}{4}S_1\right)z(t-\sigma_{12}) + 2z^{\mathrm{T}}(t-\sigma_{11}) \\
&\qquad \times \left(-\frac{\pi^2}{2h_1}S_1\right)\int_{t-\sigma_{12}}^{t-\sigma_{11}} z(s)\,\mathrm{d}s + 2z^{\mathrm{T}}(t-\sigma_{12})\left(-\frac{\pi^2}{2h_1}S_1\right)\int_{t-\sigma_{12}}^{t-\sigma_{11}} z(s)\,\mathrm{d}s \\
&\qquad + \int_{t-\sigma_{12}}^{t-\sigma_{11}} z^{\mathrm{T}}(s)\,\mathrm{d}s \quad \frac{\pi^2}{h_1^2}S_1 \int_{t-\sigma_{12}}^{t-\sigma_{11}} z(s)\,\mathrm{d}s\Bigg]. \qquad (13)
\end{aligned}$$

Similarly,

$$\begin{aligned}
&-\int_{t-\sigma_{22}}^{t-\sigma_{21}} \dot{z}^{\mathrm{T}}(s) S_2 \dot{z}(s)\,\mathrm{d}s \\
&\quad\leqslant -\frac{1}{h_2}\Bigg[z^{\mathrm{T}}(t-\sigma_{21})\left(S_2 + \frac{\pi^2}{4}S_2\right)z(t-\sigma_{21}) + 2x^{\mathrm{T}}(t-\sigma_{21})\left(-S_2 + \frac{\pi^2}{4}S_2\right) \\
&\qquad \times z(t-\sigma_{22}) + z^{\mathrm{T}}(t-\sigma_{22})\left(S_2 + \frac{\pi^2}{4}S_2\right)z(t-\sigma_{22}) + 2z^{\mathrm{T}}(t-\sigma_{21}) \\
&\qquad \times \left(-\frac{\pi^2}{2h_2}S_2\right)\int_{t-\sigma_{22}}^{t-\sigma_{21}} z(s)\,\mathrm{d}s + 2z^{\mathrm{T}}(t-\sigma_{22})\left(-\frac{\pi^2}{2h_2}S_2\right)\int_{t-\sigma_{22}}^{t-\sigma_{21}} z(s)\,\mathrm{d}s \\
&\qquad + \int_{t-\sigma_{22}}^{t-\sigma_{21}} z^{\mathrm{T}}(s)\,\mathrm{d}s \quad \frac{\pi^2}{h_2^2}S_2 \int_{t-\sigma_{22}}^{t-\sigma_{21}} z(s)\,\mathrm{d}s\Bigg]. \qquad (14)
\end{aligned}$$

$$\begin{aligned}
\dot{V}_5(t) &= \dot{z}^{\mathrm{T}}(t)\left[\frac{h_1^2}{2}T_1 + \frac{h_2^2}{2}T_2\right]\dot{z}(t) - \int_{-\sigma_{12}}^{-\sigma_{11}}\int_{t+\theta}^{t} \dot{z}^{\mathrm{T}}(s)T_1\dot{z}(s)\,\mathrm{d}s\,\mathrm{d}\theta \\
&\quad - \int_{-\sigma_{22}}^{-\sigma_{21}}\int_{t+\theta}^{t} \dot{z}^{\mathrm{T}}(s)T_2\dot{z}(s)\,\mathrm{d}s\,\mathrm{d}\theta. \qquad (15)
\end{aligned}$$

From Lemma 1 we can have

$$\begin{aligned}
&-\int_{-\sigma_{12}}^{-\sigma_{11}}\int_{t+\theta}^{t} \dot{z}^{\mathrm{T}}(s)T_1\dot{z}(s)\,\mathrm{d}s\,\mathrm{d}\theta \\
&\quad\leqslant -\frac{2}{h_1^2}\Biggl(\int_{-\sigma_{12}}^{-\sigma_{11}}\int_{t+\theta}^{t} \dot{z}(s)\,\mathrm{d}s\,\mathrm{d}\theta\Biggr)^{\mathrm{T}} T_1 \int_{-\sigma_{12}}^{-\sigma_{11}}\int_{t+\theta}^{t} \dot{z}(s)\,\mathrm{d}s\,\mathrm{d}\theta \\
&\quad= -\frac{2}{h_1^2}\Biggl[h_1 z^{\mathrm{T}}(t) - \int_{t-\sigma_{12}}^{t-\sigma_{11}} z^{\mathrm{T}}(s)\,\mathrm{d}s\Biggr] T_1 \Biggl[h_1 z(t) - \int_{t-\sigma_{12}}^{t-\sigma_{11}} z(s)\,\mathrm{d}s\Biggr] \\
&\quad= -2z^{\mathrm{T}}(t)T_1 z(t) + \frac{4}{h_1} z^{\mathrm{T}}(t)T_1 \int_{t-\sigma_{12}}^{t-\sigma_{11}} z(s)\,\mathrm{d}s \\
&\qquad - \frac{2}{h_1^2}\int_{t-\sigma_{12}}^{t-\sigma_{11}} z^{\mathrm{T}}(s)\,\mathrm{d}s\;\; T_1 \int_{t-\sigma_{12}}^{t-\sigma_{11}} z(s)\,\mathrm{d}s.
\end{aligned} \tag{16}$$

Similarly,

$$\begin{aligned}
-\int_{-\sigma_{22}}^{-\sigma_{21}}\int_{t+\theta}^{t} \dot{z}^{\mathrm{T}}(s)T_2\dot{z}(s)\,\mathrm{d}s\,\mathrm{d}\theta &\leqslant -2z^{\mathrm{T}}(t)T_2 z(t) + \frac{4}{h_2} z^{\mathrm{T}}(t)T_2 \int_{t-\sigma_{22}}^{t-\sigma_{21}} z(s)\,\mathrm{d}s \\
&\quad - \frac{2}{h_2^2}\int_{t-\sigma_{22}}^{t-\sigma_{21}} z^{\mathrm{T}}(s)\,\mathrm{d}s\;\; T_2 \int_{t-\sigma_{22}}^{t-\sigma_{21}} z(s)\,\mathrm{d}s.
\end{aligned} \tag{17}$$

$$\begin{aligned}
\dot{V}_6(t) &= \dot{z}^{\mathrm{T}}(t)\biggl[\frac{h_1^3}{6}U_1 + \frac{h_2^3}{6}U_2\biggr]\dot{z}(t) - \int_{-\sigma_{12}}^{-\sigma_{11}}\int_{\alpha}^{0}\int_{t+\theta}^{t} \dot{z}^{\mathrm{T}}(s)U_1\dot{z}(s)\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\alpha \\
&\quad - \int_{-\sigma_{22}}^{-\sigma_{21}}\int_{\alpha}^{0}\int_{t+\theta}^{t} \dot{z}^{\mathrm{T}}(s)U_2\dot{z}(s)\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\alpha.
\end{aligned} \tag{18}$$

Further, by applying Lemma 1, we have

$$\begin{aligned}
&-\int_{-\sigma_{12}}^{-\sigma_{11}}\int_{\alpha}^{0}\int_{t+\theta}^{t} \dot{z}^{\mathrm{T}}(s)U_1\dot{z}(s)\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\alpha \\
&\quad\leqslant -\frac{6}{h_1^3}\Biggl(\int_{-\sigma_{12}}^{-\sigma_{11}}\int_{\alpha}^{0}\int_{t+\theta}^{t} \dot{z}(s)\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\alpha\Biggr)^{\mathrm{T}} U_1 \int_{-\sigma_{12}}^{-\sigma_{11}}\int_{\alpha}^{0}\int_{t+\theta}^{t} \dot{z}(s)\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\alpha
\end{aligned}$$

$$= -\frac{6}{h_1^3}\left[\frac{h_1^2}{2}z^{\mathrm{T}}(t) - \int_{-\sigma_{12}}^{-\sigma_{11}}\int_{t+\alpha}^{t} z^{\mathrm{T}}(s)\,\mathrm{d}s\,\mathrm{d}\alpha\right]U_1\left[\frac{h_1^2}{2}z(t) - \int_{-\sigma_{12}}^{-\sigma_{11}}\int_{t+\alpha}^{t} z(s)\,\mathrm{d}s\,\mathrm{d}\alpha\right]$$

$$= -\frac{3}{2}h_1 z^{\mathrm{T}}(t)U_1 z(t) + \frac{6}{h_1}z^{\mathrm{T}}(t)\ U_1\int_{-\sigma_{12}}^{-\sigma_{11}}\int_{t+\alpha}^{t} z(s)\,\mathrm{d}s\,\mathrm{d}\alpha$$

$$- \frac{6}{h_1^3}\int_{-\sigma_{12}}^{-\sigma_{11}}\int_{t+\alpha}^{t} z(s)\,\mathrm{d}s\,\mathrm{d}\alpha\ U_1\int_{-\sigma_{12}}^{-\sigma_{11}}\int_{t+\alpha}^{t} z(s)\,\mathrm{d}s\,\mathrm{d}\alpha. \tag{19}$$

Similarly,

$$-\int_{-\sigma_{22}}^{-\sigma_{21}}\int_{\alpha}^{0}\int_{t+\theta}^{t} \dot{z}^{\mathrm{T}}(s)U_2\dot{z}(s)\,\mathrm{d}s\,\mathrm{d}\theta\,\mathrm{d}\alpha$$

$$\leqslant -\frac{3}{2}h_2 z^{\mathrm{T}}(t)U_2 z(t) + \frac{6}{h_2}z^{\mathrm{T}}(t)U_2\int_{-\sigma_{22}}^{-\sigma_{21}}\int_{t+\alpha}^{t} z(s)\,\mathrm{d}s\,\mathrm{d}\alpha$$

$$- \frac{6}{h_2^3}\int_{-\sigma_{12}}^{-\sigma_{11}}\int_{t+\alpha}^{t} z(s)\,\mathrm{d}s\,\mathrm{d}\alpha\ U_2\int_{-\sigma_{22}}^{-\sigma_{21}}\int_{t+\alpha}^{t} z(s)\,\mathrm{d}s\,\mathrm{d}\alpha. \tag{20}$$

$$\dot{V}_7(t) = f^{\mathrm{T}}\big(z(t)\big)[V_1 + \rho V_2]f\big(z(t)\big) - (1-\eta)f^{\mathrm{T}}\big(z\big(t-\rho(t)\big)\big)V_1 f\big(z\big(t-\rho(t)\big)\big)$$
$$- \frac{1}{\rho}\left(\int_{t-\rho(t)}^{t} z(s)\,\mathrm{d}s\right)^{\mathrm{T}} V_2\int_{t-\rho(t)}^{t} z(s)\,\mathrm{d}s. \tag{21}$$

Moreover, for any matrix Λ with appropriate dimensions, it is true that

$$0 = 2\big[z^{\mathrm{T}}(t)\Lambda + \dot{z}^{\mathrm{T}}(t)\Lambda\big]\bigg[-\dot{z}(t) + \big(-A + BK + B\Delta K(t)\big)z(t) + W_0 f\big(z(t)\big)$$
$$+ W_1 f\big(z\big(t-\sigma(t)\big)\big) + W_2\int_{t-\rho(t)}^{t} f\big(z(s)\big)\,\mathrm{d}s + B_\omega\omega(t)\bigg]. \tag{22}$$

For any $h_{1i} \leqslant 0$, $h_{2i} \leqslant 0$, $h_{3i} \leqslant 0$, $i = 1, 2, \dots, n$, it follows from (2) that

$$\big[f_i\big(z_i(t)\big) - F_i^- z_i(t)\big]h_{1i}\big[F_i^+ z_i(t) - f_i\big(z_i(t)\big)\big] \geqslant 0, \tag{23}$$

$$\big[f_i\big(z_i(t-\sigma(t))\big) - F_i^- z_i\big(t-\sigma(t)\big)\big]$$
$$\times h_{2i}\big[F_i^+ z_i\big(t-\sigma(t)\big) - f_i\big(z_i\big(t-\sigma(t)\big)\big)\big] \geqslant 0, \tag{24}$$

$$\big[f_i\big(z_i(t)\big) - f_i\big(z_i\big(t-\sigma(t)\big)\big) - F_i^-\big(z_i(t) - z_i\big(t-\sigma(t)\big)\big)\big]h_{3i}$$
$$\times \big[F_i^+\big(z_i(t) - z_i\big(t-\sigma(t)\big)\big) - f_i\big(z_i(t)\big) + f_i\big(z_i\big(t-\sigma(t)\big)\big)\big] \geqslant 0, \tag{25}$$

which imply

$$0 \leqslant \begin{bmatrix} z^{\mathrm{T}}(t) \\ f^{\mathrm{T}}(z(t)) \end{bmatrix} \begin{bmatrix} -F_1H_1 & F_2H_1 \\ * & -H_1 \end{bmatrix} \begin{bmatrix} z(t) \\ f(z(t)) \end{bmatrix}, \tag{26}$$

$$0 \leqslant \begin{bmatrix} z^{\mathrm{T}}(t-\sigma(t)) \\ f^{\mathrm{T}}(z(t-\sigma(t))) \end{bmatrix} \begin{bmatrix} -F_1H_2 & F_2H_2 \\ * & -H_2 \end{bmatrix} \begin{bmatrix} z(t-\sigma(t)) \\ f(z(t-\sigma(t))) \end{bmatrix} \tag{27}$$

and

$$0 \leqslant \begin{bmatrix} z^{\mathrm{T}}(t) \\ f^{\mathrm{T}}(z(t)) \\ z^{\mathrm{T}}(t-\sigma(t)) \\ f^{\mathrm{T}}(z(t-\sigma(t))) \end{bmatrix} \begin{bmatrix} -F_1H_3 & F_2H_3 & F_1H_3 & -F_2H_3 \\ * & -H_3 & -F_2H_3 & H_3 \\ * & * & -F_1H_3 & F_2H_3 \\ * & * & * & -H_3 \end{bmatrix} \begin{bmatrix} z(t) \\ f(z(t)) \\ z(t-\sigma(t)) \\ f(z(t-\sigma(t))) \end{bmatrix}, \tag{28}$$

where $H_1 = \operatorname{diag}\{h_{11}, h_{12}, \dots, h_{1n}\}$, $H_2 = \operatorname{diag}\{h_{21}, h_{22}, \dots, h_{2n}\}$, $H_3 = \operatorname{diag}\{h_{31}, h_{32}, \dots, h_{3n}\}$.

Next, we show that the neural network (1) is asymptotically stable with a mixed H_∞ and passivity performance γ. To this end, we define the following index:

$$J_{y\omega(t)} = \int_0^{t^*} \bigl(-\gamma^{-1}\theta y^{\mathrm{T}}(\alpha)y(\alpha) - 2(1-\theta)y^{\mathrm{T}}(\alpha)\omega(\alpha) - \gamma\omega^{\mathrm{T}}(\alpha)\omega(\alpha)\bigr)\,\mathrm{d}\alpha,$$

where $t^* \geqslant 0$.

From Eqs. (6)–(28) it can be deduced that

$$\begin{aligned} &\dot V(t) + \gamma^{-1}\theta y^{\mathrm{T}}(t)y(t) - 2(1-\theta)y^{\mathrm{T}}(t)\omega(t) - \gamma\omega^{\mathrm{T}}(t)\omega(t) \\ &\quad \leqslant \xi^{\mathrm{T}}(t)[\varXi + \Delta\varXi]\xi(t), \end{aligned} \tag{29}$$

where $\varXi = \varOmega + \gamma^{-1}\theta C^{\mathrm{T}}C$, $\Delta\varXi = \varGamma_1F(t)\varGamma_2 + \varGamma_2^{\mathrm{T}}F(t)\varGamma_1^{\mathrm{T}}$. Applying Lemma 3, there exists a scalar $\epsilon > 0$ such that

$$\Delta\varXi \leqslant \epsilon\varGamma_2^{\mathrm{T}}\varGamma_2 + \epsilon^{-1}\varGamma_1\varGamma_1^{\mathrm{T}}.$$

Then (29) becomes

$$\begin{aligned} &\dot V(t) + \gamma^{-1}\theta y^{\mathrm{T}}(t)y(t) - 2(1-\theta)y^{\mathrm{T}}(t)\omega(t) - \gamma\omega^{\mathrm{T}}(t)\omega(t) \\ &\quad \leqslant \xi^{\mathrm{T}}(t)\bigl[\varOmega + \gamma^{-1}\theta C^{\mathrm{T}}C + \epsilon\varGamma_2^{\mathrm{T}}\varGamma_2 + \epsilon^{-1}\varGamma_1\varGamma_1^{\mathrm{T}}\bigr]\xi(t), \end{aligned} \tag{30}$$

where

$$\begin{aligned} \xi^{\mathrm{T}}(t) = \Bigl[& z^{\mathrm{T}}(t),\ z^{\mathrm{T}}\bigl(t-\sigma(t)\bigr),\ z^{\mathrm{T}}\bigl(t-\sigma_1(t)\bigr),\ z^{\mathrm{T}}\bigl(t-\sigma_2(t)\bigr),\ z^{\mathrm{T}}\bigl(t-\sigma_{11}\bigr), \\ & z^{\mathrm{T}}(t-\sigma_{12}),\ z^{\mathrm{T}}(t-\sigma_{21}),\ z^{\mathrm{T}}(t-\sigma_{22}),\ \dot z^{\mathrm{T}}(t),\ f^{\mathrm{T}}\bigl(z(t)\bigr),\ f^{\mathrm{T}}\bigl(z\bigl(t-\sigma(t)\bigr)\bigr), \\ & f^{\mathrm{T}}\bigl(z\bigl(t-\sigma_1(t)\bigr)\bigr),\ f^{\mathrm{T}}\bigl(z\bigl(t-\sigma_2(t)\bigr)\bigr),\ f^{\mathrm{T}}\bigl(z\bigl(t-\rho(t)\bigr)\bigr), \end{aligned}$$

$$\int_{t-\sigma_{12}}^{t-\sigma_{11}} z^{\mathrm{T}}(s)\,\mathrm{d}s, \int_{t-\sigma_{22}}^{t-\sigma_{21}} z^{\mathrm{T}}(s)\,\mathrm{d}s, \int_{t-\rho(t)}^{t} z^{\mathrm{T}}(s)\,\mathrm{d}s, \int_{-\sigma_{12}}^{-\sigma_{11}}\int_{t+\alpha}^{t} z^{\mathrm{T}}(s)\,\mathrm{d}s\,\mathrm{d}\alpha,$$
$$\left.\int_{-\sigma_{22}}^{-\sigma_{21}}\int_{t+\alpha}^{t} z^{\mathrm{T}}(s)\,\mathrm{d}s\,\mathrm{d}\alpha,\ \omega^{\mathrm{T}}(t)\right]$$

and $\Omega = (\Omega_{i,j})_{20\times 20}$ with

$$\begin{aligned}
\Omega_{1,1} &= Q_1 + Q_4 + Q_7 + Q_{10} + Q_{11} + Q_{12} + Q_{13} + h_1R_1 + h_2R_2 - T_1 - T_1^{\mathrm{T}} - T_2 \\
&\quad - T_2^{\mathrm{T}} - \frac{3}{2}h_1U_1 - \frac{3}{2}h_2U_2 - \Lambda A - A^{\mathrm{T}}\Lambda^{\mathrm{T}} + \Lambda BK + (\Lambda BK)^{\mathrm{T}} - F_1H_1 - F_1H_3, \\
\Omega_{1,2} &= F_1H_3, \qquad \Omega_{1,5} = P_{12}, \qquad \Omega_{1,6} = -P_{12}, \qquad \Omega_{1,7} = P_{13}, \qquad \Omega_{1,8} = -P_{13}, \\
\Omega_{1,9} &= P_{11} - \Lambda - (A\Lambda)^{\mathrm{T}} + (KB\Lambda)^{\mathrm{T}}, \qquad \Omega_{1,10} = Q_2 + Q_5 + Q_8 + \Lambda W_0 + F_2H_1 + F_2H_3, \\
\Omega_{1,11} &= \Lambda W_1 - F_2H_3, \qquad \Omega_{1,15} = \frac{2}{h_1}T_1, \qquad \Omega_{1,16} = \frac{2}{h_2}T_2, \qquad \Omega_{1,17} = \Lambda W_2, \\
\Omega_{1,18} &= \frac{3}{h_1}U_1, \qquad \Omega_{1,19} = \frac{3}{h_2}U_2, \qquad \Omega_{1,20} = \Lambda B_\omega - (1-\theta)C^{\mathrm{T}}, \\
\Omega_{2,2} &= -(1-\mu)Q_1 - F_1H_2 - F_1H_3, \qquad \Omega_{2,10} = -H_3^{\mathrm{T}}F_2^{\mathrm{T}}, \\
\Omega_{2,11} &= -(1-\mu)Q_2 + F_2H_2 + F_2H_3, \qquad \Omega_{3,3} = -(1-\mu_1)Q_4, \\
\Omega_{3,12} &= -(1-\mu_1)Q_5, \qquad \Omega_{4,4} = -(1-\mu_2)Q_7, \qquad \Omega_{4,13} = -(1-\mu_2)Q_8, \\
\Omega_{5,5} &= -Q_{10} - \frac{1}{h_1}\left(S_1 + \frac{\pi^2}{4}S_1\right), \qquad \Omega_{5,6} = -\frac{1}{h_1}\left(-S_1 + \frac{\pi^2}{4}S_1\right), \\
\Omega_{5,15} &= P_{22}^{\mathrm{T}} + \frac{1}{h_1^2}\frac{\pi^2}{2}S_1, \qquad \Omega_{6,6} = -Q_{11} - \frac{1}{h_1}\left(S_1 + \frac{\pi^2}{4}S_1\right), \\
\Omega_{6,15} &= -P_{22}^{\mathrm{T}} + \frac{1}{h_1^2}\frac{\pi^2}{2}S_1, \qquad \Omega_{7,7} = -Q_{12} - \frac{1}{h_2}\left(S_2 + \frac{\pi^2}{4}S_2\right), \\
\Omega_{7,8} &= -\frac{1}{h_2}\left(-S_2 + \frac{\pi^2}{4}S_2\right), \qquad \Omega_{7,15} = P_{23}^{\mathrm{T}}, \qquad \Omega_{7,16} = P_{33}^{\mathrm{T}} + \frac{1}{h_2^2}\frac{\pi^2}{2}S_2, \\
\Omega_{8,8} &= -Q_{13} - \frac{1}{h_2}\left(S_2 + \frac{\pi^2}{4}S_2\right), \qquad \Omega_{8,15} = -P_{23}^{\mathrm{T}}, \qquad \Omega_{8,16} = \frac{1}{h_2^2}\frac{\pi^2}{2}S_2, \\
\Omega_{8,16} &= -P_{33}^{\mathrm{T}}, \qquad \Omega_{9,9} = h_1S_1 + h_2S_2 + \frac{h_1^2}{2}T_1 + \frac{h_2^2}{2}T_2 + \frac{h_1^3}{6}U_1 + \frac{h_2^3}{6}U_2 - \Lambda - \Lambda^{\mathrm{T}}, \\
\Omega_{9,10} &= \Lambda W_0, \qquad \Omega_{9,11} = \Lambda W_1, \qquad \Omega_{9,17} = \Lambda W_2, \qquad \Omega_{9,20} = \Lambda B_\omega, \\
\Omega_{10,10} &= Q_3 + Q_6 + Q_9 + V_1 + \rho V_2 - H_1 - H_3, \qquad \Omega_{10,11} = H_3, \\
\Omega_{11,11} &= -(1-\mu)Q_3 - H_2 - H_3, \qquad \Omega_{12,12} = -(1-\mu_1)Q_6, \\
\Omega_{13,13} &= -(1-\mu_2)Q_9, \Omega_{14,14} = -V_1, \qquad \Omega_{15,15} = -\frac{1}{h_1}R_1 - \frac{\pi^2}{h_1^3}S_1 - \frac{2}{h_1^2}T_1, \\
\Omega_{16,16} &= -\frac{1}{h_2}R_2 - \frac{\pi^2}{h_2^3}S_2 - \frac{2}{h_2^2}T_2, \qquad \Omega_{17,17} = -\frac{1}{\rho}V_2, \qquad \Omega_{18,18} = -\frac{6}{h_1^3}U_1, \\
\Omega_{19,19} &= -\frac{6}{h_2^3}U_2, \qquad \Omega_{20,20} = -\gamma I,
\end{aligned}$$

$$C_1 = [C \ \overbrace{0 \ldots 0}^{19 \text{ times } 19}\ ], \qquad \Gamma_1^{\mathrm{T}} = \big[(\Lambda B)^{\mathrm{T}} \overbrace{0 \ldots 0}^{7 \text{ times}} (\Lambda B)^{\mathrm{T}} \overbrace{0 \ldots 0}^{11 \text{ times}}\big]^{\mathrm{T}} H_a, \qquad \Gamma_2^{\mathrm{T}} = [E_a \overbrace{0 \ldots 0}^{19 \text{ times}}]^{\mathrm{T}}.$$

Now, (30) using Schur complement, it is easy to obtain

$$\Phi = \begin{bmatrix} \Omega & \sqrt{\theta} C_1 & \Gamma_1 & \epsilon \Gamma_2 \\ * & -\gamma I & 0 & 0 \\ * & * & -\epsilon I & * \\ * & * & * & -\epsilon I \end{bmatrix} < 0, \tag{31}$$

Then, pre- and post- multiplying both sides of (31) by $\operatorname{diag}(\overbrace{X, \ldots, X}^{20 \text{ times}} IXI)$ and its transpose, respectively. Define the following variables:

$$\begin{aligned}
&F = KX^T, \quad \bar{H}_1 = XH_1X^{\mathrm{T}}, \quad \bar{H}_2 = XH_2X^{\mathrm{T}}, \quad \bar{H}_3 = XH_3X^{\mathrm{T}}, \\
&\Lambda = X^{-1}, \quad \bar{P} = XPX^{\mathrm{T}}, \quad \bar{Q}_i = XQ_iX^{\mathrm{T}} \quad (i = 1, 2, \ldots, 13), \\
&\bar{R}_1 = XR_1X^{\mathrm{T}}, \quad \bar{R}_2 = XR_2X^{\mathrm{T}}, \quad \bar{S}_1 = XS_1X^{\mathrm{T}}, \quad \bar{S}_2 = XS_2X^{\mathrm{T}}, \\
&\bar{T}_1 = XT_1X^{\mathrm{T}}, \quad \bar{T}_2 = XT_2X^{\mathrm{T}}, \quad \bar{U}_1 = XU_1X^{\mathrm{T}}, \quad \bar{U}_2 = XU_2X^{\mathrm{T}}, \\
&\bar{V}_1 = XV_1X^{\mathrm{T}}, \quad \bar{V}_2 = XV_2X^{\mathrm{T}}.
\end{aligned}$$

We can obtain (31) is equivalent to (4). Therefore, if $\Phi < 0$, then

$$\dot{V}(t) + \gamma^{-1}\theta y^{\mathrm{T}}(t)y(t) - 2(1-\theta)y^{\mathrm{T}}(t)\omega(t)^{\mathrm{T}} - \gamma\omega^{\mathrm{T}}(t)\omega(t) < 0 \tag{32}$$

for any $\xi(t) \neq 0$. We integrate (32) from 0 to t^* and get

$$\begin{aligned}
&\int_0^{t^*} \big(-\gamma^{-1}\theta y^{\mathrm{T}}(\alpha)y(\alpha) - 2(1-\theta)y^{\mathrm{T}}(\alpha)\omega(\alpha) - \gamma\omega^{\mathrm{T}}(\alpha)\omega(\alpha)\big)\,\mathrm{d}\alpha \\
&\quad \leqslant -V(t^*) + V(0),
\end{aligned}$$

therefore,

$$J_{y\omega(t)} \leqslant -V(t^*) + V(0),$$

which implies

$$J_{y\omega(t)} \leqslant 0$$

because $V(0) = 0$ under zero initial condition and $V(t^*) \geqslant 0$. Therefore, (3) holds for all $t^* > 0$. By Definition 1 the neural network (1) is asymptotically stable with a mixed H_∞ and passivity performance γ. This concludes the proof. □

Remark 1. In Corollary 1, when $W_2 = 0$, $B = 0$ and $B_\omega = 0$, the neural network (1) will be reduced to the following system:

$$\dot{z}(t) = -Az(t) + W_0 f\big(z(t)\big) + W_1 f\big(z\big(t - \sigma(t)\big)\big). \tag{33}$$

Then setting $S_1 = S_2 = S_3 = S_4 = 0$ in the proof of Theorem 1.

Corollary 1. *Under assumption* (A1)*, for given scalers* σ_{11}*,* σ_{12}*,* σ_{21}*,* σ_{22}*,* μ_1 *and* μ_2*, the neural network* (33) *is asymptotically stable if there exists positive definite matrices* P*,* Q_i $(i = 1, 2, \dots, 13)$*,* R_1*,* R_2*,* S_1*,* S_2*,* T_1*,* T_2*,* U_1*,* U_2*,*

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ * & P_{22} & P_{23} \\ * & * & P_{33} \end{bmatrix},$$

and positive diagonal matrices H_1*,* H_2*,* H_3 *such that the following LMIs hold*:

$$\tilde{\Omega}_{17\times 17} < 0, \tag{34}$$

where

$$\begin{aligned}
\tilde{\Omega}_{1,1} &= Q_1 + Q_4 + Q_7 + Q_{10} + Q_{11} + Q_{12} + Q_{13} + h_1 R_1 + h_2 R_2 - T_1 - T_1^{\mathrm{T}} - T_2 \\
&\quad - T_2^{\mathrm{T}} - \frac{3}{2} h_1 U_1 - \frac{3}{2} h_2 U_2 - F_1 H_1 - F_1 H_3,
\end{aligned}$$

$\tilde{\Omega}_{1,2} = F_1 H_3, \quad \tilde{\Omega}_{1,5} = P_{12}, \quad \tilde{\Omega}_{1,6} = -P_{12}, \quad \tilde{\Omega}_{1,7} = P_{13},$

$\tilde{\Omega}_{1,8} = -P_{13}, \quad \tilde{\Omega}_{1,9} = P_{11}, \quad \tilde{\Omega}_{1,10} = Q_2 + Q_5 + Q_8 + \Lambda W_0 + F_2 H_1 + F_2 H_3,$

$\tilde{\Omega}_{1,11} = -F_2 H_3, \tilde{\Omega}_{1,14} = \frac{2}{h_1} T_1, \tilde{\Omega}_{1,15} = \frac{2}{h_2} T_2, \quad \tilde{\Omega}_{1,16} = \frac{3}{h_1} U_1,$

$\tilde{\Omega}_{1,17} = \frac{3}{h_2} U_2, \quad \tilde{\Omega}_{2,2} = -(1-\mu) Q_1 - F_1 H_2 - F_1 H_3, \quad \tilde{\Omega}_{2,10} = -H_3^{\mathrm{T}} F_2^{\mathrm{T}},$

$\tilde{\Omega}_{2,11} = -(1-\mu) Q_2 + F_2 H_2 + F_2 H_3, \quad \tilde{\Omega}_{3,3} = -(1-\mu_1) Q_4,$

$\tilde{\Omega}_{3,12} = -(1-\mu_1) Q_5, \quad \tilde{\Omega}_{4,4} = -(1-\mu_2) Q_7, \quad \tilde{\Omega}_{4,13} = -(1-\mu_2) Q_8,$

$\tilde{\Omega}_{5,5} = -Q_{10} - \frac{1}{h_1}\left(S_1 + \frac{\pi^2}{4} S_1\right), \quad \tilde{\Omega}_{5,6} = -\frac{1}{h_1}\left(-S_1 + \frac{\pi^2}{4} S_1\right),$

$\tilde{\Omega}_{5,14} = P_{22}^{\mathrm{T}} + \frac{1}{h_1^2}\frac{\pi^2}{2} S_1, \quad \tilde{\Omega}_{6,6} = -Q_{11} - \frac{1}{h_1}\left(S_1 + \frac{\pi^2}{4} S_1\right),$

$\tilde{\Omega}_{6,14} = -P_{22}^{\mathrm{T}} + \frac{1}{h_1^2}\frac{\pi^2}{2} S_1, \quad \tilde{\Omega}_{7,7} = -Q_{12} - \frac{1}{h_2}\left(S_2 + \frac{\pi^2}{4} S_2\right),$

$\tilde{\Omega}_{7,8} = -\frac{1}{h_2}\left(-S_2 + \frac{\pi^2}{4} S_2\right), \quad \tilde{\Omega}_{7,14} = P_{23}^{\mathrm{T}}, \quad \tilde{\Omega}_{7,15} = P_{33}^{\mathrm{T}} + \frac{1}{h_2^2}\frac{\pi^2}{2} S_2,$

$\tilde{\Omega}_{8,8} = -Q_{13} - \frac{1}{h_2}\left(S_2 + \frac{\pi^2}{4} S_2\right), \quad \tilde{\Omega}_{8,14} = -P_{23}^{\mathrm{T}}, \quad \tilde{\Omega}_{8,15} = \frac{1}{h_2^2}\frac{\pi^2}{2} S_2, \quad \tilde{\Omega}_{8,16} = -P_{33}^{\mathrm{T}},$

$\tilde{\Omega}_{9,9} = h_1 S_1 + h_2 S_2 + \frac{h_1^2}{2} T_1 + \frac{h_2^2}{2} T_2 + \frac{h_1^3}{6} U_1 + \frac{h_2^3}{6} U_2 - \Lambda - \Lambda^{\mathrm{T}}, \quad \tilde{\Omega}_{9,10} = \Lambda W_0,$

$\tilde{\Omega}_{9,11} = \Lambda W_1, \quad \tilde{\Omega}_{10,10} = Q_3 + Q_6 + Q_9 - H_1 - H_3, \quad \tilde{\Omega}_{10,11} = H_3,$

$\tilde{\Omega}_{11,11} = -(1-\mu) Q_3 - H_2 - H_3, \quad \tilde{\Omega}_{12,12} = -(1-\mu_1) Q_6, \quad \tilde{\Omega}_{13,13} = -(1-\mu_2) Q_9,$

$\tilde{\Omega}_{14,14} = -\frac{1}{h_1} R_1 - \frac{\pi^2}{h_1^3} S_1 - \frac{2}{h_1^2} T_1, \quad \tilde{\Omega}_{15,15} = -\frac{1}{h_2} R_2 - \frac{\pi^2}{h_2^3} S_2 - \frac{2}{h_2^2} T_2,$

$\tilde{\Omega}_{16,16} = -\frac{6}{h_1^3} U_1, \quad \tilde{\Omega}_{17,17} = -\frac{6}{h_2^3} U_2.$

Remark 2. In general, computational complexity will be a big issue based on how large are the LMIs and how more are the decision variables. The results in Theorem 3.1 and Corollary 1 are derived based on the construction of proper Lyapunov? Krasovskii functional with triple and four integral terms and by using Wirtinger-based inequality, Jensen's inequality. It should be mentioned that the derived nonfragile mixed H_∞ passivity criteria for the considered neural networks with time-varying delays is less conservative. Meanwhile, it should also be noticed that the relaxation of the derived results is acquired at the cost of more number of decision variables. As far the results to be efficient enough, it is more comfortable to have larger maximum allowable upper bounds, but still in order to reduce computational burden and time consumption, our future work will be focused on reducing the number of decision variables.

Remark 3. In [1,21,30,37,56], the authors discussed stability, passivity and dissipativity of various models such as Markov jump NNs, BAM NNs, neutral-type NNs, genetic regulatory networks. These models are dealt with only one time-varying delay, but in [25,28,35,41,46,55], the systems are based on a new type of time-varying delay model proposed recently, which contains two time-varying delay components in the state of the dynamical systems, because the system with two additive time-varying delay has a physically powerful application background in a networked control system. Here, it should be mentioned that the passivity control for NNs with the additive time-varying delays. This criterion is derived by defining LKF in (5), which makes full use of the information about $\sigma_1(t)$ and $\sigma_2(t)$. Therefore, it is of significance to consider nonfragile mixed H_∞ and passivity control problem for neural networks with two additive time-varying delay components.

Remark 4. In [50], the problem of nonfragile robust finite-time H_∞ control for a class of uncertain nonlinear stochastic Itô systems via neural network is addressed. The problem of H_∞ control system with parametric uncertainty in all matrices of the system and output equations have been investigated in [47]. In [10], the author addressed the problem of nonfragile observer-based passive control for a class of Markovian jumping systems (MJSs) subjected to uncertainties, nonlinearities and time-delays. In the literature, many control methods have been used such as nonfragile H_∞ control, observer-based passive control and output feedback H_∞ control. However, investigation on nonfragile mixed H_∞ and passivity with successive time-varying delay components have yet to be found in the literature. Motivated by the above discussion, a nonfragile controller mixed H_∞ and passivity for neural networks with successive time-varying delay components, which is different from other existing literature, has been developed in this paper.

4 Numerical examples

In this section, we present two numerical examples to demonstrate the effectiveness and less conservativeness of the proposed results.

Example 1. Consider the neural network (1) with the following parameters:

$$A = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.2 \end{bmatrix}, \qquad B = \begin{bmatrix} 0.4 & 0.1 \\ 0.01 & 0.2 \end{bmatrix}, \qquad B_\omega = \begin{bmatrix} 0.3 & 0.1 \\ 0.02 & 0.03 \end{bmatrix},$$
$$C = \begin{bmatrix} 0.2 & 0.01 \\ 0.01 & 0.4 \end{bmatrix}, \qquad E_a = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad H_a = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad F_2 = \begin{bmatrix} 0.60 & 0 \\ 0 & 0.55 \end{bmatrix},$$

$$W_0 = \begin{bmatrix} 0.9 & 0.7 \\ -0.04 & -0.01 \end{bmatrix}, \qquad W_1 = \begin{bmatrix} 0.4 & 0.03 \\ 0.01 & 0.07 \end{bmatrix}, \qquad W_2 = \begin{bmatrix} 0.2 & 0.01 \\ 0.5 & 0.01 \end{bmatrix}.$$

For three different values of θ (three different cases), by Theorem 1, we can obtain the desire nonfragile state feedback controller as follows:

Case 1 (*mixed H_∞ and passivity case*). When $\sigma_{11} = 0.2$, $\sigma_{12} = 1.5$, $\sigma_{21} = 0.2$, $\sigma_{22} = 1.9$, $\rho = 1.1$, $\mu_1 = 0.1$, $\mu_2 = 0.2$, $\eta = 0.3$ and $\theta = 0.5$, by applying Theorem 1 and *Matlab LMI* toolbox to solve LMI (4), the optimized minimum mixed H_∞ and passivity performance can be obtained as $\gamma = 0.6011$, and the corresponding nonfragile state feedback controller gain is given by

$$K = \begin{bmatrix} 0.6716 & -0.0162 \\ -0.0159 & 0.7128 \end{bmatrix}.$$

Figure 1(a) denotes the state response of $z(t)$ with the obtained controller gain. Figure 1(b) represents the state response $z(t)$ without controller. It is concluded from Figs. 1(a) and 1(b) that the state trajectories converges to zero quickly, and it demonstrates the efficiency of the proposed controller.

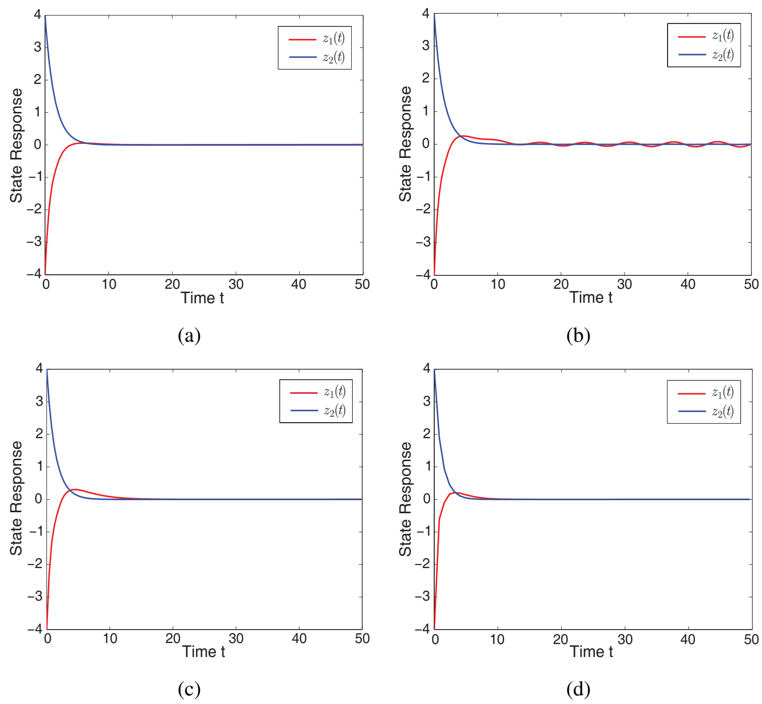

Figure 1. State response curves for (1) in Example 1: (a) mixed H_∞ and passivity control; (b) without control; (c) passivity control; (d) H_∞ control.

Case 2 (*passivity case*). When $\sigma_{11} = 0.2$, $\sigma_{12} = 1.5$, $\sigma_{21} = 0.2$, $\sigma_{22} = 1.9$, $\rho = 1.1$, $\mu_1 = 0.1$, $\mu_2 = 0.2$, $\eta = 0.3$ and $\theta = 0$, by applying Theorem 1 and *Matlab LMI* toolbox to solve LMI (4), the optimized minimum passivity performance index can be obtained as $\gamma = 0.6371$, and the associated nonfragile state feedback controller gain is given by

$$K = \begin{bmatrix} 0.6746 & -0.0178 \\ -0.0176 & 0.7160 \end{bmatrix}.$$

The state response curve for passive control is provided in Fig. 1(c).

Case 3 (H_∞ *case*). When $\sigma_{11} = 0.2$, $\sigma_{12} = 1.5$, $\sigma_{21} = 0.2$, $\sigma_{22} = 1.9$, $\rho = 1.1$, $\mu_1 = 0.1$, $\mu_2 = 0.2$, $\eta = 0.3$ and $\theta = 1$, by applying Theorem 1 and *Matlab LMI* toolbox to solve LMI (4), the optimized minimum H_∞ performance index is calculated as $\gamma = 0.9901$, and the corresponding nonfragile state feedback controller gain is given by

$$K = \begin{bmatrix} 0.6941 & -0.0076 \\ -0.0074 & 0.7280 \end{bmatrix}$$

and the state response curves for H_∞ control is given in Fig. 1(d).

Example 2. Consider the neural network (33) as discussed in [25, 28, 35, 41, 46, 55] with the following parameters:

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \qquad W_0 = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, \qquad W_1 = \begin{bmatrix} 0.88 & 1 \\ 1 & 1 \end{bmatrix},$$
$$F_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad F_2 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.8 \end{bmatrix}.$$

Let us consider σ_{12} and σ_{22} be the upper bounds of time-varying delays $\sigma_1(t)$ and $\sigma_2(t)$, respectively, and $\dot{\sigma_1}(t) \leqslant \mu_1$, $\dot{\sigma_2}(t) \leqslant \mu_2$. Solving LMI (34) in Corollary 1 by using Matlab LMI toolbox, we can calculate admissible upper bounds of σ_{12} and σ_{22}, which are given in Tables 1, 2. When $\sigma_{11} = 0$, $\sigma_{21} = 0$, $\mu_1 = 0.7$ and $\mu_2 = 0.1, 0.2$, Table 1 illustrates admissible upper bounds of σ_{22} for different values of σ_{12}. Similarly, admissible upper bounds of σ_{12} for different values of σ_{22} with $\sigma_{11} = 0$, $\sigma_{21} = 0$, $\mu_1 = 0.7$, $\mu_2 = 0.1, 0.2$ are described in Table 2. For system (33) with the above parameters, Fig. 2 shows, for the state responses $z(t)$, when $\sigma_{12} = 0.8$, $\sigma_{22} = 2.7651$, and the initial condition $(-0.2, 0.2)^{\mathrm{T}}$. This figure shows that the state signal converges to zero, which verifies the effectiveness of Corollary 1.

Table 1. Admissible upper bounds of σ_{22} for different values of μ_2 and σ_{12} with $\mu_1 = 0.7$.

Methods	$\mu_2 = 0.1$			$\mu_2 = 0.2$		
	$\sigma_{12} = 0.8$	$\sigma_{12} = 1$	$\sigma_{12} = 1.2$	$\sigma_{12} = 0.8$	$\sigma_{12} = 1$	$\sigma_{12} = 1.2$
[35]	1.5666	1.3668	1.1664	0.8515	0.6596	0.4616
[46]	1.9528	1.7992	1.6441	0.8703	0.6713	0.4715
[41]	2.0164	1.8203	1.6197	1.1364	0.9454	0.7207
[55]	1.9666	1.8351	1.6803	1.1296	0.9603	0.7443
[28]	2.2448	1.9642	1.8591	1.1684	1.0079	0.8856
[25]	2.3547	2.0053	1.9217	1.2012	1.1303	0.9754
Corollary 1	2.7651	2.4331	2.2156	1.7259	1.3992	1.0895

Table 2. Admissible upper bounds of σ_{12} for different values of μ_2 and σ_{22} with $\mu_1 = 0.7$.

Methods	$\mu_2 = 0.1$			$\mu_2 = 0.2$		
	$\sigma_{22} = 0.8$	$\sigma_{22} = 1$	$\sigma_{22} = 1.2$	$\sigma_{22} = 0.8$	$\sigma_{22} = 1$	$\sigma_{22} = 1.2$
[35]	2.6928	2.2389	2.0639	1.8474	1.5292	1.3455
[46]	2.7248	2.3325	2.2187	1.8710	1.5973	1.4782
[41]	2.8545	2.4856	2.4579	1.9851	1.8881	1.6203
[55]	2.9792	2.5684	2.5104	2.0740	1.9021	1.8003
Corollary 1	3.4567	3.0017	2.9604	2.4102	2.0135	1.9136

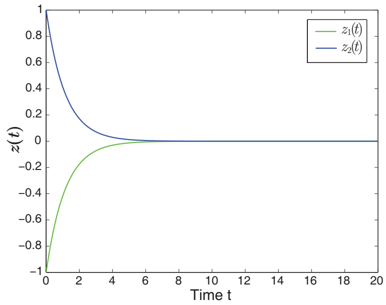

Figure 2. State trajectories system (33) in Example 2.

5 Conclusion

The problem of nonfragile mixed H_∞ and passivity control for neural networks with successive time-varying delay components have been presented in this paper. We construct a suitable Lyapunov–Krasovskii function (LKF) with triple and quadruple integral terms and using Wirtinger-type inequality technique. Sufficient conditions are established to ensure the existence of nonfragile mixed H_∞ and passivity analysis. The results are proposed in terms of linear matrix inequalities, which can guarantee the asymptotic stable of the considered neural networks and its nonfragile controller. Finally two examples are presented to illustrate the effectiveness of the proposed criteria. This work can be extended to complex networks and dissipative with Markovian jumping parameter and using delay partitioning approach. This will be done in the near future.

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