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Synchronization of a class of fractional-order neural networks with multiple time delays by comparison principles*

Weiwei Zhang[a], **Ranchao Wu**[b], **Jinde Cao**[c,d,1], **Ahmed Alsaedi**[d], **Tasawar Hayat**[d,e]

[a]School of Mathematics and Computational Science,
Anqing Normal University,
AnQing 246011, China
wwzhahu@aliyun.com

[b]School of Mathematics, Anhui University,
Hefei 230039, China
rcwu@ahu.edu.cn

[c]School of Mathematics, Southeast University,
Nanjing 210096, China
jdcao@seu.edu.cn

[d]Department of Mathematics, Faculty of Science,
King Abdulaziz University,
Jeddah 21589, Saudi Arabia
aalsaedi@hotmail.com

[e]Department of Mathematics, Quaid-I-Azam University,
Islamabad 44000, Pakistan
tahaksag@yahoo.com

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Abstract. This paper studies the synchronization of fractional-order neural networks with multiple time delays. Based on an inequality of fractional-order and comparison principles of linear fractional equation with multiple time delays, some sufficient conditions for synchronization of master-slave systems are obtained. Example and related simulations are given to demonstrate the feasibility of the theoretical results.

Keywords: fractional-order neural networks, comparison principles, multiple time delays, synchronization.

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[1]Corresponding author.

1 Introduction

Neural networks are important nonlinear systems, which have been studied in scientific research and have been applied to the signal processing, parallel computation, optimization, artificial intelligence, and so on. It is known that fractional calculus mainly deals with a generalization of differentiation and integration of arbitrary orders. Compared with classical integer-order models, the fractional-order calculus owns the description of memory and hereditary properties of a variety of processes. Fractional-order models are far better regard the dynamical behaviors of systems [2, 6, 13, 15, 16, 23, 24]. Taking these factors into account, many researchers have incorporated fractional calculus to neural networks and formed fractional-order neural networks [3, 7–9, 14, 25, 27].

Time-delayed models unavoidable exist in biological, engineering systems, and neural networks, see [19–21, 28, 29]. Generally, time delays will affect oscillation and instability behavior of a network. Nowadays, the synchronization of fractional-order delayed neural networks has attracted more and more attention. Some synchronization results have been obtained, for instance, in [26], authors studied the finite-time synchronization of fractional-order memristor-based neural networks with time delay. Paper [10] investigated the stability and synchronization of memristor-based fractional-order delayed neural networks. In [4], the synchronization of fractional-order complex-valued neural networks with time delay was studied. Paper [5] discussed the adaptive synchronization of fractional-order memristor-based neural networks with time delay.

However, most existing results related to synchronization of delayed neural networks have been considered with single time delay [17, 18]. In fact, the differential models with multiple time delays unavoidable exist in neural networks. It is worthy to point out that study about the stability of nonlinear fractional order with multiple time delays seems quite difficulty. There are only few results on the stability and synchronization of fractional-order neural networks with multiple time delays in existing literatures [12, 22]. Motivated by the above discussions, the aim here is to study the synchronization of fractional-order neural networks with multiple time delays. By constructing a Lyapunov function, applying an inequality of fractional-order and comparison principles of linear fractional equation with multiple time delays, we obtain some sufficient conditions, which can achieve synchronization. The obtained results are novel.

The remainder of this article is organized as follows. In Section 2, some definitions and lemmas are introduced. The model description is also given. In Section 3, the sufficient conditions for synchronization are obtained. Numerical simulations are presented in Section 4. Some conclusions are drawn in Section 5.

2 Preliminaries and model description

There are some definitions of the fractional-order integrals and derivatives, such as Riemann–Liouville definition and the Caputo definition. From the Laplace transform of fractional derivative, the advantage of the Caputo fractional derivative is that it only requires initial conditions given in terms of integer-order derivatives. Here, the definition of Caputo derivative is adopted.

Definition 1. (See [24].) The fractional integral with noninteger order $\alpha > 0$ for a function $x(t)$ is defined as

$$I^\alpha x(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t-\tau)^{\alpha-1} x(\tau)\,\mathrm{d}\tau,$$

where $t \geqslant t_0$, t_0 is the initial time, $\Gamma(\cdot)$ is the gamma function, given by $\Gamma(s) = \int_0^\infty t^{s-1}\mathrm{e}^{-t}\,\mathrm{d}t$.

Definition 2. (See [24].) The Caputo fractional derivative of order α for a function $x(t)$ is defined as

$$D^\alpha x(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^{t} (t-\tau)^{n-\alpha-1} x^{(n)}(\tau)\,\mathrm{d}\tau,$$

in which $t \geqslant t_0$, t_0 is the initial time, $n-1 < \alpha < n \in Z^+$.

We now consider the following fractional-order neural networks with time delays as master system:

$$D^\alpha x_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j\bigl(x_j(t)\bigr) + \sum_{j=1}^{n} b_{ij} g_j\bigl(x_j(t-\tau_j)\bigr) + I_i, \tag{1}$$

$i, j = 1, 2, \dots, n$, where $0 < \alpha < 1$, n is the number of units in a neural network, $x_i(t)$ denotes the pseudostate variable of the ith unit of master system, $c_i > 0$ is the self-regulating parameters of the ith unit. I_i represents the external input of the ith unit, a_{ij} and b_{ij} denote the strength of the jth unit on the ith unit at time t and $t-\tau_j$, respectively, $\tau_j > 0$ is the transmission delay, $f_j(x_j(t))$ and $g_j(x_j(t))$ denote the output of the jth unit at time t and $t-\tau_j$, respectively.

The slave system is given by

$$D^\alpha y_i(t) = -c_i y_i(t) + \sum_{j=1}^{n} a_{ij} f_j\bigl(y_j(t)\bigr) + \sum_{j=1}^{n} b_{ij} g_j\bigl(y_j(t-\tau_j)\bigr) - u_i(t) + I_i, \tag{2}$$

$i, j = 1, 2, \dots, n$. Note that $y_i(t)$ is the pseudostate vector of slave system and $u_i(t)$ is a suitable controller.

To ensure the main results, we present the following assumption and lemmas.

Assumption 1. The neuron activation functions $f_j(x)$, $g_j(x)$ satisfy the following Lipschitz conditions with Lipschitz constants $l_j > 0$, $h_j > 0$:

$$\bigl|f_j(u) - f_j(v)\bigr| \leqslant l_j|u-v|, \qquad \bigl|g_j(u) - g_j(v)\bigr| \leqslant h_j|u-v|$$

for all $u, v \in \mathbb{R}$.

Lemma 1. (*See* [1, 11].) *Suppose that $x(t) \in \mathbb{R}^n$ is a continuous and differentiable vector-value function. Then for any time instant $t \geqslant t_0$, we have*

$$\frac{1}{2} D^\alpha x^2(t) \leqslant x(t) D^\alpha x(t) \tag{3}$$

when $0 < \alpha < 1$.

Lemma 2. (*See* [22].) *Suppose that $V(t) \in \mathbb{R}$ is a continuous, differentiable, and non-negative function satisfying*

$$\begin{aligned} &D^\alpha V(t) \leqslant -aV(t) + \sum_{j=1}^n b_j V(t-\tau_j), \quad 0<\alpha<1,\ 1 \leqslant j \leqslant n, \\ &V(t) = \varphi(t) \geqslant 0, \quad t \in [-\tau, 0]. \end{aligned} \tag{4}$$

If $a > \sqrt{2}\sum_{j=1}^n b_j$ and $b_j > 0$, $j = 1, 2, \dots, n$, then for all $\varphi(t) \geqslant 0, \tau_j > 0$, $\lim_{t\to+\infty} V(t) = 0$.

3 Synchronization of fractional-order neural networks with multiple time delays

We will discuss the master-slave synchronization of fractional-order neural networks. The purpose is to choose an effective controller to achieve the synchronization of the master-slave systems.

Let $e_i(t) = y_i(t) - x_i(t)$, $i = 1, 2, \dots, n$, be the synchronization errors.

Select the control input function as follows:

$$u_i(t) = k_i\bigl(y_i(t) - x_i(t)\bigr), \quad i = 1, 2, \dots, n, \tag{5}$$

where k_i denotes the controller feedback gain.

Then the error systems are obtained in the form

$$\begin{aligned} D^\alpha e_i(t) = {}& -c_i e_i(t) + \sum_{j=1}^n a_{ij}\bigl[f_j\bigl(y_j(t)\bigr) - f_j\bigl(x_j(t)\bigr)\bigr] \\ &+ \sum_{j=1}^n b_{ij}\bigl[g_j\bigl(y_j\bigl(t-\tau_j\bigr)\bigr) - g_j\bigl(x_j(t-\tau_j)\bigr)\bigr] - k_i e_i(t). \end{aligned} \tag{6}$$

Theorem 1. *Under Assumption* 1 *and since the control function satisfies* (5)*, if the controller feedback gain k_i satisfies the inequality*

$$\lambda > \frac{\sqrt{2}}{2} \sum_{j=1}^n b_j, \tag{7}$$

then the fractional-order delayed neural networks system (1) *synchronizes system* (2)*, where $\lambda = \min_{1\leqslant i\leqslant n}\{2(c_i + k_i) - \sum_{j=1}^n l_j|a_{ij}| - \sum_{j=1}^n l_i|a_{ji}| - \sum_{j=1}^n h_j|b_{ij}|\}$, $b_j = \sum_{i=1}^n h_j|b_{ij}|$.*

Proof. We define the following Lyapunov function:

$$V(t) = \frac{1}{2} e^{\mathrm{T}}(t) e(t), \quad e(t) = \left(e_1(t), \dots, e_n(t)\right)^{\mathrm{T}}. \tag{8}$$

From Lemma 1,

$$\begin{aligned}
D^\alpha V(t) &= D^\alpha \left[\frac{1}{2} e^{\mathrm{T}}(t) e(t)\right] = D^\alpha \left[\frac{1}{2} \sum_{i=1}^n e_i^2(t)\right] \leqslant \sum_{i=1}^n e_i(t) D^\alpha e_i(t) \\
&= \sum_{i=1}^n \Bigg\{ -c_i e_i^2(t) + e_i(t) \sum_{j=1}^n a_{ij} \left[f_j\big(y_j(t)\big) - f_j\big(x_j(t)\big)\right] \\
&\quad + e_i(t) \sum_{j=1}^n b_{ij} \left[g_j\big(y_j(t-\tau_j)\big) - g_j\big(x_j(t-\tau_j)\big)\right] - k_i e_i^2(t) \Bigg\} \\
&\leqslant \sum_{i=1}^n \Bigg\{ -c_i e_i^2(t) + \sum_{j=1}^n \big|e_i(t)\big| |a_{ij}| \big| f_j\big(y_j(t)\big) - f_j\big(x_j(t)\big)\big| \\
&\quad + \sum_{j=1}^n \big|e_i(t)\big| |b_{ij}| \big| g_j\big(y_j(t-\tau_j)\big) - g_j\big(x_j(t-\tau_j)\big)\big| - k_i e_i^2(t) \Bigg\} \\
&\leqslant \sum_{i=1}^n \Bigg\{ -(c_i + k_i) e_i^2(t) + \sum_{j=1}^n \big|e_i(t)\big| |a_{ij}| l_j \big|e_j(t)\big| \\
&\quad + \sum_{j=1}^n \big|e_i(t)\big| |b_{ij}| h_j \big|e_j(t-\tau_j)\big| \Bigg\}.
\end{aligned} \tag{9}$$

Note that

$$\begin{aligned}
\big|e_i(t)\big| |a_{ij}| l_j \big|e_j(t)\big| &\leqslant \frac{1}{2} l_j |a_{ij}| \big(e_i^2(t) + e_j^2(t)\big), \\
\big|e_i(t)\big| |b_{ij}| h_j \big|e_j(t-\tau_j)\big| &\leqslant \frac{1}{2} h_j |b_{ij}| \big(e_i^2(t) + e_j^2(t-\tau_j)\big), \\
e_j^2(t-\tau_j) &\leqslant e_j^{\mathrm{T}}(t-\tau_j) e(t-\tau_j) = 2V(t-\tau_j).
\end{aligned}$$

Substitute these into (9), one has

$$\begin{aligned}
D^\alpha V(t) &\leqslant \sum_{i=1}^n \Bigg\{ -(c_i + k_i) e_i^2(t) + \frac{1}{2} \sum_{j=1}^n l_j |a_{ij}| \big(e_i^2(t) + e_j^2(t)\big) \\
&\quad + \frac{1}{2} \sum_{j=1}^n h_j |b_{ij}| \big(e_i^2(t) + e_j^2(t-\tau_j)\big) \Bigg\} \\
&= -\sum_{i=1}^n \left\{ (c_i + k_i) - \frac{1}{2} \sum_{j=1}^n l_j |a_{ij}| - \frac{1}{2} \sum_{j=1}^n l_i |a_{ji}| - \frac{1}{2} \sum_{j=1}^n h_j |b_{ij}| \right\} e_i^2(t)
\end{aligned}$$

$$+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}h_j|b_{ij}|e_j^2(t-\tau_j)$$
$$\leqslant -\min_{1\leqslant i\leqslant n}\left\{2(c_i+k_i)-\sum_{j=1}^{n}l_j|a_{ij}|-\sum_{j=1}^{n}l_i|a_{ji}|-\sum_{j=1}^{n}h_j|b_{ij}|\right\}V(t)$$
$$+\sum_{j=1}^{n}\sum_{i=1}^{n}h_j|b_{ij}|V(t-\tau_j). \tag{10}$$

Let

$$\lambda=\min_{1\leqslant i\leqslant n}\left\{2(c_i+k_i)-\sum_{j=1}^{n}l_j|a_{ij}|-\sum_{j=1}^{n}l_i|a_{ji}|-\sum_{j=1}^{n}h_j|b_{ij}|\right\},$$
$$b_j=\sum_{i=1}^{n}h_j|b_{ij}|,$$

According to Lemma 2, when $\lambda > (\sqrt{2}/2)\sum_{j=1}^{n}b_j$, system (1) synchronizes system (2). □

For convenience, we rewrite (10) as

$$D^\alpha V(t)\leqslant \lambda V(t)+\sum_{j=1}^{n}LV(t-\tau_j), \tag{11}$$

where $L=\max\{b_j\}$.

According to Theorem 1, the following corollary holds.

Corollary 1. *Under Assumption* 1 *and since the control function satisfies* (5)*, if the controller feedback gain* k_i *satisfies the inequality*

$$\lambda>\frac{\sqrt{2}}{2}\sum_{j=1}^{n}L, \tag{12}$$

then the fractional-order delayed neural networks system (1) *synchronizes system* (2).

Remark 1. If $\tau_1=\tau_2=\cdots=\tau_j=\tau$, the model will reduce to system with single time delay.

Remark 2. Here, we have used comparison principles of fractional-order couple system with multiple time delays. Some synchronization results of fractional-order neural networks with multiple time delays are derived. The method is novel.

Remark 3. Generally, the differential models with multiple time delays unavoidable exist in neural networks. However, the research of synchronization of fractional-order neural networks with multiple time delays has seldom seen. A comparative study reveals that our results are with multiple time delays. Hence, the results obtained in this paper are more general than the existing results dealing with the single time delay [3–5, 10, 12].

4 Numerical simulation

We now consider the following two-dimensional fractional-order neural networks with two time delays as drive system:

$$\begin{aligned} D^\alpha x_1(t) &= -c_1 x_1 + a_{11} f_1\big(x_1(t)\big) + a_{12} f_2\big(x_2(t)\big) + b_{11} g_1\big(x_1(t-\tau_1)\big) \\ &\quad + b_{12} g_2\big(x_1(t-\tau_2)\big) + I_1, \\ D^\alpha x_2(t) &= -c_2 x_2 + a_{21} f_1\big(x_1(t)\big) + a_{22} f_2\big(x_2(t)\big) + b_{21} g_1\big(x_1(t-\tau_1)\big) \\ &\quad + b_{22} g_2\big(x_1(t-\tau_2)\big) + I_2 \end{aligned} \tag{13}$$

with $\alpha = 0.92$, $c_1 = c_2 = 1$, $a_{11} = 2.0$, $a_{12} = -0.1$, $a_{21} = -5.0$, $a_{22} = 2.0$, $b_{11} = -1.5$, $b_{12} = -0.1$, $b_{21} = -0.2$, $b_{11} = -1.5$, $I_1 = I_2 = 0$, $\tau_1 = 1$, $\tau_2 = 2$, $f_i(s) = g_i(s) = \tanh(s)$ for $s \in \mathbb{R}$. Obviously, $f_i(x_i)$ and $g_i(x_i)$ satisfy Assumption 1 with $l_i = h_i = 1$.

The response system is given as follows:

$$\begin{aligned} D^\alpha y_1(t) &= -c_1 y_1 + a_{11} f_1\big(y_1(t)\big) + a_{12} f_2\big(y_2(t)\big) + b_{11} g_1\big(y_1(t-\tau_1)\big) \\ &\quad + b_{12} g_2\big(y_1(t-\tau_2)\big) + I_1 - k_1(y_1 - x_1), \\ D^\alpha y_2(t) &= -c_2 y_2 + a_{21} f_1\big(y_1(t)\big) + a_{22} f_2\big(y_2(t)\big) + b_{21} g_1\big(y_1(t-\tau_1)\big) \\ &\quad + b_{22} g_2\big(y_1(t-\tau_2)\big) + I_2 - k_2(y_2 - x_2). \end{aligned} \tag{14}$$

If we select the control gain $k_1 = k_2 = 11$, by simple computing, the condition of Theorem 1 is satisfied. We denote the initial values of state vector $x_1(t)$, $x_2(t)$, $y_1(t)$, $y_2(t)$ of the master-slave systems as follows: $\tilde{x}_1(t)$, $\tilde{x}_2(t)$, $\tilde{y}_1(t)$, $\tilde{y}_2(t)$, select the initial values in such a way that when $t \in [-2, 0]$, $\tilde{x}_1(t) = 10\tanh(\pi(t+2)/2)$, $\tilde{x}_2(t) = 10\cos(\pi(t+2)/2)$, $\tilde{y}_1(t) = 10(\pi(t+2)/2)$, $\tilde{y}_2(t) = 10(\pi(t+2)/2)$. Under these parameters, the state trajectories of system without controller and with the control gain are shown in Fig. 1. The synchronization errors are shown in Fig. 2. The state synchronization trajectories of master-slave systems are shown in Fig. 3.

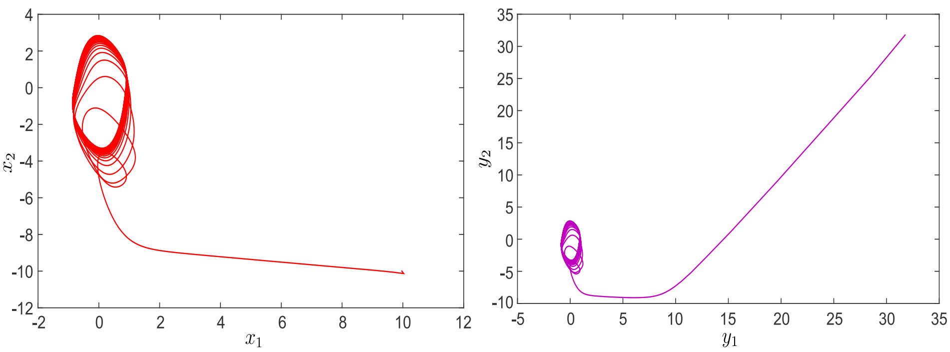

Figure 1. Trajectories of the system without controller (13) and with the control gain (14).

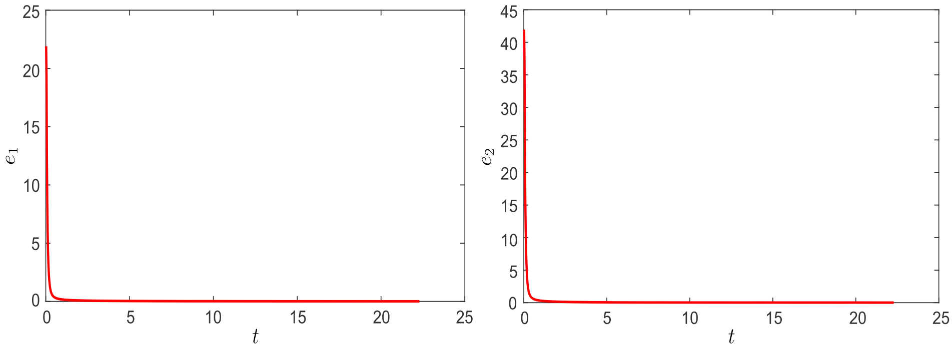

Figure 2. Synchronization error trajectories of e_i, $i = 1, 2$.

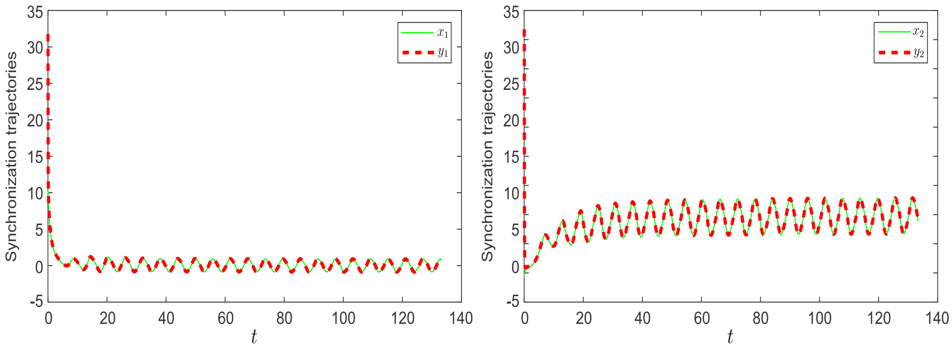

Figure 3. State synchronization trajectories of x_i, y_i, $i = 1, 2$.

5 Conclusions

Here, we investigated the synchronization of fractional-order neural networks with multiple time delays. By using Lyapunov function and the comparison principles of linear fractional equation with multiple time delays, some sufficient conditions are derived to ensure the synchronization of the master-slave systems. A numerical example is presented to verify the effectiveness of the theoretical results.

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