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A wire transducer in a system with a van der Pol oscillator and velocity feedback*

Grzegorz Cieplok

AGH University of Science and Technology,
al. Mickiewicza 30, Krakow, Poland
cieplok@agh.edu.pl

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Abstract. The work concerns wire transducers, which use a system with a van der Pol oscillator with the aim of maintaining nondecreasing natural vibrations in the wire. As opposed to classical solutions, in which the feedback signal of the oscillator contains the course of displacement and velocity of the vibrating mass, a simple solution based only on the course of velocity is used. Such a solution is more advantageous from a practical perspective as regards to physical systems because, in the case of velocity transducers, it eliminates the need to integrate the signal and the problems connected with it.

Particular places for this solution may be found in wire tensometer systems designated for the long-term constant measurement, as well as for the measurement of time-variable courses, including those of a chaotic character.

In the work, analysis was conducted on a modified van der Pol equation adapted for the movement of a discrete mass for the determination conditions of the existence of a limit cycle, the vibration course, as well as the definition of the capabilities of adapting the results in the case of a wire transducer.

The results of theoretical analysis were confirmed by the results of the experimental tests conducted on a laboratory model constructed for this purpose.

Keywords: wire transducer, van der Pol equation, self-oscillation, perturbation method.

1 Introduction

The van der Pol equation

$$\ddot{x} - \epsilon\big(1 - x^2\big)\dot{x} + x = 0, \tag{1}$$

presented in scientific literature, represents a great example for illustrating widely in nature occurring self-oscillation. Its original form and its later modifications and generalizations, among which the van der Pol–Rayleigh equation should be mentioned, have been and are still a subject to numerous analysis.

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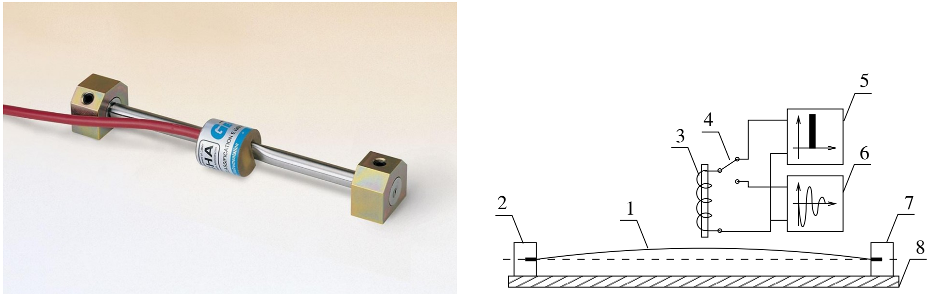


(a) Photograph of a transducer (made available by Geokon).

(b) Principles of action. 1 – wire, 2,7 – wire terminal, 3 – electromagnet, 4 – switch, 5 – impulsator, 6 – analyzer, 8 – base.

Figure 1. Wire transducer.

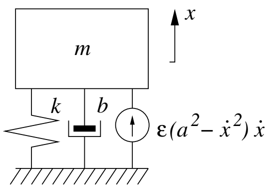


Figure 2. Model of the mass with an actuator realizing a component of the van der Pol equation.

The equation has become the object of interest of physicists, electronic engineers, biologists, neurologists, sociologists, and economists as a basic model for describing oscillation processes. In electronics, the equation has become a foundation for the development of radio and telecommunications. In medicine, it has been used to model the action of the heart [13, 14, 19], in economics for economic cycles [3, 11, 17], and in geotechnics, it has helped to describe the mutual interaction of tectonic plates [2]. The application of a system of coupled oscillators [7] was used to recreate the action of CPGs (Central Pattern Generators – a sub-system of the central nervous system) in animals [6, 12] responsible for the generation of rhythmic movement, including the supervision of walking. On this basis, automatic solutions for mobile robots have been created [8, 9], as have appliances used during the physiotherapy of accident victims.

A particular role in the successful analysis of the equation has been played by the perturbation method [10, 16].

The idea of maintaining self-oscillating vibrations in a mechanical system with the application of the van der Pol oscillator was the inspiration for the author to conduct research into the wire transducer designated for the continuous measurement of displacement, including fast-changing displacement. When using traditional wire sensors (Fig. 1), measurements of this type represent a problem excited to vibrations by cyclical jerking of the wire with an impulse of force generated by an electromagnet [15].

Before, however, the van der Pol oscillator was used for driving vibrations in wire transducer, a discrete system presented in Fig. 2 was analyzed. In this case, equation (1) provides meaning to the equation of motion of a mass mounted on an elastic suspension excited into vibration with the help of an actuator realizing a nonlinear component of the van der Pol equation $\epsilon(a^2 - \dot{x}^2)\dot{x}$.

Completing equation (1) with physical parameters, in accordance with Fig. 2, we obtain

$$m\ddot{x} - \epsilon\bigl(a^2 - \dot{x}^2\bigr)\dot{x} + b\dot{x} + kx = 0. \tag{2}$$

In contrast with the original equation, it contains a nonlinear component dependent only on the velocity of the vibrating mass motion, as well as an additional component presenting damping force. From the application perspective, these changes are significant for the functioning of a wire transducer because the velocity signal directly obtained from the clamp of the electromagnet may be used, without the need for integrating, to generate the force realizing the van der Pol component. In turn, taking damping into consideration provides the opportunity to research the influence of friction on the vibration amplitude or to check its influence on the existence of a limit cycle.

2 Solution of the van der Pol equation with velocity feedback

The solution of equation (2) may be conducted using perturbation methods in the variant formulated by Bogoliubov [1, 10]. With this aim, it was transformed into a normal form

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} v \\ x \end{bmatrix} = \begin{bmatrix} \frac{\epsilon}{m}(a^2 - v^2)v - \frac{b}{m}v - \frac{k}{m}x \\ v \end{bmatrix}, \tag{3}$$

in which the unknown quantities were velocity v of mass m, as well as its location x. The van der Pol transformation [18] allows system (3) to be shaped into a standard form of perturbation methods. With this goal, as a solution of the nondisturbed system, for which $\epsilon = 0$ and $b = 0$, the following dependency was assumed:

$$\begin{bmatrix} v_0 \\ x_0 \end{bmatrix} = \begin{bmatrix} A\omega\cos(\omega t + \varphi_0) \\ A\sin(\omega t + \varphi_0) \end{bmatrix}. \tag{4}$$

It is possible to concisely define the influence of variations in the constant of integration A and φ_0 on the assumed solution, that is,

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} v_0(t,A,\varphi_0) \\ x_0(t,A,\varphi_0) \end{bmatrix} &= \frac{\partial}{\partial t}\begin{bmatrix} v_0(t,A,\varphi_0) \\ x_0(t,A,\varphi_0) \end{bmatrix} + \frac{\partial}{\partial A}\begin{bmatrix} v_0(t,A,\varphi_0) \\ x_0(t,A,\varphi_0) \end{bmatrix}\dot{A} \\ &\quad + \frac{\partial}{\partial \varphi_0}\begin{bmatrix} v_0(t,A,\varphi_0) \\ x_0(t,A,\varphi_0) \end{bmatrix}\dot{\varphi}_0. \end{aligned} \tag{5}$$

In order to assure a more distinct form of further transformations (at a simultaneous increasing generality of considerations), equation (3) can be presented in a form

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} v \\ x \end{bmatrix} = \begin{bmatrix} f_1(x,v,\epsilon,b) \\ f_2(x,v,\epsilon,b) \end{bmatrix}. \tag{6}$$

Basing on the expansion of the right member of the equation into the Taylor series in surroundings of $\epsilon = 0$ and $b = 0$

$$\begin{bmatrix} f_1(x,v,\epsilon,b) \\ f_2(x,v,\epsilon,b) \end{bmatrix} = \begin{bmatrix} f_1(x_0,v_0,0,0) \\ f_2(x_0,v_0,0,0) \end{bmatrix} + \epsilon \frac{\partial}{\partial \epsilon} \begin{bmatrix} f_1(x,v,\epsilon,b) \\ f_2(x,v,\epsilon,b) \end{bmatrix} \Bigg|_{\substack{\epsilon=0 \\ b=0}} + b \frac{\partial}{\partial b} \begin{bmatrix} f_1(x,v,\epsilon,b) \\ f_2(x,v,\epsilon,b) \end{bmatrix} \Bigg|_{\substack{\epsilon=0 \\ b=0}} + \cdots, \tag{7}$$

it is possible, by means of comparing right members of equations (5) and (7) and omitting small terms of higher orders, to obtain the dependence

$$\underbrace{\frac{\partial}{\partial t} \begin{bmatrix} v_0(t,A_0,\varphi_0) \\ x_0(t,A_0,\varphi_0) \end{bmatrix}}_{[A]} + \frac{\partial}{\partial A_0} \begin{bmatrix} v_0(t,A_0,\varphi_0) \\ x_0(t,A_0,\varphi_0) \end{bmatrix} \dot{A}_0 + \frac{\partial}{\partial \varphi_0} \begin{bmatrix} v_0(t,A_0,\varphi_0) \\ x_0(t,A_0,\varphi_0) \end{bmatrix} \dot{\varphi}_0 = \underbrace{\begin{bmatrix} f_1(x_0,v_0,0,0) \\ f_2(x_0,v_0,0,0) \end{bmatrix}}_{[B]} + \epsilon \frac{\partial}{\partial \epsilon} \begin{bmatrix} f_1(x,v,\epsilon,b) \\ f_2(x,v,\epsilon,b) \end{bmatrix} \Bigg|_{\substack{\epsilon=0 \\ b=0}} + b \frac{\partial}{\partial b} \begin{bmatrix} f_1(x,v,\epsilon,b) \\ f_2(x,v,\epsilon,b) \end{bmatrix} \Bigg|_{\substack{\epsilon=0 \\ b=0}}. \tag{8}$$

This dependence allows to determine integration constants $A(t)$, $\varphi_0(t)$, which, substituted to the solution of the undisturbed equation (4), will present the disturbed equation solution (6). Taking into account the equality of matrices $[A]$ and $[B]$, we can finally derive equation

$$\begin{bmatrix} \dot{A} \\ \dot{\varphi_0} \end{bmatrix} = \begin{bmatrix} \frac{\partial v_0}{\partial A} & \frac{\partial v_0}{\partial \varphi_0} \\ \frac{\partial x_0}{\partial A} & \frac{\partial x_0}{\partial \varphi_0} \end{bmatrix}^{-1} \left\{ \epsilon \frac{\partial}{\partial \epsilon} \begin{bmatrix} f_1(x,v,\epsilon,b) \\ f_2(x,v,\epsilon,b) \end{bmatrix} \Bigg|_{\substack{\epsilon=0 \\ b=0}} + b \frac{\partial}{\partial b} \begin{bmatrix} f_1(x,v,\epsilon,b) \\ f_2(x,v,\epsilon,b) \end{bmatrix} \Bigg|_{\substack{\epsilon=0 \\ b=0}} \right\}. \tag{9}$$

In the physical system described by equations (3), we are able, after introducing dependencies for v_0 and x_0 into (9), to obtain

$$\begin{bmatrix} \dot{A} \\ \dot{\phi}_0 \end{bmatrix} = \frac{1}{A\omega} \begin{bmatrix} A\cos(\omega t + \varphi_0) & A\omega \sin(\omega t + \varphi_0) \\ -\sin(\omega t + \varphi_0) & \omega \cos(\omega t + \varphi_0) \end{bmatrix} \begin{bmatrix} \frac{\epsilon}{m}(a^2 - v_0^2)v_0 - \frac{b}{m}v_0 \\ 0 \end{bmatrix}, \tag{10}$$

and further

$$\dot{A} = -\frac{A^3\epsilon\omega^2\cos^4(\omega t + \varphi_0) + A(b - a^2\epsilon)\cos^2(\omega t + \varphi_0)}{m}, \tag{11a}$$

$$\dot{\varphi_0} = \frac{[A^2\epsilon\omega^2\cos^3(\omega t + \varphi_0) + (b - a^2\epsilon)\cos(\omega t + \varphi_0)]\sin(\omega t + \varphi_0)}{m}. \tag{11b}$$

It is now possible to define the average amplitude and vibration phase. Thanks to the oscillating character of the variable x, the averaging can be narrowed to the period of vibration. Designating respectively by $g_1(t)$ and $g_2(t)$ the right-hand side of the dependency (11), we can write

$$\dot{A} = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} g_1(t)\,\mathrm{d}t = -\frac{3\epsilon\omega^2 A^3 + 4(b - a^2\epsilon)A}{8m}, \tag{12a}$$

$$\dot{\varphi}_0 = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} g_2(t)\,\mathrm{d}t = 0. \tag{12b}$$

As a result of the averaging, we obtain two independent differential equations

$$-\frac{m\,\mathrm{d}A}{\frac{3}{8}\epsilon\omega^2 A^3 + \frac{1}{2}(b - a^2\epsilon)A} = \mathrm{d}t, \tag{13a}$$

$$\frac{\mathrm{d}\varphi_0}{\mathrm{d}t} = 0, \tag{13b}$$

which can be integrated thanks to the opportunity of the variable separation method. For equation (13a), we have:

$$-\int \frac{m}{\frac{3}{8}\epsilon\omega^2 A^3 + \frac{1}{2}(b - a^2\epsilon)A}\,\mathrm{d}A = \int \mathrm{d}t \tag{14a}$$

and after integrating:

$$\ln \frac{3\epsilon\omega^2 A^2 + 4(b - a^2\epsilon)}{A^2} + \ln C_1 = \frac{b - a^2\epsilon}{m} t. \tag{14b}$$

For equation (13b), we obtain

$$\varphi_0 = \mathrm{const.} \tag{15}$$

In this way, we can designate a formula

$$A = 2\sqrt{\frac{\epsilon a^2 - b}{3\omega^2\epsilon - \frac{1}{C_1}\mathrm{e}^{-(\epsilon a^2 - b)t/m}}} \tag{16}$$

describing the course of vibration amplitude from the values of the initially-defined constant of integration C_1 to the values set defined by the formula

$$A_{x_{\mathrm{cykl}}} = \frac{2}{\omega\sqrt{3}}\sqrt{a^2 - \frac{b}{\epsilon}}, \tag{17}$$

as well as the formula of the vibration velocity amplitude

$$A_{v_{\mathrm{cykl}}} = \frac{2}{\sqrt{3}}\sqrt{a^2 - \frac{b}{\epsilon}}. \tag{18}$$

To allow the dynamics of the transition processes to be characterized, the constant of time T_{time} was defined by the formula

$$T_{\text{time}} = \frac{m}{\epsilon a^2 - b}. \tag{19}$$

In order to quantify the influence of the damping coefficient b, the vibration reduction coefficient p, expressing the relation between the vibration amplitude of a damped system and that of a nondamped system, was defined:

$$p = \sqrt{1 - \frac{b}{\epsilon a^2}} \tag{20}$$

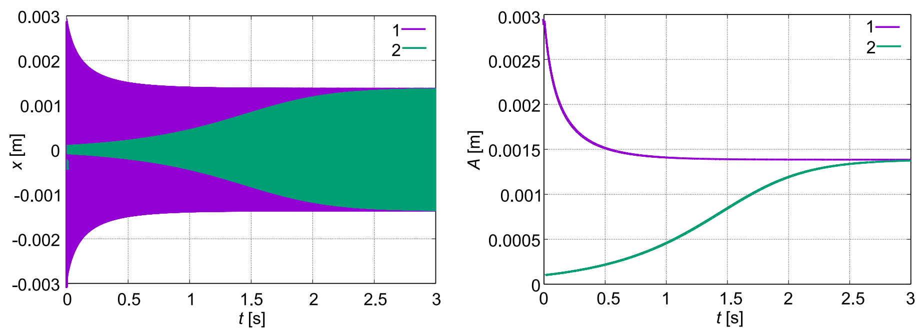
As can be seen, the course of the variable x reaches a set state with a nonzero vibration amplitude with the lack of external harmonic force stimulation, which allows this motion to be qualified as self-oscillation. As opposed to vibrations described by equation (1), the limit cycle may develop only after crossing a certain value of critical reinforcement ϵ_{crit}. This value, in accordance with formula (20), equals:

$$\epsilon_{\text{crit}} = \frac{b}{a^2}. \tag{21}$$

With the aim of presenting the obtained results, computer simulations of equations (3) were conducted. The results for sample physical parameters (Table 1) are presented in Fig. 3.

Table 1. Parameters used in computer simulations.

m	$= 5.43 \cdot 10^{-4}$	[kg]
b	$= 5.71 \cdot 10^{-4}$	[Ns/m]
k	$= 1.44 \cdot 10^{+4}$	[N/m]
a	$= 7.14$	[m/s]
ϵ	$= 4.48 \cdot 10^{-5}$	[Ns^3/m^3]



(a) Computer simulation results. Time chart. (b) Analitycal results. Time chart.

Figure 3. The course of the variable x for two various values of C_1 with reference to amplitude A defined by formula (16). $1/C_1 = 2805.1$ Ns/m^3 – line 1; $1/C_1 = -6.8106 \cdot 10^{+5}$ Ns/m^3 –line 2.

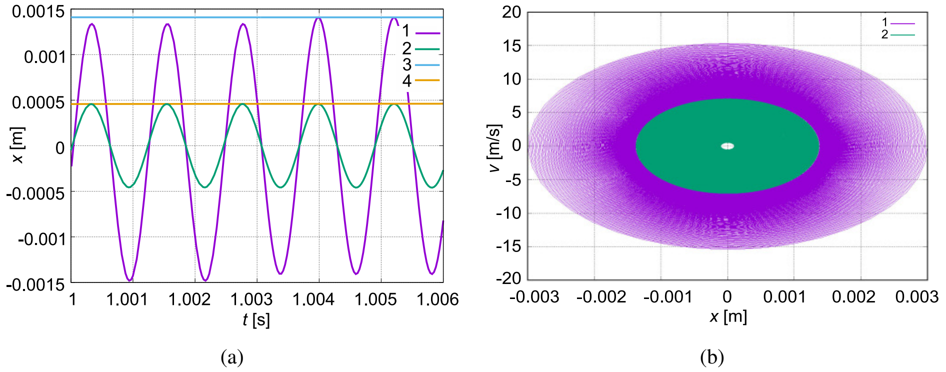


Figure 4. (a) Comparison of the results presented in Fig. 3(a) (lines 1 and 2) and Fig. 3(b) (lines 3 and 4). (b) Phase chart.

As can be observed, the graphs designated on the basis of formula (16) are in accordance with the results of the computer simulations, both in the description of the set state and transition process phases. Figure 4(b) presents the existence of a limit cycle.

3 Adaptation of the idea to the case of a wire transducer

The wire of a wire transducer tied between two terminals behaves like a body with a continuous distribution of mass. Stimulated with the help of an external force, it may be introduced into vibrations with various harmonic participation. However, as simulation and laboratory tests have shown [4], a wire, stimulated by a force generated in accordance with the formula of the nonlinear component of the van der Pol equation placed in its centre, vibrates with the frequency of the first mode. The wire is also especially resistant to the formation of undesired transition states, which appear as a result of violent changes in its tension, which takes place e.g. during the installation of gauges on constructions subject to violent deformation.

In order to transfer the theoretical dependencies introduced in the previous section to the case of a wire, it is possible to compare the discrete model presented in Fig. 2 with a model of a replacement wire designated for the particular mode of vibrations.

Let us examine the model presented in Fig. 5.

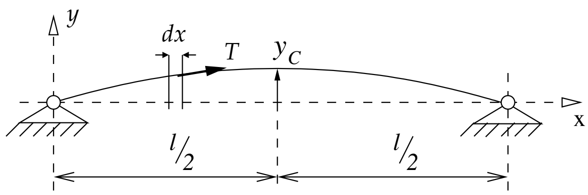


Figure 5. Wire model.

Assuming a sinusoidal form of vibration

$$y(x,t) = y_C(t) \cdot \sin\frac{\pi}{l}x, \tag{22}$$

it is possible to designate the kinetic energy E and potential energy V stored in the wire:

$$E = \frac{1}{2}\rho S \int\limits_0^l \dot{y}^2 \,\mathrm{d}t = \frac{1}{4}m_s\dot{y}_C^2, \tag{23}$$

$$V = \frac{1}{2}T \int\limits_0^l \left(\frac{\partial y}{\partial x}\right)^2 \mathrm{d}x = \frac{T\pi^2}{4l}y_C^2. \tag{24}$$

This energy also representing the system of a discrete harmonic oscillator is described by the following equation:

$$\frac{1}{2}m_s\ddot{y_C} + \frac{T\pi^2}{2l}y_C = 0, \tag{25}$$

in which the mass, the coefficient of elasticity, and vibration frequency equal respectively:

$$m_* = \frac{m_s}{2} = \frac{\rho Sl}{2}, \qquad k_* = \frac{T\pi^2}{2l}, \qquad \omega_0 = \frac{\pi}{l}\sqrt{\frac{T}{\rho S}}. \tag{26}$$

Comparing equation (25) with the dependencies (17) and (18), it is possible to define a formula for the vibration amplitude and wire velocity:

$$A_{x_{\mathrm{cykl}}} = \frac{2l}{\pi\sqrt{3}}\sqrt{\frac{\rho S}{T}}\sqrt{a^2 - \frac{b}{\epsilon}}, \tag{27}$$

$$A_{v_{\mathrm{cykl}}} = \frac{2}{\sqrt{3}}\sqrt{a^2 - \frac{b}{\epsilon}}. \tag{28}$$

Parameters b and ϵ occurring in equations (27) and (28) should be substituted by their wire equivalents. Parameter b can be determined on the bases of comparing the work of the discrete system damping force W_b (Fig. 2) with the work of the resistance overcame by wire W_s (determined by a linear damping β [$\mathrm{Ns/m^2}$]).

Assuming the motion of the mid-point of the wire as sinusoidal

$$y_C = A\sin(\omega t), \tag{29}$$

we can write

$$W_b = \int\limits_{t=0}^{T} -b\dot{y}_C^2 \,\mathrm{d}t = -\pi bA^2\omega, \tag{30}$$

$$W_s = \int\limits_{x=0}^{l}\int\limits_{t=0}^{T} -\beta\left[\dot{y}_C \sin\left(\frac{\pi}{l}x\right)\right]^2 \mathrm{d}t\,\mathrm{d}x = -\frac{\pi}{2}\beta lA^2\omega, \tag{31}$$

Table 2. Physical parameters of the wire.

$l = 0.275$	[m]
$D = 0.0008$	[m]
$\rho = 7860$	[kg/m^3]

Table 3. Parameters used in computer simulations.

$a^2 = 1.0$	[m^2/s^2]
$\epsilon_1 = 2.0 \cdot 10^{-3}$	[Ns^3/m^3]
$\epsilon_2 = 8.0 \cdot 10^{-4}$	[Ns^3/m^3]

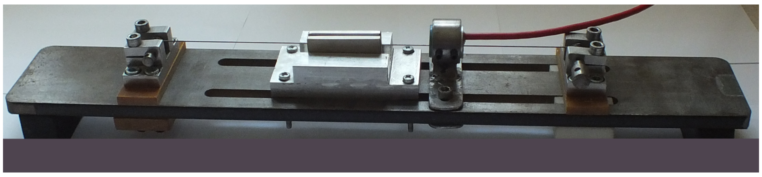


Figure 6. Photograph of a wire transducer.

which provides

$$b = \frac{1}{2}\beta l. \tag{32}$$

Parameter ϵ depends on the exact technical solution of the actuator realising the van der Pol component. The way of determining the equation for the substitute value of parameter ϵ for the actuator applied in experimental investigations is given in Section 4.

The condition of self-oscillation of the limit cycle (21) remains unchanged.

In order to picture the influence of reinforcement on the appearance of limit cycles, computer simulations of equation (25) were conducted. The simulations were based on the physical parameters of the real wire transducer (Table 2, Fig. 6) performed in the research unit for the needs of verification of experiments.

On their basis, the mass of the replacement model was designated, which, in accordance with formula (26), equaled:

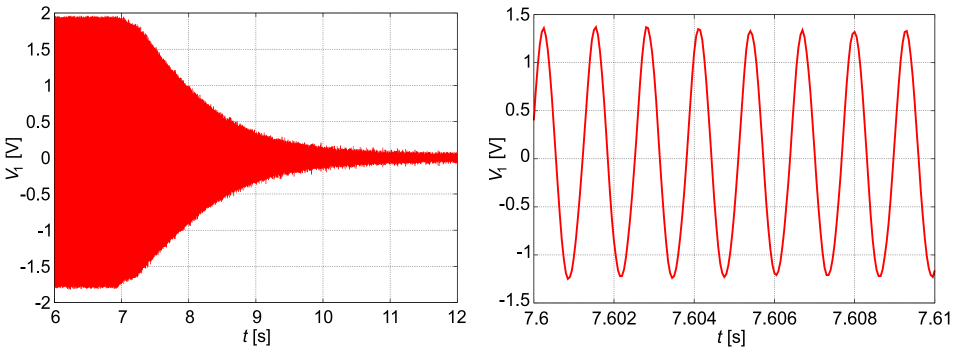
$$m_* = \frac{1}{2}\rho S l = 5.43 \cdot 10^{-5}\ \text{kg}. \tag{33}$$

In order to designate the other replacement parameters, the course of natural vibrations of the wire's mid-point was used (Fig. 7). Based on the logarithmic decrement of damping, the damping coefficient b_* and, in turn, on the basis of vibration frequency, the coefficient of elasticity k_* were designated:

$$b_* = \frac{2m_* \ln \frac{A_1}{A_{nT}}}{nT} = 0.001\ \frac{\text{Ns}}{\text{m}}, \tag{34}$$

$$k_* = m_*(2\pi f_0)^2 = 1.28 \cdot 10^{+4}\ \frac{\text{N}}{\text{m}}. \tag{35}$$

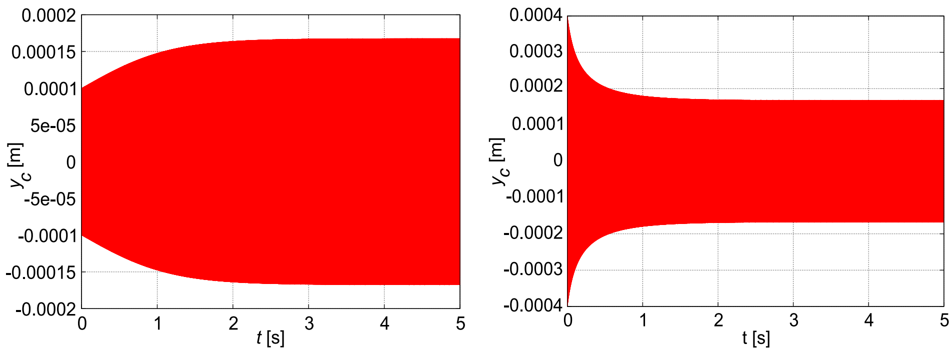
Figure 8 presents vibration of the replacement mass m_* for two various values of reinforcement ϵ and a constant value of the parameter a^2 (Table 3). Reinforcement ϵ_1, as well as ϵ_2 equaled 200% and 80% of the critical value, respectively. In the case of simulations 8(a) and 8(b), reinforcement ensured the maintenance of a limit cycle independent of the initial state. In the case of simulation 8(d), a wire vibration decreased until it finally disappeared completely, thus confirming the suitability of the formula for the existence of critical reinforcement.



(a) Voltage course in the electromagnet coil.

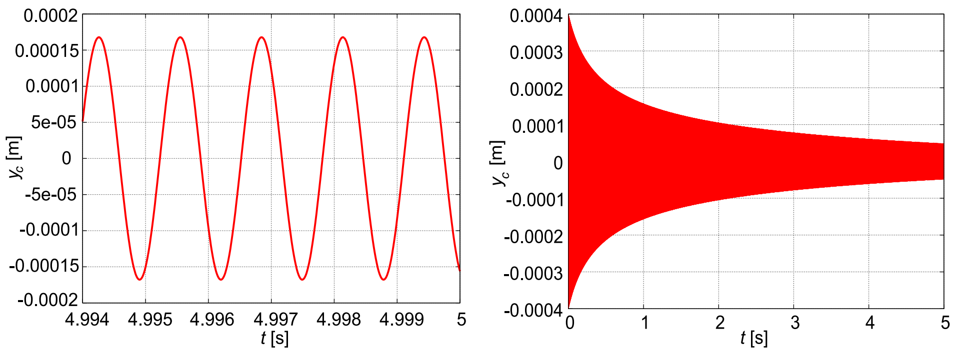
(b) Enlargement. Vibration frequency $f_0 = 773.48$ Hz.

Figure 7. Course of natural vibrations of the wire's mid-point.



(a) Course for ϵ_1 and initial conditions contained inside the limit cycle.

(b) Course for ϵ_1 and initial conditions outside the limit cycle.



(c) Enlargement of Fig.8(a).

(d) Course for ϵ_2 and initial conditions lying outside the limit cycle.

Figure 8. Courses of vibration of the reduced mass m_* in a system with a van der Pol oscillator.

4 Experiments

In order to verify the theoretical results, the experiments were conducted. The wire presented in Fig. 6 was excited using a Lorentz force created as a result of the interaction of the magnetic fields of a constant magnet (located in the middle of the wire) and current flowing through the wire. The current was formed in an electronic summing-multiplying system, it is schematically presented in Fig. 9. The input signal was the electric voltage induced in the electromagnetic transducer proportional to the vibration velocity at the mid-point of the wire. A detailed description of the experimental workstation can be found in the work [5].

The results of the experimental tests are presented in Fig. 10. In the first of them, the courses of vibration velocity in the wire registered in the output of the electric voltage amplifier at the measurement point U1 can be seen. These recordings are the basis for designating the characteristics connecting the wire's vibration amplitude with the parameter a^2 (Fig. 10(b)). Based on figure, it can be stated that the experimental results are in accordance with the theoretical formula (18), and also that there is a point of the extrapolated curve that designates the critical value of the parameter a^2. In Fig. 10(c), the start of the wire is presented beginning with the stopped state until the set working state. The signal with a nonzero value visible in the beginning phase of the recordings represents the noise of the electromagnetic transducer (enlarged in Fig. 10(d)).

Having the technical solution of the force actuator it is possible, as it was shown in Section 3, to determine the wire equivalent of parameter ϵ. In this case, the work $W_{\epsilon-mbk}$ of the van der Pol component force in the discret system should be balanced with work the $W_{\epsilon-s}$ of the van der Pol component force in the system with the wire.

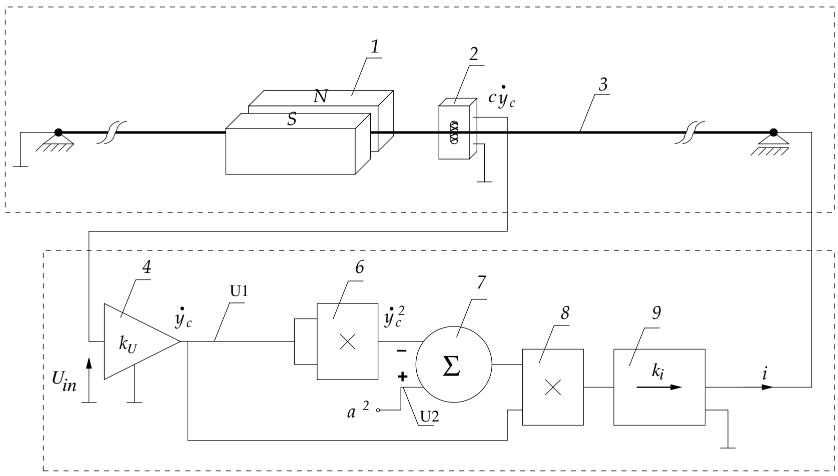
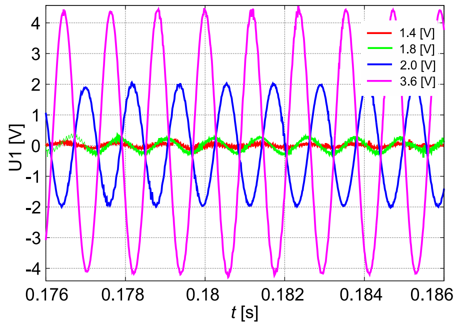
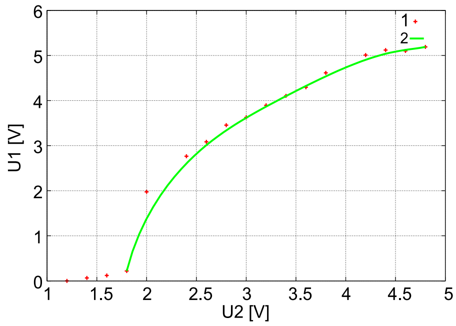


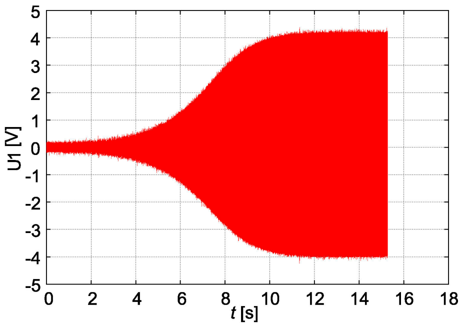
Figure 9. Plan of a wire transducer system with the van der Pol oscillator with a velocity feedback. 1 – electromagnet, 2 – magnetic pickup, 3 – wire, 4, 9 – amplifier, 6, 8 – multiplier, 7 – adder, U1, U2 – measuring point.



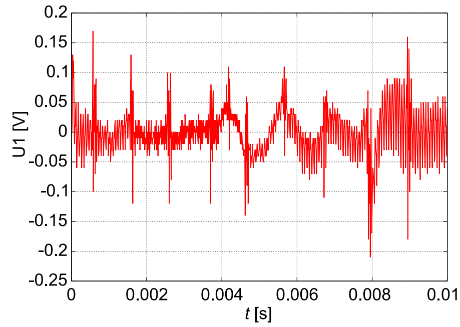
(a) Time chart.



(b) Vibration amplitude – a^2 dependence.



(c) Start.



(d) Noise.

Figure 10. Courses of vibrations at the mid-point of a wire for various values of the parameter a^2.

In the first case, we obtain:

$$W_{\epsilon-mbk} = \int_{t=0}^{T} \epsilon\left(a^2 - \dot{y}_C^2\right)\dot{y}_C \,\mathrm{d}t = -\frac{1}{4}\pi\epsilon A^2\omega\left(3A^2\omega^2 - 4a^2\right). \tag{36}$$

In the second case, additional calculations are needed. Basing on designations from Fig. 9, the dependence between the input voltage u_{in} and the actuator output current i can be written in a form

$$i = k_U k_i\left(a^2 - (k_U u_{\mathrm{in}})^2\right)u_{\mathrm{in}}, \tag{37}$$

where $u_{\mathrm{in}} = c \cdot \dot{y}_C$. The magnet interaction force, at omitting the wire curvature within the interaction range, can be determined as follows:

$$F = Bil^*, \tag{38}$$

where B – magnetic induction in the magnet clearance, l^* – wire segment of the efficient influence of the magnetic field. Work force in the period of the van der Pol component equals there:

$$W_{\epsilon\text{-}s} = \int\limits_0^T F\dot{y}_C\,\mathrm{d}t = -\frac{1}{4}BA^2ck_ik_Ul^*\omega\pi\bigl(3A^2c^2k_U^2\omega^2 - 4a^2\bigr). \tag{39}$$

Assuming

$$ck_U = 1\ \frac{\mathrm{Vs}}{\mathrm{m}}, \tag{40}$$

we obtain

$$W_{\epsilon\text{-}s} = -\frac{1}{4}BA^2k_il^*\omega\pi\bigl(3A\omega^2 - 4a^2\bigr). \tag{41}$$

Comparing equations (36) and (41), we finally determine

$$\epsilon = Bk_il^*\ \frac{\mathrm{Ns}^3}{\mathrm{m}^3}. \tag{42}$$

5 Conclusions

Based on the theoretical analysis and experimental tests conducted in the work, it is possible to state that the application of the van der Pol oscillator with a velocity feedback with the objective of maintaining nondecreasing natural vibrations, no matter if there is a discrete mass or a wire, is possible. Vibrations created in this way are resistant to the loss of motion stability caused by external factors and obtain a set state with constant vibration amplitude and frequency, regardless of the initial conditions.

The work presents the simple method for designating the required reinforcing of a system for maintaining vibrations, as well as a simple method for regulating the vibration amplitude. In addition, it was shown in [4] that wire properties, based on the fact that the transition processes of longitudinal vibrations take place significantly faster than the transition processes of transverse vibrations, may be used to build wire transducers for the continuous measurement of body deformations, including fast-changing deformations. The fast process of straining taking place along the wire transfers almost immediately to changes in the frequency of transverse vibrations of the wire without clear oscillation of undesirable transition states, through which it is possible to obtain, almost immediately, useful measurement signals representing the value of wire strain or body deformation.

The theoretical formulas introduced in Section 3 may be used in the design of a wire transducer in a system with the van der Pol oscillator and when combined with a discrete model of a replacement wire for simulating their dynamic properties.

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