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pth moment exponential stability of stochastic fuzzy Cohen–Grossberg neural networks with discrete and distributed delays*

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Abstract. In this paper, stochastic fuzzy Cohen–Grossberg neural networks with discrete and distributed delays are investigated. By using Lyapunov function and the Itô differential formula, some sufficient conditions for the pth moment exponential stability of such stochastic fuzzy Cohen–Grossberg neural networks with discrete and distributed delays are established. An example is given to illustrate the feasibility of our main theoretical findings. Finally, the paper ends with a brief conclusion. Methodology and achieved results is to be presented.

Keywords: stochastic fuzzy Cohen–Grossberg neural networks, global pth moment exponential stability, discrete delays, distributed delay, Itô differential formula.

1 Introduction

It is well known that Cohen–Grossberg neural networks have been widely applied in various fields such as signal processing, associative memory and optimization problems [6]. Many scholars argue that in these applications for neural networks, it is of prime importance to ensure that the designed neural networks are stable [26]. In hardware implementation, time delays inevitably occur due to the finite switching speed of the amplifiers and communication time. The qualitative research and analysis of Cohen–Grossberg neural networks with delays has been investigated by numerous authors. Much richer dynamics has been reported [20,21,23,55,58]. Considering that the synaptic transmission is a noisy process brought about by random fluctuations from the release of neurotransmitters and other probabilistic causes, we think that it is of great significance to consider stochastic effects on the stability of neural networks described by stochastic functional differential equations [6]. In recent years, numerous authors deal with the dynamical behavior of stochastic neural networks, see, e.g. [10, 11, 40, 60]. Since Yang and Yang [50] first

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introduced fuzzy cellular neural networks, a lot of scholars have found that fuzzy neural networks have important applications in image processing, and many results have been reported on stability and periodicity of fuzzy neural networks [1–5, 12, 14, 15, 18, 19, 22, 24, 25, 27, 28, 31, 33–37, 41, 42, 44, 45, 48, 50–52, 54, 56]. In addition, we shall point out that neural networks usually have a spatial nature due to the presence of an amount of parallel pathways of variety of axon sizes and length. A distribution of conduction velocities along these pathways will lead to a distribution of propagation delays. Thus, the time-varying delays and continuous distributed delays are more appropriate to fuzzy cellular networks [18, 19, 31, 44, 48]. To the best of our knowledge, there are very few papers that deal with the stability of stochastic fuzzy Cohen–Grossberg neural networks with discrete and distributed delays [9, 13, 17, 29, 30, 39, 49].

Inspired by the analysis above, in this paper, we consider the following stochastic fuzzy Cohen–Grossberg neural networks with discrete and distributed delays:

$$\begin{aligned}
\mathrm{d}x_i(t) = -a_i\big(x_i(t)\big)\Bigg[& b_i\big(x_i(t)\big) - \sum_{j=1}^{n} c_{ij}(t) f_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) \\
&- \bigwedge_{j=1}^{n} \alpha_{ij}(t) \int_{-\infty}^{t} K_{ij}(t-s) g_j\big(x_j(s)\big)\,\mathrm{d}s \\
&- \bigvee_{j=1}^{n} \beta_{ij}(t) \int_{-\infty}^{t} K_{ij}(t-s) g_j\big(x_j(s)\big)\,\mathrm{d}s + I_i(t)\Bigg]\,\mathrm{d}t \\
&+ \sum_{j=1}^{n} \sigma_{ij}\big(x_j(t)\big)\,\mathrm{d}\omega_j(t),
\end{aligned} \tag{1}$$

where n corresponds to the number of units in the neural networks, respectively, $x_i(t)$ corresponds to the state of the ith neuron, f_j and g_j are signal transmission functions, $\tau_{ij}(t)$ denotes the transmission delay along the axon of the jth unit from the ith unit and satisfies $0 \leqslant \tau_{ij}(t) \leqslant \tau_{ij}$($\tau_{ij}$ is a positive constant). $a_i(x_i(t))$ denotes an amplification function at time t, $b_i(x_i(t))$ is an appropriately behaved function at time t such that the solutions of model (1) remain bounded, $I_i(t) = \tilde{I}(t) + \bigwedge_{j=1}^{n} T_{ij}(t)u_j(t) + \bigvee_{j=1}^{n} H_{ij}(t)u_j(t)$. $\alpha_{ij}(t), \beta_{ij}(t), T_{ij}$ and $H_{ij}(t)$ are elements of fuzzy feedback MIN template and fuzzy feedback MAX template, fuzzy feed-forward MIN template and fuzzy feed-forward MAX template, respectively, $\bigwedge$ and $\bigvee$ stands for the fuzzy AND and fuzzy OR operation, respectively, $u_j(t)$ denotes the external input of the ith neurons. $\tilde{I}(t)$ is the external bias of ith unit. $K_{ij}(\cdot)$ is the delay kernel function, $\sigma_{ij}(\cdot)$ is the diffusion coefficient, $\sigma_i = (\sigma_{i1}, \sigma_{i2}, \dots, \sigma_{in})$, $\omega(t) = (\omega_1(t), \omega_2(t), \dots, \omega_n(t))^{\mathrm{T}}$ is an n-dimensional Brownian motion defined on a complete probability space $(\Omega, F, \{F_t\}_{t\geqslant 0}, \mathbf{P})$ with a filtration $\{F_t\}_{t\geqslant 0}$ satisfying the usual conditions (i.e., it is right continuous and F_0 contains all $\mathbf{P}$-null sets).

Here we would like to emphasize that pth moment exponential stability of stochastic delayed fuzzy neural networks plays an important role in biological and artificial neural networks. It can effectively portray the dynamics of neural networks [8, 16, 38, 38, 53, 59].

Thus, the research on pth moment exponential stability of stochastic delayed fuzzy neural networks has important practical meanings. In addition, we point out that the exponential stability in general sense and the pth moment exponential stability are different. The former is aimed at all differential equations, and the latter is aimed at stochastic differential equations. General speaking, a *stochastic differential equation is exponentially stable* traditionally implies a *stochastic differential equation is pth moment exponentially stable*. In particular, if $p = 2$, then we say that a stochastic differential equation is exponentially stable in mean square.

The key task of this article is to discuss the pth moment exponential stability of system (1). In recent years, there are many papers that deal with pth moment exponential stability of stochastic neural networks [32,43,46]. It is worth pointing out that most neural networks involve negative feedback terms or fuzzy terms and do not possess amplification functions, behaved functions and fuzzy terms. Model (1) of this paper has amplifications function and behaved functions, which differ from most neural networks with negative feedback term. Up to now, there are rare papers that consider pth moment exponential stability this kind of stochastic fuzzy neural networks.

The main advantages of this article consist of four aspects: (i) the study of pth moment exponential stability for stochastic delayed fuzzy Cohen–Grossberg neural networks with amplification functions and behaved functions is proposed; (ii) a set of new sufficient criteria that ensure the pth moment exponential stability of system (1) by using Lyapunov function and the Itô differential formula are established; (iii) the key ideas of this article are also suitable for handling some other similar stochastic fuzzy Cohen–Grossberg neural networks; (iv) to the best of our knowledge, it is the first time to deal with the pth moment exponential stability for stochastic delayed fuzzy Cohen–Grossberg neural networks with amplification functions, behaved functions and fuzzy terms.

The remainder of the paper is organized as follows: in Section 2, the basic definitions and lemmas are introduced. In Section 3, the sufficient condition for the pth moment ($p \geqslant 2$) exponential stability for system (1) is established by using the Lyapunov function method and Itô differential inequality. In Section 4, an illustrative example is given. A brief conclusion is drawn in Section 5.

2 Preliminaries

For convenience, we introduce some notations. Let $C = C([-\infty, 0], \mathbb{R}^n)$ be the Banach space of continuous function, which map into $\mathbb{R}^n$ with the topology of uniform convergence. For any $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^{\mathrm{T}} \in \mathbb{R}^n$, we define $\|x\| = \|x\|_p = (\sum_{i=1}^n |x_i(t)|^p)^{1/p}$ $(1 < p < \infty)$.

The initial conditions of system (1) are $x(s) = \varphi(s)$, $-\tau \leqslant s \leqslant 0$, $\varphi \in L_F^p((-\tau, 0], \mathbb{R}^n)$, where $L_F^p((-\tau, 0], \mathbb{R}^n)$ is $\mathbb{R}^n$-value stochastic process $\varphi(s)$, $-\tau \leqslant s \leqslant 0$, $\varphi(s)$ is F_0 measurable, $\int_{-\tau}^0 \mathbf{E}[|\varphi(s)|^p] \,\mathrm{d}s < \infty$.

Throughout this paper, we always make the following assumptions:

(H1) There exist positive constants $\underline{a}_i$ and $\bar{a}_i$ such that $0 < \underline{a}_i \leqslant a_i(x) \leqslant \bar{a}_i$ for $x \in \mathbb{R}$, $i = 1, 2, \ldots, n$.

(H2) $f_j(\cdot)$ and $g_j(\cdot)$ are Lipschitz continuous on $\mathbb{R}$ with Lipschitz constants L_j^f, L_j^g, $j = 1, 2, \dots, n$, i.e., for all $x, y \in \mathbb{R}$, one has

$$\left|f_j(x) - f_j(y)\right| \leqslant L_j^f |x - y|, \qquad \left|g_j(x) - g_j(y)\right| \leqslant L_j^g |x - y|.$$

(H3) $b_i(\cdot) \in C(\mathbb{R}, \mathbb{R})$ and there exist positive constants μ_i such that

$$\frac{b_i(u) - b_i(v)}{u - v} \geqslant \mu_i$$

for $u \neq v$, $i = 1, 2, \dots, n$.

(H4) $\sigma(x(t)) = (\sigma_{ij}(x_j(t)))_{n\times n}$ $(i, j = 1, 2, \dots, n)$, there exist nonnegative numbers ϱ_i $(i = 1, 2, \dots, n)$ such that $\operatorname{tr}[\sigma^{\mathrm{T}}(x)\sigma(x)] \leqslant \sum_{i=1}^n \varrho_i x_i^2$.

(H5) The delay kernel $K_{ij} : [0, +\infty) \to [0, +\infty)$ is a real-valued nonnegative continuous function and satisfies $\int_{-\infty}^t K_{ij}(t - s)\,\mathrm{d}s \leqslant \rho_{ij}$, where ρ_{ij} is a positive constant and $i, j = 1, 2, \dots, n$.

Let $C^{1,2}([-\tau, \infty)\times\mathbb{R}^n; \mathbb{R}^+)$ denote the family of all nonnegative functions $V(t, x)$ on $[-\tau, \infty) \times \mathbb{R}^n$, which are continuous once and differentiable in t and twice differentiable in x. If $V(t, x) \in C^{1,2}([-\tau, \infty) \times \mathbb{R}^n; \mathbb{R}^+)$, in view of the Itô formula, we define an operator LV associated with (1) as

$$\begin{aligned} LV(t, x) = {} & V_t(t, x) + \sum_{i=1}^n V_x(t, x)\Biggl\{-a_i(t)\Biggl[b_i(t) - \sum_{j=1}^n c_{ij}(t) f_j\bigl(x_j\bigl(t - \tau_{ij}(t)\bigr)\bigr) \\ & - \bigwedge_{j=1}^n \alpha_{ij}(t) \int_{-\infty}^t K_{ij}(t - s) g_j\bigl(x_j(s)\bigr)\,\mathrm{d}s \\ & - \bigvee_{j=1}^n \beta_{ij}(t) \int_{-\infty}^t K_{ij}(t - s) g_j\bigl(x_j(s)\bigr)\,\mathrm{d}s + I_i(t)\Biggr]\,\mathrm{d}t\Biggr\} \\ & + \frac{1}{2}\operatorname{tr}\bigl[\sigma^{\mathrm{T}} V_{xx}(t, x)\sigma\bigr], \end{aligned}$$

where

$$V_t(t, x) = \frac{\partial V(t, x)}{\partial t}, \qquad V_x(t, x) = \frac{\partial V(t, x)}{\partial x_i}, \qquad V_{xx}(t, x) = \left(\frac{\partial V(t, x)}{\partial x_i \partial x_j}\right)_{n\times n}.$$

Definition 1. The equilibrium x^* of system (1) is said to be global pth moment exponentially stable if there exist positive constants $M \geqslant 1$, $\lambda > 0$ such that

$$\mathbf{E}\bigl(\bigl\|x(t) - x^*\bigr\|^p\bigr) \leqslant M\|\varphi - x^*\|_L^p \mathrm{e}^{-\lambda(t - t_0)}, \quad t > t_0,\ \forall x_0 \in \mathbb{R}^n,$$

where $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^{\mathrm{T}}$ is any solution of system (1), $p \geqslant 2$ is a constant when $p = 2$, it is said to be exponential stability in mean square.

Lemma 1. (*See* [50].) *Let x and y be two states of system* (1). *Then*

$$\left|\bigwedge_{j=1}^{n}\alpha_{ij}(t)g_j(x)-\bigwedge_{j=1}^{n}\alpha_{ij}(t)g_j(y)\right|\leqslant\sum_{j=1}^{n}\left|\alpha_{ij}(t)\right|\left|g_j(x)-g_j(y)\right|,$$
$$\left|\bigvee_{j=1}^{n}\beta_{ij}(t)g_j(x)-\bigvee_{j=1}^{n}\beta_{ij}(t)g_j(y)\right|\leqslant\sum_{j=1}^{n}\left|\beta_{ij}(t)\right|\left|g_j(x)-g_j(y)\right|.$$

Lemma 2. (*See* [7].) *If $a_i>0$ ($i=1,2,\dots,n$), denote p^* nonnegative real numbers, then*

$$a_1a_2\cdots a_m\leqslant\frac{a_1^{p^*}+a_2^{p^*}+\dots+a_m^{p^*}}{p^*},$$

where $p^*\geqslant 1$ denotes an integer. A particular form of the above inequality is

$$a_1^{p^*-1}a_2\leqslant\frac{(p-1)a_1^{p^*}}{p^*}+\frac{a_2^{p^*}}{p^*}.$$

Lemma 3 [Hölder inequality]. (*See* [38].) *Let $f(x)$ and $g(x)$ be two continuous functions and Ω a set, a and b satisfy $1/b+1/a=1$ for any $a\geqslant 0, b\geqslant 0$ if $a>1$, then the following inequality holds*:

$$\int\limits_{\Omega}\left|f(x)g(x)\right|\mathrm{d}s\leqslant\left(\int\limits_{\Omega}\left|f(x)\right|^a\mathrm{d}s\right)^{1/a}\left(\int\limits_{\Omega}\left|g(x)\right|^b\mathrm{d}s\right)^{1/b}.$$

3 pth moment exponential stability

In this section, we shall present sufficient conditions for the global pth moment exponential stability of system (1).

Theorem 1. *Suppose that* (H1)–(H5) *and the following assumption hold true*:

(H6) *there exist a positive diagonal matrix $M=\operatorname{diag}(\theta_1,\theta_2,\dots,\theta_n)$ and two constants $0<\Pi_2, 0<u<1$ such that $0<\Pi_2\leqslant\Pi_2(t)\leqslant u\Pi_1(t)$, $t\geqslant t_0$, where*

$$\begin{aligned}\Pi_1(t)=\min_{1\leqslant i\leqslant n}\Biggl\{&p\underline{a}_i\mu_i-\sum_{j=1}^{n}\theta_i(p-1)\bar{a}_i\left|c_{ij}(t)\right|L_j^f\\&+\sum_{j=1}^{n}\theta_i\bar{a}_i\left|c_{ij}(t)\right|L_j^f\bigl((p-1)\bigr)\\&-\sum_{j=1}^{n}\theta_i(p-1)\bar{a}_i\bigl(\left|\alpha_{ij}(t)\right|+\left|\beta_{ij}(t)\right|\bigr)\rho_{ij}L_j^g\\&-\sum_{j=1}^{n}\frac{(p-1)(p-2)}{2}\varrho_j-\sum_{j=1}^{n}\frac{\theta_j}{\theta_i}(p-1)\varrho_i\Biggr\},\end{aligned}$$

$$\Pi_2(t) = \max_{1\leqslant i\leqslant n}\left\{\sum_{j=1}^{n}\frac{\theta_j}{\theta_i}\bar{a}_i\bigl|c_{ij}(t)\bigr|L_j^f\bigl((p-1)\bigr)\right\},$$

then $x^* = (x_1^*, x_2^*, \dots, x_n^*)^{\mathrm{T}}$ *is a unique equilibrium, which is globally pth moment exponentially stable, where* $p \geqslant 2$ *denotes a positive constant. When* $p = 2$*, the equilibrium* x^* *of system* (1) *has exponential stability in mean square.*

Proof. Similar to [47, 57], we can easily prove the existence and uniqueness of the equilibrium for system (1). Here we omit it.

Let $x^* = (x_1^*, x_2^*, \dots, x_n^*)^{\mathrm{T}}$ be the unique equilibrium of system (1). Set $y_i(t) = x_i(t) - x_i^*$, $\bar{\sigma}_{ij} = \sigma_{ij}(y_i(t) + x_j^*) - \sigma_{ij}(x_j^*)$. Then it follows from (1) that

$$\begin{aligned}
\mathrm{d}y_i(t) = {} & -a_i\bigl(y_i(t) + x_i^*\bigr)\Biggl[b_i\bigl(y_i(t) + x_i^*\bigr) - b_i(x_i^*) \\
& - \sum_{j=1}^{n} c_{ij}(t)\bigl(f_j(x_j\bigl(t - \tau_{ij}(t)\bigr)\bigr) - f_j(x_j^*)\bigr) \\
& - \bigwedge_{j=1}^{n}\alpha_{ij}(t)\int_{-\infty}^{t} K_{ij}(t-s)\bigl(g_j(x_j(s)\bigr) - g_j(x_j^*)\bigr)\,\mathrm{d}s \\
& - \bigvee_{j=1}^{n}\beta_{ij}(t)\int_{-\infty}^{t} K_{ij}(t-s)\bigl(g_j\bigl(x_j(s)\bigr) - g_j(x_j^*)\bigr)\,\mathrm{d}s\Biggr]\,\mathrm{d}t \\
& + \sum_{j=1}^{n}\bar{\sigma}_{ij}\bigl(y_j(t)\bigr)\,\mathrm{d}\omega_j(t), \quad t \geqslant t_0,\ i = 1, 2, \dots, n.
\end{aligned} \tag{2}$$

Define a Lyapunov function V by

$$V\bigl(t, y(t)\bigr) = \sum_{i=1}^{n}\theta_i\bigl|y_i(t)\bigr|^p = \sum_{i=1}^{n}\theta_i\bigl|x_i(t) - x_i^*\bigr|^p, \quad p \geqslant 2. \tag{3}$$

Calculating the operator $LV(t, y(t))$ and using Lemma 2 associated with system (2), we have

$$\begin{aligned}
LV\bigl(t, y(t)\bigr) = {} & p\sum_{i=1}^{n}\theta_i\bigl|y_i(t)\bigr|^{p-1}\operatorname{sgn}\bigl\{y_i(t)\bigr\}\Biggl\{-a_i\bigl(y_i(t) + x_i^*\bigr) \\
& \times\Biggl[b_i\bigl(y_i(t) + x_i^*\bigr) - b_i(x_i^*) - \sum_{j=1}^{n} c_{ij}(t)\bigl(f_j\bigl(x_j\bigl(t - \tau_{ij}(t)\bigr)\bigr) - f_j(x_j^*)\bigr) \\
& - \bigwedge_{j=1}^{n}\alpha_{ij}(t)\int_{-\infty}^{t} K_{ij}(t-s)\bigl(g_j(x_j(s)\bigr) - g_j(x_j^*)\bigr)\,\mathrm{d}s
\end{aligned}$$

$$
\begin{aligned}
&\quad - \bigvee_{j=1}^{n} \beta_{ij}(t) \int_{-\infty}^{t} K_{ij}(t-s)\big(g_j\big(x_j(s)\big) - g_j(x_j^*)\big)\,\mathrm{d}s\Bigg]\Bigg\} \\
&\quad + \frac{p(p-1)}{2} \sum_{i=1}^{n} \big|y_i(t)\big|^{p-2} \sum_{j=1}^{n} \bar{\sigma}_{ij}\big(y_i(t)\big) \\
&\leqslant -p \sum_{i=1}^{n} \theta_i \big|y_i(t)\big|^{p-1} a_i\big(y_i(t)+x_i^*\big)\mu_i y_i(t)\operatorname{sgn}\big\{y_i(t)\big\} \\
&\quad + p \sum_{i=1}^{n} \theta_i \big|y_i(t)\big|^{p-1} a_i\big(y_i(t)+x_i^*\big)\Bigg[\sum_{j=1}^{n} c_{ij}(t) f_j\big(y_j\big(t-\tau_{ij}(t)\big)\big) \\
&\quad \times \operatorname{sgn}\big\{y_i(t)\big\} + \sum_{j=1}^{n} \big|\alpha_{ij}(t)\big|\rho_{ij}\big|g_j\big(x_j(s)\big) - g_j(x_j^*)\big|\operatorname{sgn}\big\{y_i(t)\big\} \\
&\quad + \sum_{j=1}^{n} \big|\beta_{ij}(t)\big|\rho_{ij}\big|g_j\big(x_j(s)\big) - g_j(x_j^*)\big|\operatorname{sgn}\big\{y_i(t)\big\}\Bigg] \\
&\quad + \frac{p(p-1)}{2} \sum_{i=1}^{n} \theta_i \big|y_i(t)\big|^{p-2} \sum_{j=1}^{n} \sigma_{ij}^2 \operatorname{sgn}\big\{y_i(t)\big\} \\
&\leqslant -p \sum_{i=1}^{n} \theta_i \big|y_i(t)\big|^{p-1} \underline{a}_i \mu_i \big|y_i(t)\big| \\
&\quad + p \sum_{i=1}^{n} \theta_i \big|y_i(t)\big|^{p-1} \bar{a}_i \sum_{j=1}^{n} \big|c_{ij}(t)\big| L_j^f \big|y_i\big(t-\tau_{ij}(t)\big)\big| \\
&\quad + p \sum_{i=1}^{n} \theta_i \big|y_i(t)\big|^{p-1} \bar{a}_i \sum_{j=1}^{n} \big(\big|\alpha_{ij}(t)\big| + \big|\beta_{ij}(t)\big|\big)\rho_{ij} L_j^g \big|y_i(t)\big| \\
&\quad + \frac{p(p-1)}{2} \sum_{i=1}^{n} \theta_i \big|y_i(t)\big|^{p-2} \sum_{j=1}^{n} \varrho_j y_i^2(t) \\
&\leqslant -p \sum_{i=1}^{n} \theta_i \big|y_i(t)\big|^{p-1} \underline{a}_i \mu_i \big|y_i(t)\big| \\
&\quad + \sum_{i=1}^{n} \sum_{j=1}^{n} \theta_i \bar{a}_i \big|c_{ij}(t)\big| L_j^f (p-1)\big|y_i(t)\big|^p + \big|y_i\big(t-\tau_{ij}(t)\big)\big|^p \\
&\quad + p \sum_{i=1}^{n} \theta_i \big|y_i(t)\big|^{p-1} \bar{a}_i \sum_{j=1}^{n} \big(\big|\alpha_{ij}(t)\big| + \big|\beta_{ij}(t)\big|\big)\rho_{ij} L_j^g \big|y_i(t)\big| \\
&\quad + \frac{p(p-1)}{2} \sum_{i=1}^{n} \theta_i \big|y_i(t)\big|^{p-2} \sum_{j=1}^{n} \varrho_j y_i^2(t)
\end{aligned}
$$

$$
\begin{aligned}
&\leqslant -\sum_{i=1}^{n}\theta_i\Biggl\{p\underline{a}_i\mu_i - \sum_{j=1}^{n}\theta_i(p-1)\bar{a}_i\bigl|c_{ij}(t)\bigr|L_j^f \\
&\quad + \sum_{j=1}^{n}\theta_i\bar{a}_i\bigl|c_{ij}(t)\bigr|L_j^f(p-1) - \sum_{j=1}^{n}\theta_i(p-1)\bar{a}_i\bigl(\bigl|\alpha_{ij}(t)\bigr|+\bigl|\beta_{ij}(t)\bigr|\bigr)\rho_{ij}L_j^g \\
&\quad - \sum_{j=1}^{n}\frac{(p-1)(p-2)}{2}\varrho_j - \sum_{j=1}^{n}\frac{\theta_j}{\theta_i}(p-1)\varrho_i\Biggr\}\bigl|y_i(t)\bigr|^p \\
&\quad + \sum_{i=1}^{n}\theta_i\sum_{j=1}^{n}\frac{\theta_j}{\theta_i}\bar{a}_i\bigl|c_{ij}(t)\bigr|L_j^f(p-1)\bigl|y_i\bigl(t-\tau_{ij}(t)\bigr)\bigr|^p \\
&\leqslant -\Pi_1(t)V\bigl(t,y(t)\bigr) + \Pi_2(t)\sup_{t-\tau\leqslant s\leqslant t}V\bigl(s,y(s)\bigr),
\end{aligned}
\tag{4}
$$

where

$$
\begin{aligned}
\Pi_1(t) &= \min_{1\leqslant i\leqslant n}\Biggl\{p\underline{a}_i\mu_i - \sum_{j=1}^{n}\theta_i(p-1)\bar{a}_i\bigl|c_{ij}(t)\bigr|L_j^f \\
&\quad + \sum_{j=1}^{n}\theta_i\bar{a}_i\bigl|c_{ij}(t)\bigr|L_j^f\bigl((p-1)\bigr) - \sum_{j=1}^{n}\theta_i(p-1)\bar{a}_i\bigl(\bigl|\alpha_{ij}(t)\bigr|+\bigl|\beta_{ij}(t)\bigr|\bigr)\rho_{ij}L_j^g \\
&\quad - \sum_{j=1}^{n}\frac{(p-1)(p-2)}{2}\varrho_j - \sum_{j=1}^{n}\frac{\theta_j}{\theta_i}(p-1)\varrho_i\Biggr\}, \\
\Pi_2(t) &= \max_{1\leqslant i\leqslant n}\Biggl\{\sum_{j=1}^{n}\frac{\theta_j}{\theta_i}\bar{a}_i\bigl|c_{ij}(t)\bigr|L_j^f\bigl((p-1)\bigr)\Biggr\}.
\end{aligned}
$$

Applying the Itô formula, for $t \geqslant t_0$, we have

$$
\begin{aligned}
&V\bigl(t+\xi, y(t+\xi)\bigr) - V\bigl(t,y(t)\bigr) \\
&\quad = \int_0^{t+\xi} LV\bigl(s,y(s)\bigr)\,\mathrm{d}s + \int_0^{t+\xi} V_y\bigl(s,y(s)\bigr)\sigma\bigl(s,y(s)\bigr)\,\mathrm{d}\omega(s).
\end{aligned}
\tag{5}
$$

Since $\mathbf{E}[V_x(s,y(s))\sigma(s,y(s))\,\mathrm{d}\omega(s)] = 0$, taking expectations on both sides of (5) and applying (4), we get

$$
\begin{aligned}
&V\bigl(t+\xi, y(t+\xi)\bigr) - V\bigl(t,y(t)\bigr) \\
&\quad \leqslant \int_t^{t+\xi}\Bigl[-\Pi_1(t)\mathbf{E}\bigl(V\bigl(s,y(s)\bigr)\bigr) + \Pi_2(t)\mathbf{E}\Bigl(\sup_{s-\tau\leqslant\varsigma\leqslant s}V\bigl(\varsigma,y(\varsigma)\bigr)\Bigr)\Bigr]\,\mathrm{d}s.
\end{aligned}
\tag{6}
$$

The Dini derivative D^+ is

$$
D^+\mathbf{E}\bigl(V\bigl(t,y(t)\bigr)\bigr) = \lim_{\xi\to 0}\sup\frac{\mathbf{E}(V(t+\xi,y(t+\xi)) - V(t,y(t)))}{\xi}.
$$

Denote $z(t) = \mathbf{E}(V(t, y(t)))$. It follows from (6) that

$$D^+ z(t) \leqslant -\varPi_1(t)z(t) + \varPi_2(t)\|z_t\|^p.$$

In view of Lemma of [17], we obtain

$$z(t) \leqslant \big\|z(t_0)\big\|^p \mathrm{e}^{-\lambda(t-t_0)}.$$

That is

$$\mathbf{E}\big[\big\|x(t) - x^*\big\|^p\big] \leqslant M\|\varphi - x^*\|^p \mathrm{e}^{-\lambda(t-t_0)}, \quad t \geqslant t_0,$$

where

$$M = \frac{\max_{1\leqslant i\leqslant n} \theta_i}{\min_{1\leqslant i\leqslant n} \theta_i} > 1,$$

and λ is the unique positive solution of the following equation:

$$\lambda = \varPi_1(t) - \varPi_2(t)\mathrm{e}^{\lambda\tau}.$$

Thus, the equilibrium x^* of system (1) is pth moment exponentially stable. The proof of Theorem 1 is completed. □

4 An illustrate example

In this section, we present numerical examples to illustrate the effectiveness of the obtained results. Consider the following stochastic fuzzy Cohen–Grossberg neural networks with discrete and distributed delays:

$$\begin{aligned}
\mathrm{d}x_1(t) = -a_1\big(x_1(t)\big)\Bigg[& b_1\big(x_1(t)\big) - \sum_{j=1}^{2} c_{1j}(t) f_j\big(x_j\big(t - \tau_{1j}(t)\big)\big) \\
&- \bigwedge_{j=1}^{2} \alpha_{1j}(t) \int_{-\infty}^{t} K_{ij}(t-s) g_j\big(x_j(s)\big)\,\mathrm{d}s \\
&- \bigvee_{j=1}^{2} \beta_{1j}(t) \int_{-\infty}^{t} K_{1j}(t-s) g_j\big(x_j(s)\big)\,\mathrm{d}s + I_1(t)\Bigg]\,\mathrm{d}t \\
&+ \sum_{j=1}^{2} \sigma_{1j}\big(x_j(t)\big)\,\mathrm{d}\omega_j(t), \qquad \text{(7a)} \\
\mathrm{d}x_2(t) = -a_2\big(x_2(t)\big)\Bigg[& b_2\big(x_2(t)\big) - \sum_{j=1}^{2} c_{2j}(t) f_j\big(x_j\big(t - \tau_{2j}(t)\big)\big) \\
&- \bigwedge_{j=1}^{2} \alpha_{2j}(t) \int_{-\infty}^{t} K_{2j}(t-s) g_j\big(x_j(s)\big)\,\mathrm{d}s
\end{aligned}$$

$$
\begin{aligned}
&- \bigvee_{j=1}^{2} \beta_{2j}(t) \int\limits_{-\infty}^{t} K_{2j}(t-s) g_j\bigl(x_j(s)\bigr)\,\mathrm{d}s + I_2(t)\Biggr]\,\mathrm{d}t \\
&+ \sum_{j=1}^{2} \sigma_{2j}\bigl(x_j(t)\bigr)\,\mathrm{d}\omega_j(t),
\end{aligned} \tag{7b}
$$

where $f_j(x) = g_j(x) = (|x+1| - |x-1|)/2$, $K_{ij}(t) = t\mathrm{e}^{-t}$ and

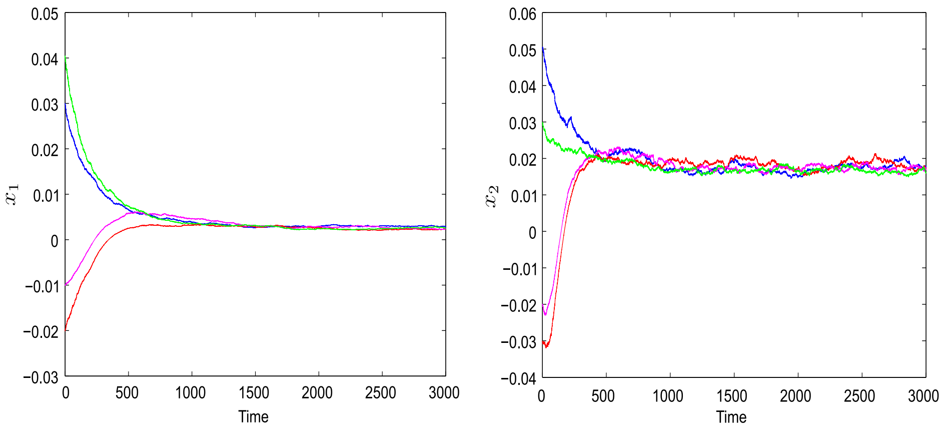
$$
\begin{bmatrix} a_1(x_1(t)) & a_2(x_2(t)) \\ b_1(x_1(t)) & b_2(x_2(t)) \end{bmatrix} = \begin{bmatrix} 4 + 2\cos x_1(t) & 3 + 2\sin x_2(t) \\ 12x_1(t) & 14x_2(t) \end{bmatrix},
$$

$$
\begin{bmatrix} c_{11}(t) & c_{12}(t) \\ c_{21}(t) & c_{22}(t) \end{bmatrix} = \begin{bmatrix} 0.1 & 0.5 \\ 0.4 & 0.6 \end{bmatrix}, \qquad \begin{bmatrix} \alpha_{11}(t) & \alpha_{12}(t) \\ \alpha_{21}(t) & \alpha_{22}(t) \end{bmatrix} = \begin{bmatrix} 1.1 & 1.3 \\ 1.5 & 1.1 \end{bmatrix},
$$

$$
\begin{bmatrix} \beta_{11}(t) & \beta_{12}(t) \\ \beta_{21}(t) & \beta_{22}(t) \end{bmatrix} = \begin{bmatrix} 1.5 & 1.1 \\ 2.1 & 1.8 \end{bmatrix}, \qquad \begin{bmatrix} \sigma_{11}(t) & \sigma_{12}(t) \\ \sigma_{21}(t) & \sigma_{22}(t) \end{bmatrix} = \begin{bmatrix} 0.3x & 0.2x \\ 0.1x & 0.4x \end{bmatrix},
$$

$$
\begin{bmatrix} \tau_{11}(t) & \tau_{12}(t) \\ \tau_{21}(t) & \tau_{22}(t) \end{bmatrix} = \begin{bmatrix} 1.1 & 1.3 \\ 1.5 & 1.1 \end{bmatrix}, \qquad \begin{bmatrix} I_1(t) \\ I_1(t) \end{bmatrix} = \begin{bmatrix} 3 + 4t \\ 1 + 2t \end{bmatrix}.
$$

Let $\varrho_1 = 0.04$, $\varrho_2 = 0.8$, then it is easy to see that that (H1)–(H5) are satisfied. Let $p = 2$, then we can obtain $\varPi_1 = 16.77$, $\varPi_2 = 8.43$. There exists a positive constant $0 < u = 0.8 < 1$ such that $0 < \varPi_2 = 8.43 < u\varPi_1 = 0.8 \times 16.77 = 13.416$. Thus, all the assumptions in Theorem 1 are fulfilled. Thus, we can conclude that system (7) has a unique equilibrium point x^*, which is pth moment exponentially stable. The results are illustrated in Fig. 1



(a) (b)

Figure 1. Transient response of state variables: (a) $x_1(t)$, (b) $x_2(t)$.

5 Conclusions

In this paper, applying Lyapunov function and the Itô differential formula, we investigate the pth moment exponential stability for a class of stochastic fuzzy Cohen–Grossberg neural networks with discrete and distributed delays. Some simple sufficient conditions checking the pth moment exponential stability of the stochastic fuzzy Cohen–Grossberg neural networks with discrete and distributed delays have been obtained. The obtained criteria play an important role in designing pth moment exponential stability of stochastic fuzzy Cohen–Grossberg neural networks.

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