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On economic-technological optimization of high-voltage electric cables*

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Abstract. In this paper, mathematical modelling of high voltage cables for power transmission line design is presented. The Finite Volume Method (FVM) is used to approximate the developed mathematical model (a system of nonlinear multi-physic differential equations) and OpenFOAM (Open source Field Operation And Manipulation) tool is used to implement the obtained parallel finite volume schemes.

In order to optimize the design of power lines with respect to technological parameters, different cases of nonstationary load dynamics are investigated and the influence of system nonlinearity and external day, month and years periodical boundary conditions and the source function regimes are simulated. The main aim of this paper is to include into the mathematical model also economic requirements and to optimize sizes of cables with respect to both technologic and economic requirements. Numerical algorithms targeted to solve PDE-constrained optimization problems are developed. Results of computational experiments are presented.

Keywords: mathematical modelling, finite volume method, differential equation, OpenFOAM, high voltage cables.

1 Introduction

High voltage electrical lines make an important part of all infrastructure for electrical energy supply systems. During design of such lines, two constraints essentially influence the selection of optimal sizes of cable. The first one is thermal (technological), it should guarantee that during exploitation the maximal temperature of cables is not exceeding some nominal value. This thermal constraint is intensively investigated by many groups

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and companies, and there exist and constantly are developed new mathematical models, algorithms and software tools to simulate accurately heat transfer in electrical cables [1, 2, 7, 8].

The second constraint is economical and attention to it have arisen only recently [5, 11]. It is well known that cables have heat resistance and therefore energy is lost in the electricity supply systems. The amount of generated heat is inversely proportional to the size of a cable. At the same time, the total cost of electrical cable system depends on the sizes of cables. Thus the optimal balance must be obtained in order to minimize the cost and satisfy the thermal constraint.

The first investigation on optimal sizes of cables, when economical requirements are also included into the total cost model, was done in [5]. In this report, a simple mathematical model is used to calculate the energy losses in electrical cables and it is shown by analytical calculations that optimal sizes of cables should be increased in comparison with sizes defined by the international technical standards. Similar problems are investigated in [11].

It is interesting to note that the requirement to achieve a balance among technological and economical aims starts to be important in many other technological fields. Here we only mention "the green computing" challenge when the aim to reach high speed of computations, as well as the energy cost constraint, should be balanced.

In our paper, we extend the analysis given in [5] and investigate much more accurate mathematical models that take into account all basic factors of heat transfer in electrical cables. Numerical simulation of heat transfer in and around cables is done by using special numerical solvers, developed to solve heat conduction problems for multiphysics models. Finite Volume Method (FVM) is used to approximate systems of differential equations and OpenFOAM (Open source Field Operation And Manipulation tool is used to implement the obtained finite volume schemes [3]. This approach enabled us to investigate different cases of nonstationary load dynamics and to estimate influence of day, month and years periodical regimes. Optimal sizes of cables are obtained by applying numerical algorithms targeted to solve PDE-constrained optimization problems.

The rest of this paper is organized as follows. In Section 2, the detailed mathematical models of nonstationary heat transfer in and around electrical cables are formulated. Results of computational experiments are presented in order to show possibilities of the presented models and developed numerical solvers. In Section 3, the economic optimization of cable sizes is investigated. It is shown that optimal sizes should be increased in comparison with sizes defined by the international technical standards. A detailed analysis of simplified and full models is done, results of computational experiments are presented. Some final conclusions are given in Section 4.

2 Mathematical models of heat transfer

With regard to heat transfer in underground cables, we assume the diffusion to be the main transfer mechanism. The heat source in the conductors is described by the Joule–Lenz law. The mathematical model of nonstationary heat-transfer is given by the parabolic differential equation [1,2]:

$$
\begin{aligned}
&c\rho\frac{\partial T}{\partial t} = \nabla\cdot(\lambda\nabla T) + F(x,t,T), \quad x\in\Omega,\\
&T(x,0) = T_0, \quad x\in\Omega,\\
&T \text{ and } \lambda\nabla T \text{ are continuous}, \quad x\in\Omega,
\end{aligned}
\tag{1}
$$

here $x = (x_1, x_2)$, $T(x,t)$ is temperature in the Kelvin scale, $\lambda(x) > 0$ is the heat conductivity coefficient, $F(x,t,T)$ defines the source function. Coefficient $\rho(x) > 0$ is the mass density, $c(x) > 0$ is the specific heat capacity, T_0 is the initial temperature.

The heat source in the conductors is described by the Joule–Lenz law:

$$
F(x,t,T) = \begin{cases} q_0(1+\alpha(T^*)(T(x,t)-T^*))(\frac{I_j(t)}{S_j})^2, & t\in[0,t_{\max}],\ x\in\Omega_{\mathrm{cond},j},\\ 0 & \text{otherwise.}\end{cases}
$$

In the model, we take into account the nonlinear dependence of the resistance q on temperature, T^* is the reference temperature, I_j is the electrical current in the jth conductor and S_j is the cross section area of the jth conductor $j = 1,\dots,M$.

We note that this model describes the heat transfer when a set of M cables is simulated and therefore the coefficients of the model are functions of space coordinates $x = (x_1, x_2)$. All coefficients are continuous in different geometrical subregions, but they are discontinuous across these subregions. A geometry of regions can be very complicated and the values of coefficients may vary hundreds and even thousands times, therefore numerical simulation of such systems is a very challenging task.

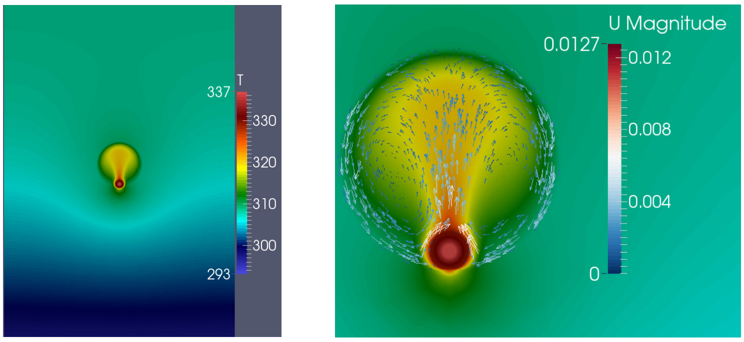
For more complicated technological situations multi-physic models should be used.

2.1 Model A

Let us consider a single cable placed in the PVC tube directly buried in the soil. A tube is placed in the center of the soil domain. Due to its relatively low heat conduction coefficient, a tube represents a significant thermal resistance for the cooling of the cable. The main heat transfer mechanism in air is described by air circulation inside the tube. Velocities of the free convection process are computed by solving the Navier–Stokes equations in the air area. The model of laminar incompressible flow is given by the following system of equations [3]:

$$
\begin{aligned}
&\nabla\cdot\mathbf{u} = 0,\\
&\rho\frac{\partial\mathbf{u}}{\partial t} + \rho\mathbf{u}\nabla\cdot\mathbf{u} - \nabla\cdot(\eta\nabla\mathbf{u}) = -\nabla p - \rho\widetilde{\alpha}\mathbf{g}(T-T_0),\\
&\rho c\left(\frac{\partial T}{\partial t} + \nabla\cdot(\mathbf{u}T)\right) - \nabla\cdot(\lambda\nabla T) = F,
\end{aligned}
\tag{2}
$$

where $\rho(x) > 0$ is the density of material in particular area, $\mathbf{u}(x,t)$ is velocity of the flow, p is the pressure, η is the dynamic viscosity, α is the thermal expansion coefficient.



(a) (b)

Figure 1. A single cable in a PVC tube: (a) distribution of temperature, (b) distribution of velocity fields.

Next, some results of simulations are presented. Let us consider the case of one cable, which is placed in a PVC tube buried in the soil. The heat transfer is simulated during three summer months, when the soil is assumed to be semi-dry, boundary conditions are taken $T = 293$ K on all boundaries, and the electrical current is equal to 470 A. Figure 1 shows a distribution of temperature and velocity fields in the domain of computations. More results are presented in [3].

2.2 Model B

Another multi-physic model should be used when main properties of the soil start to be important for accurate simulation of industrial installations of cables. Then we should take into account the effect of soil thermal resistivity. It is known from engineering experiments that in a well-designed high power electrical lines system, the soil may account for half or more of the total thermal resistance. The heat conductivity (or resistance) of the soil essentially depends on the moisture (water) content in the soil.

The macroscopic equation that describes the conservation of water in soil both in liquid and vapor states is defined by the Darcy law:

$$\frac{\partial \Theta}{\partial t} = \nabla \cdot \big(D_\Theta(\Theta, T)\nabla\Theta + D_T(\Theta, T)\nabla T - K(\Theta, T)\vec{g}\big), \tag{3}$$

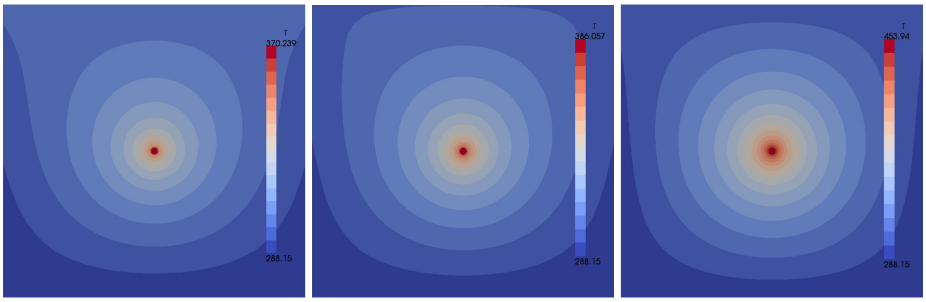
where Θ is the volumetric moisture (water) content. The macroscopic equation that describes the conservation of energy in soil is given by

$$\rho c_{\mathrm{eff}}(\Theta, T)\frac{\partial T}{\partial t} = \nabla \cdot \big(\lambda_{\mathrm{eff}}(\Theta, T)\nabla T\big) + \nabla \cdot \big(L(T)J_v\big), \tag{4}$$

where $L(T)$ is the latent heat of vaporization.

In Fig. 2, we present distribution of temperature for different types of soil. The results essentially depend on the moisture (water) content in the soil.

The obtained systems of PDEs are solved by using the Finite Volume Method (FVM). OpenFOAM (Open source Field Operation And Manipulation) tool is used to implement



(a) (b) (c)

Figure 2. Distribution of temperature for different types of soil: (a) moist soil $\lambda = 1.11$, (b) average soil $\lambda = 0.83$, (c) dry soil $\lambda = 0.41$.

the given finite volume schemes. OpenFOAM is a C++ toolbox (library) targeted for the development of customized numerical solvers for partial differential equations [1] The efficiency and accuracy of this tool are analyzed in [3]. The parallel version of this tool is presented. The efficiency analysis of parallel algorithms for solving the given multiphysics problem is investigated in [4, 10].

2.3 Temperature constraints of cable optimization

The developed software tool enables users to simulate the distribution of temperature $T(x,t)$ in and around cables. All main factors influencing this distribution can be simulated in accurate way.

- The maximum temperature of all cables during the whole working cycle of the electrical power system is defined:

  $$T_C = \max_{x \in D,\, 0 \leqslant t \leqslant t_F} T(x,t),$$

  where D denotes the metal area of the cable. The main technological constraint

  $$T_C \leqslant T_{\max} \tag{5}$$

  must be fulfilled during the whole working cycle. This constraint can be satisfied by appropriate selection of cross sections S_j of cables and/or regulating the values of applied electrical currents I_j.
- A distribution of temperature is important for computation of heat sources F since the heat resistance coefficient $q(T)$ depends on temperature.

3 Economic optimization of cable sizes

The total life cycle cost C_T of the cables can be estimated as

$$C_T = C_I + C_L, \tag{6}$$

here C_I defines the installation cost and C_L defines the cost of energy losses.

3.1 Full economic model

The installation cost (or investment cost) per meter of cable length ($L = 1$ m) is defined as

$$C_I = (C_0 + C_P S)(1 + p)^n, \tag{7}$$

where C_0 is a constant part of installation cost, C_P is the price of cable conductor in euro per m^3 and $S = \sum_{j=1}^{M} S_j$ is the total metal area of all cables. The cost also includes the interest rate p and the economic lifetime of the cable n (in years).

When a copper cable is dismantled, the cooper can be recycled and the scrap value $C_{PS}S$ should be subtracted from the investment value

$$C_I = (C_0 + C_P S - C_{PS} S)(1 + p)^n. \tag{8}$$

Next, we define the cost of energy losses:

$$\begin{aligned} C_L = q_0 \sum_{j=1}^{n} (1+p)^{n-j} \tau_T(t_j) \\ \times \sum_{m=1}^{M} \int_{t_{j-1}}^{t_j} \bigl(1 + \alpha(T^*)\bigl(T(x_m, t) - T^*\bigr)\bigr) \frac{I_m^2(t)}{S_m} \, \mathrm{d}t, \end{aligned} \tag{9}$$

where $T(x_m, t)$ defines the temperature at the center of the mth cable at time t, and τ_T defines the electricity tariff. Here the interest rate p is assumed to be constant during the lifetime of the cable. Temperatures $T(x_m, t)$ are defined from the appropriate multi-physic mathematical model used to simulate heat conduction in electrical cables.

3.2 Simplified economic model

Let us assume that energy losses of all cables are constant during each year time, the resistance q is not depending on temperature, the electricity tariff τ_T is constant, $C_{PS} = 0$. We also assume that the sizes of all cables are equal $S_m = S_0$, $m = 1, \ldots, M$. Then the optimal value of $S = MS_0$ is obtained by minimizing the simplified function $P(S)$ (see, also [5]):

$$P(S) := C_0 + C_P S + N_C(n) q_0 \tau_T \frac{ME}{S}, \tag{10}$$

where E defines the total amount of currents that flow through all cables per year

$$E = M \max_{1 \leqslant m \leqslant M,\, 1 \leqslant j \leqslant n} \int_{t_{j-1}}^{t_j} I_m^2(t) \, \mathrm{d}t$$

and N_C is the so called capitalization factor

$$N_C(n) = \frac{(1+p)^n - 1}{p(1+p)^n}.$$

If the scrap value of recycled copper is taken into account, then we modify coefficient C_P:

$$\widetilde{C}_P = C_P - C_{PS}. \qquad (11)$$

The optimal value of S is obtained from the equation

$$P'(S) := C_P - N_C(n) q_0 \tau_T \frac{ME}{S^2} = 0$$

and it is given by

$$S_{\mathrm{opt}} = \left(\frac{N_C(n) q_0 \tau_T ME}{C_P}\right)^{1/2}.$$

3.3 Analysis of the simplified model

In this paragraph, the dependence of the total cost on the size of conductor is investigated. Let us assume that one electrical cable is put into the soil, $M = 1$, and the constant current $I_0(t) = 740$ A is applied during the whole working cycle. In model (10), the following values of parameters are used:

$$C_0 = 56.7\,\frac{\text{EUR}}{\text{m}}, \qquad C_P = 0.125 \cdot 10^6\,\frac{\text{EUR}}{\text{m}^3},$$
$$\tau_T = 0.066\,\frac{\text{EUR}}{\text{kWh}}, \qquad p = 0.05.$$

The value of copper resistance q_0 is taken at $90\,^\circ$C temperature and equal to $1.98 \times 10^{-8}\ \Omega\,$m.

In Table 1, for different values of n, optimal cable conductor sizes and the total life cycle cost $P(S)$ per meter for optimal cables S_{opt} are presented. For comparison, the total cost of a standard cable with $S_T = 400$ mm^2 is given. The cable size S_T is defined by technological requirements that temperature of cables should be bounded by some specified value T_M. Thus the technologic-economic optimal size of cables is obtained from the modified formula

$$S^*_{\mathrm{opt}} = \max(S_T, S_{\mathrm{opt}}). \qquad (12)$$

It follows from the presented results that the economic approach can help to save up to 90 euro per one meter of cable during 40 years life cycle.

Table 1. Analysis of total life cycle cost of the cables.

n	$N(n)$	S_{opt}, mm^2	$P(400)$	$P(S_{\mathrm{opt}})$
1	0.952	219	121.6	111.3
5	4.329	466	174.6	173.2
10	7.722	622	227.7	212.3
20	12.462	791	302.0	254.3
30	15.373	878	347.6	276.2
40	17.159	928	375.6	288.6

Remark 1. Here we can make one important conclusion. The economic approach starts to be important only if total cost is optimized for a sufficiently long life cycle of cables. For short time investments strategy (up to 5 years), the technological approach leads to the main restriction of cable sizes.

The same conclusion is valid with respect to a possibility to recycle copper at the end of life cycle of cables. As it follows from (11), this step reduces installation cost and therefore leads to larger optimal sizes of cables and larger savings of operation cost. Again, this possibility is important only if this deposit money is returned to an initial investor.

Remark 2. The international technical standards define minimum allowed sizes of cables and such sizes are always defined with some safety margin. As it follows from economic analysis, a strategy to increase sizes of cables leads to reduction of the total cost.

3.4 Analysis of the full model of electrical cables

In this section, we take into account one important property of the mathematical model that the conductor heat resistance $q(T)$ depends on temperature and this effect is simulated by the linearized approximation [1, 8]:

$$q(T) = q_0\bigl(1 + \alpha(T^*)\bigl(T(x,t) - T^*\bigr)\bigr),$$

where T^* is the reference temperature. Thus the full mathematical model (see, e.g. (1)) enables us to compute the distribution of temperature in an accurate way and to analyze the effects of the following factors:

- The dependence of temperature on dynamics of electrical current $I(t)$;
- The influence of different boundary conditions during summer and winter seasons;
- The dependence of temperature dynamics on the selected layout of cables;
- The dependence of heat transfer dynamics on surrounding porous media and air properties and geometry of the region.

In all computations, we simulate the system of three cables arranged horizontally and directly buried in the soil. A schematic description of the layout of cables is shown in Fig. 3.

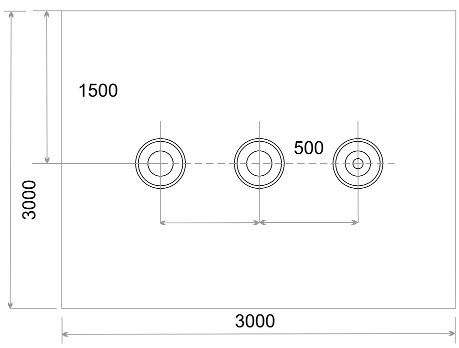


Figure 3. The layout of three cables in the soil.

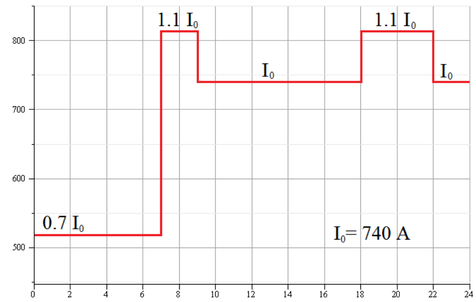


Figure 4. The daily dynamics of electrical currents.

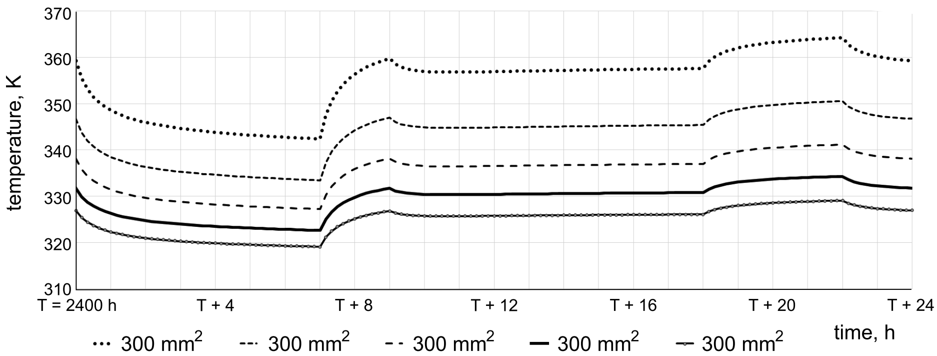


Figure 5. The daily dynamics of temperature in the conductor region of the central cable for different cable cross sections S.

All three cables are loaded by the same current $I(t)$ during the total life cycle. We have considered a case when the periodicity of $I(t)$ is equal to 24 hours. The daily dynamics of the electrical current is shown in Fig. 4.

The developed software solver POWEROPT is used to simulate a distribution of temperature in and around cables for given parameters of cables, porous media and air [3, 10]. The results depend on the geometry of cable grid and specified boundary conditions. In the presented computational experiments, we have used the following parameters of cables and soil: cables are buried in averaged moisture soil at the 1500 mm depth, distance between cables center is 500 mm (Fig. 3).

First, it is important to investigate the dependence of temperature on the cross sections of cables. In Fig. 5, we present the dynamics of temperature in the conductor region of the second cable for different values of cross sections S. The boundary conditions are specified for summer months.

The influence of boundary conditions is analyzed in Fig. 6, where dynamics of temperature is presented for summer and winter months. The boundary condition $T_0 = 293$ is specified for summer case and $T_0 = 278$ for winter case. The cross section of all three cables is $S = 500$ mm^2, and only temperature of the central cable is presented.

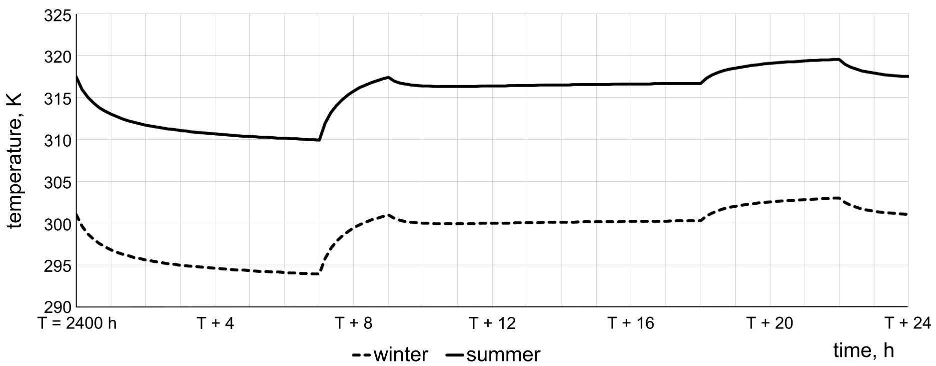


Figure 6. The daily dynamics of temperature for summer and winter cases.

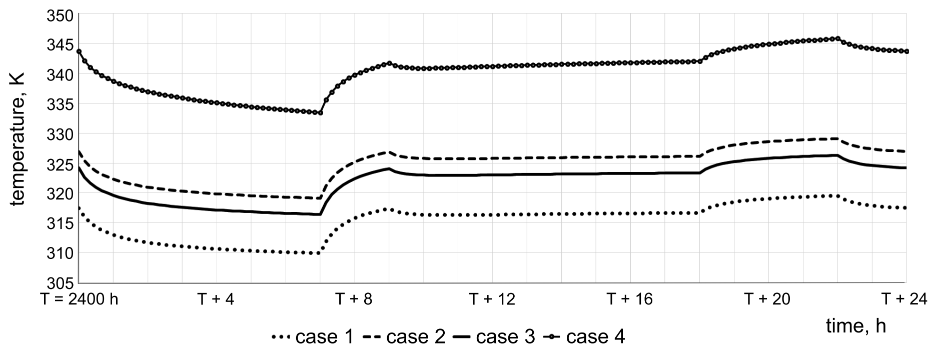


Figure 7. The daily dynamics of temperature for different placements of cables.

Next, we have investigated the influence of different placements of cables. In Fig. 7, the dynamics of temperature is presented for cables with cross sections $S = 500$ mm^2 and the following cases: 1) one cable, 2) three cables in line, the central cable, 3) three cables in line, the last cable, 4) a trefoil case, the upper cable.

3.5 Determination of optimal sizes of cables

In this section, we solve the PDE-constrained optimization problem to find optimal sizes of cables. The full cost function

$$C_F(S) := (C_0 + C_P S - C_{PS} S)(1+p)^n + q_0 \sum_{j=1}^{n} (1+p)^{n-j} \tau_T(t_j) \\ \times \sum_{m=1}^{M} \int_{t_{j-1}}^{t_j} \big(1 + \alpha(T^*)\big(T(x_m, t) - T^*\big)\big) \frac{I_m^2(t)}{S_m} \, \mathrm{d}t, \tag{13}$$

is minimized, here $S = \sum_{m=1}^{M} S_m$ and M is the number of cables, temperature $T(x_m, t)$ is obtained by solving the full mathematical model of electrical cables. All main technologic details (including material properties, geometry of the domain, boundary conditions) are taken into account in POWEROPT software tool. For simplicity, in all computational experiments, we assume that $S_m = S_0$, $m = 1, \dots, M$, i.e. the sizes of all cables are the same.

3.6 Optimization algorithm

It is shown in previous computational experiments that the temperature distribution depends monotonically on sizes of cables. Thus the minimization of functional (13) can be done efficiently by using the golden section search algorithm [6]. It is well known that the convergence rate of this algorithm is geometrical and the error reduction factor $r = 0.618$.

The temperature of cables is simulated for $T = 40$ years duration. At each step of the minimization algorithm, a new value of $C_F(S)$ is computed. A system of nonstationary nonlinear PDE equations should be solved for the specified value of S in order to compute temperature of cables. This step of the optimization algorithm is very CPU time demanding, up to 24 hours is required to solve optimization problem on one processor. Thus a parallel version of the solver is used to reduce the computation time [4, 10]. The scalability analysis, which is presented in these papers shows a quite good efficiency of the developed parallel solvers of the basic direct problem on heat conduction in and around the cables.

Results of computational experiments on optimization of cables for two specific values of I_0 are presented in Table 2.

The optimization analysis have started from the size of cables $S = 300$ mm^2 which is sufficient to guarantee the technological requirements. It clearly seen from the presented results that the full cost C_F of operating system can be reduced essentially if sizes of cables are increased above the standard size $S_T = 300$ mm^2. The reduction factor of 10 percents is obtained for $I_0 = 740$.

The presented optimization results are showing that the function of total cost $C_F(S)$ is changing slowly in the neighbourhood of the optimal point. Thus for industrial applications engineers can select sizes of cables from given specifications of cables presented in the market. It is possible that a demand for larger size cables can justify changes of these specification in future.

Table 2. Optimization results for the full mathematical model.

$I_0 = 570$		$I_0 = 740$	
S, mm^2	$C_F(S)$	S, mm^2	$C_F(S)$
300	993.5	300	1287.15
326.3	985.4	490	1168.64
365	982.7	494	1168.78
366.4	**982.5**	**495**	**1167.84**
371	983.28	497	1169.13
391	985.26	566	1184.79
541	1043.0	734	1256.41
700	1140.7	1000	1427.40

4 Conclusions

In order to define optimal sizes of cables in new electrical power supply systems, it is necessary to take into account both technologic and economic requirements. Technologic requirements mainly should guarantee that during full life cycle and for all possible scenarios, the maximum temperature will remain bellow some specified critical value $T_{\max}$. The minimal values of allowed sizes of cables are defined by the international technical standards [9]. We have developed a specialized software tool POWEROPT, which is used to simulate a distribution of temperature in and around cables for given parameters of cables, porous media and air [3, 10]. The mathematical model is restricted to nonstationary two-dimensional approximation of the full three-dimensional problem. Even such an approximation gives a big challenge for fast, robust and accurate simulation of distribution of temperature in real world engineering applications. POWEROPT tool enables engineers to define optimal sizes of cables S_T such that, for cables with sizes $S \geqslant S_T$, the temperature inside cables never overshoot the critical value $T_{\max}$. In comparison with recommendations given by the international technical standards, this tool gives a possibility to simulate accurately the influence of boundary conditions (summer and winter conditions), the dynamics of variations in electrical power consumption during day/week/month cycles, the dependence of the surrounding material properties on temperature, moister content and similar factors. Thus the obtained recommendations improve and optimize the existing recommendations defined by the technical standards.

We note that the recommendations of technical standards are still important, but users should use them in a proper way and they should understand that these estimates are obtained from simplified models and they have a safety margin.

In this paper, we have considered also economic approach in determination of optimal sizes of electrical cables. The generated heat defines the energy loses and increases essentially total life cycle cost C_T of the electric network, and as a consequence, increases the price of electricity for consumers. We note that generated heat (i.e. lost energy) also influences seriously the ecological quality of our environment. It is typical that, in most countries, energy companies should pay eco-taxes and fines in order to reduce such a pollution.

The presented economic-technological optimization technology gives a possibility to estimate the length of time required to cover expanses in using a larger size cables and to get a profit of such a strategy. It is shown that this time duration is reduced and larger sizes of cables are recommended if utilization of copper and the scrap value of recycled copper is taken into account in the economic model.

At the end of conclusions, we present two important notes. First, for a long duration life cycle, it is impossible to predict accurate parameter values of the economic model (the electricity tariff τ_T, the interest rate p and the price of copper C_p). Thus the developed optimization tool POWEROPT can be used to simulated various scenarios and give recommendations for engineers, city administration and ecologist. Second, in most big energy supply projects, different companies, offices and operators are responsible for different stages of exploitation of such lines. It is easy to see that they have different time windows of participation and their profit strategies can be quite different from the optimal strategy of clients (city population or a whole society of country). The proposed technology can help to find consensus in reaching the optimal solution.

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