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An almost learning curve model for manual assembly performance improvement

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Abstract. In this paper, an almost learning curve (ALC) model is presented. This provides a more accurate approximation of the production data than the traditional log-linear learning curve model. The proposed ALC model is based on the solution of differential equations and still has all the necessary log-linear learning curve function properties. The ALC model was tested on the wiring harness manufacturer production data. Findings suggest that the ALC model approximates data accurately and is superior to the classical learning curve (CLC) for various manufacturing situations. Moreover, the use of the ALC showed an additional insight into the analysis of learning and skill development.

Keywords: mathematical modeling, differential equation, optimization of data fitting, learning curve.

1 Introduction

The performance of a manual task improves as the task is repeated until maximum performance is reached. Mathematically the learning is defined by a certain function i.e. learning curve (in this article classical learning curve – CLC) which shows a time (or cost) decrement as the argument (number of units) increases. This learning phenomenon was firstly reported by Wright [30] after studying the assembly of airplanes. Since then, CLC has become an important industrial engineering topic and has been used for predicting future costs, analyzing and controlling the performance and efficiency of certain individuals, groups, organizations etc. The usage of CLC has spread from industrial manufacturing to other fields, such as healthcare institutions, military, education, training and other sectors, however, manufacturing, especially manual assembly based industry, is at the top of the interest. Initially, CLC was used to predict and forecast operating time and production cost decrement as production continues [3]. Since manufacturing is shifting from mass production with high production volume and low diversity to LEAN production and Mass Customization with small production quantity and almost endless variety of products, the manual assembly based production systems are encountering serious issues, mainly caused by the never ending learning phase; quantities are just too small to complete it and the time designated for learning constitutes a major part of the total task processing time [19]. As reported in various articles [1], this is the reason why CLC problems are re-emerging as the important issue among production researchers. This new production environment has several remarkable aspects such as prototype production, order quantity fluctuations, demand based production, production orders in random and long-time intervals. Therefore, traditional CLC models became inapplicable for such situations and there is an obvious need for more accurate models [6].

The goal of this article is to create a versatile CLC model based on the generalized power model. The new model should be applicable to most manufacturing situations. Classical power model used by many researchers possess several drawbacks. This model is unbounded, it lacks flexibility and these reasons make CLC model insufficient to deal with certain issues in the new production environment, such as planning, scheduling, optimal order quantity calculation, managing ramp-ups in production etc. The main idea about new learning curve model is to add one or several additional parameters to the model, while keeping all the properties of the CLC. These new parameters would improve curve fitting, on the other hand, there would be the possibility to assign for these parameters an appropriate meaning from the particular manufacturing situation.

The proposed generalized model is based on the solutions of special (with perturbation parameter) differential equations. These solutions define the almost learning curves (ALC). This definition was proposed by Lowenthal [21]. However, Lowenthal did not determine sufficient conditions for the perturbation parameter values which enable ALC to have all main properties of the power model (CLC). In this research, sufficient conditions for the perturbation parameter and other parameters of the ALC are determined. Also, mathematical analysis of the ALC is performed to explore the versatility possibilities and establish the foundation of such ALC modeling. Proposed model is tested by experimental data from the wiring harness manufacturing company.

The article is organized in the following order: the brief review of CLC models is presented in the following section, mathematical modeling of the ALC is presented in Section 3, application results and discussion is given in Section 4, and lastly the conclusions are presented in Section 5.

2 Brief review of the CLC models

Wright [30] proposed a cumulative average learning curve (CLC) based on power function. During the next decades many other models were proposed such as Crawford model, DeJong model, Plateau model etc. [4, 11]. All mentioned traditional models are widely applied and available in the literature [1, 2, 4, 6, 11, 31].

Wright model is the most popular and has very broad application possibilities, however many drawbacks of this model exists, therefore CLC research is open for improvements. Superiority of power function as the best fit for a production data has been proved by intensive study and data [23], although some authors [10] proposed exponential function to define learning. Other authors [20] reported the limitations of the traditional CLC models and proposed analytical model to calculate impact of knowledge depreciation and plateauing phenomena to total processing time. Smunt [27] continued to unravel shortcomings of the conventional CLC by eliminating misunderstandings of the CLC application and proposed mid-unit CLC model on the result. The research [29] estimated that errors due to misunderstandings and misapplications of traditional CLC might reach up to 30% and proposed a theory for the correct application. On the other hand, a universal calculation algorithm was proposed in work by Janiak and Rudek [17]. This algorithm avoids major drawbacks of CLC fitting to particular production data, because it is open to any CLC model. Shortcomings and drawbacks of traditional CLC were being solved by using a dual phase learning assumption. This was initially reported by Dar-El et al. [5]. The idea is based on cognitive and motor improvements with different CLC's combined into the one model. The proposed model was further improved [13, 15]. Some other authors [14] emphasized traditional CLC limitations arising with production stops due to reworks and re-adjustments and also proposed the newly developed model. In addition to this, Monfared and Jenab [22] proposed a CLC model to be applied in the demand-based manufacturing, where traditional CLC might be not applicable. Proposed model consists of double segment CLC with breakpoint.

Another group of researches encompasses forgetting phenomenon. Problems and stoppages in manufacturing environment are causing a production breaks which appear as forgetting phenomenon. The work [7] reported significant impact of breaks to the forgetting. The next research [12] proposed the analytical method to predict this impact and the performance after forgetting. The comparison of three different potential learning forgetting models could be found in [16].

The studies [1, 2, 6] declared the need for the multivariate models. Even the Wright based univariate CLC models dominate in the most literature, the advanced multivariate learning model was presented recently in the work [25]. It seems that multivariate models demand for rigorous research, although Badiru in [2] declared limitations of such a model application. The majority of presented works deal with the individual learning curve. However, group learning curves (GLC) are also considered [8]. Such curves recall multivariate learning models, because they combine different parameters of the individuals to the general group parameters. Even this GLC model is promising it has limitations related to quality issues and uncertainty.

Two main conclusions follow from the literature review. First of all, it is clear that a large variety of different models is available in the literature. The majority of them are based on classical power model and the major question arise which model to use on particular manufacturing situation. Recent work [9] proposed a meta-analysis to answer this question i.e. to facilitate decision of learning curve selection. However, selecting one of existing models does not improve the model itself and the need to propose better learning curve model still exists. Such an improvement might be achieved by implementing ALC.

3 Mathematical problem formulation

In this section, a definition of the classical learning curve (CLC) and almost learning curve (ALC) as solutions of the differential equations are presented. Since the CLC [27] is defined as $y(x,\alpha,\beta) = \beta x^{-\alpha}, \beta > 0,\ 0 < \alpha < 1,\ x \geqslant 1$, CLC is the solution of Cauchy problem

$$L_\alpha(y) = 0, \qquad y(1,\alpha) = \beta, \tag{1}$$

where $L_\alpha(y) = y' + \alpha x^{-1} y$. Let the solution

$$y(x,\alpha,\beta) = \beta x^{-\alpha} \tag{2}$$

of problem (1) be a definition of CLC. Considering the more general Cauchy problem

$$L_\alpha(y) = 0, \qquad y(x_1,\alpha) = y_1, \tag{3}$$

whose solution is

$$y(x,\alpha) = x_1^\alpha y_1 x^{-\alpha}, \tag{4}$$

then $\beta = y(1,\alpha)$. The properties of the bundle (with respect to the α) of solutions (4) follows from direct differentiation:

$$y \in C^{(2)}[1,+\infty) \quad \text{if } x_1 \geqslant 1,\ y_1 > 0,\ \alpha \in (0,1),$$

$$y(x,\alpha) > 0, \qquad y'_x(x,\alpha) \leqslant 0, \qquad y''_x(x,\alpha) \geqslant 0, \tag{5}$$

$$y_a = \lim_{x\to+\infty} y_h(x,\alpha) \geqslant 0. \tag{6}$$

Analyzing the Cauchy problem

$$L_\alpha(w) = \varepsilon x^{-r}, \quad r \geqslant 1, \qquad y(x_1,\alpha) = y_1, \tag{7}$$

then the function

$$w(x,\alpha,\varepsilon,r) = x_1^{s(\alpha,r)} \left[\frac{c(\alpha,r)-\varepsilon}{s(\alpha,r)}\right] x^{-\alpha} + \frac{\varepsilon}{s(\alpha,r)} x^{1-r} \tag{8}$$

is a solution of (7). Here $s(\alpha,r) = \alpha + (1-r)$, $c(\alpha,r) = x_1^{r-1} y_1 s(\alpha,r)$.

Stating sufficient conditions for parameters α, ε and r such that solution (8) satisfy (5)–(6). Such solutions are called the ALC. In addition to this, conditions when solution (8) have positive horizontal asymptote (i.e. plateauing phenomenon [18, 28]). Note that, under conditions $x \in [1,+\infty)$, $\alpha \in (0,1)$, $r \in [1,+\infty)$ and $\varepsilon \in (-\infty,+\infty)$, solution (8) exists and is unique, because $\alpha/x, \varepsilon/x^r \in C^{(0)}[1,+\infty)$ [26].

Proposition 1. *Under the condition*

$$(\alpha,\varepsilon,r) \in D_1 = \bigl\{1 \leqslant r < 2,\ \alpha_0(r,\varepsilon) < \alpha < 1,\ 0 \leqslant \varepsilon \leqslant c(\alpha,r)\bigr\} \tag{9}$$

(*where* $\alpha_0 = \varepsilon(x_1^{r-1}y_1)^{-1} + (r-1)$), *the Cauchy problem* (7) *solution* (8) *is ALC.*

Proof. Introducing the following notation:

$$C_1 = \frac{c(\alpha, r) - \varepsilon}{s(\alpha, r)}, \qquad C_2 = \frac{\varepsilon}{s(\alpha, r)}, \qquad C_3 = \frac{\varepsilon(r-1)}{s(\alpha, r)}, \tag{10}$$

then from (8) we have

$$w(x, \alpha, \varepsilon, r) = x_1^s C_1 x^{-\alpha} + C_2 x^{1-r}, \tag{11}$$

$$w'_x(x, \alpha, \varepsilon, r) = -\alpha x_1^s C_1 x^{-(\alpha+1)} - C_3 x^{-r}, \tag{12}$$

$$w''_x(x, \alpha, \varepsilon, r) = \alpha(\alpha+1) x_1^s C_1 x^{-(\alpha+2)} + C_3 r x^{-(r+1)}. \tag{13}$$

From (11)–(13) follows that if

$$C_1 > 0, \qquad C_2 \geqslant 0, \qquad C_3 \geqslant 0 \tag{14}$$

for all $x \geqslant 1$, then functions $w > 0$, $w'_x \leqslant 0$, $w''_x \geqslant 0$ for all $x \geqslant 1$.

If $(\alpha, \varepsilon, r) \in D_1$, then $\varepsilon \geqslant 0$; $c(\alpha, r) \geqslant 0$, $s(\alpha, r) \geqslant 0$ if $\alpha \geqslant r-1$; $\alpha_0 \geqslant r-1$ and $c(\alpha, r) - \varepsilon > 0$.

$C_1 > 0$, because C_1 is a strictly monotone increasing with respect to α ($(C_1)'_\alpha = \varepsilon / s(\alpha, r)^2 > 0$, because $\varepsilon > 0$), and $C_1 = 0$ when $\alpha = \alpha_0$. From $s(\alpha, r) > 0$ and $(r-1) \geqslant 0$ follows that $C_2 \geqslant 0$, $C_3 \geqslant 0$.

Showing that $D_1 \neq \emptyset$. By integrating the function $c(\alpha, r)$ with respect to α, results in

$$\operatorname{mes} D_1 = \int_{\alpha_0}^{1} c(\alpha, r)\, \mathrm{d}\alpha > 0, \tag{15}$$

because $c(\alpha, r) > 0$ and always exist $\varepsilon > 0$, that $0 < \alpha_0 < 1$. The equation of horizontal asymptote for solution (11)

$$wa = \lim_{x \to +\infty} \frac{\varepsilon}{s(\alpha, r)} x^{1-r} = \begin{cases} \varepsilon/\alpha > 0 & \text{if } r = 1, \\ 0 & \text{if } 1 < r < 2. \end{cases} \qquad \square \tag{16}$$

Proposition 2. *Under the condition*

$$(\alpha, \varepsilon, r) \in D_2(r) = \bigl\{ r \geqslant 2,\ 0 < \alpha < \alpha_0(r, \varepsilon),\ 0 \geqslant \varepsilon \geqslant c(\alpha, r) \bigr\}, \tag{17}$$

Cauchy problem (7) *solution* (8) *is ALC.*

Proof. If $(\alpha, \varepsilon, r) \in D_2$, then $\varepsilon \leqslant 0$; $c(\alpha, r) \leqslant 0$, $s(\alpha, r) \leqslant 0$ if $\alpha \leqslant r-1$; $\alpha_0 \leqslant r-1$, and $c(\alpha, r) - \varepsilon \leqslant 0$. $C_1 > 0$, because C_1 is a strictly monotone decreasing with respect to α ($(C_1)'_\alpha = \varepsilon / s(\alpha, r)^2 < 0$, because $\varepsilon < 0$) and $C_1 = 0$ when $\alpha = \alpha_0$. From $s(\alpha, r) < 0$ and $(r-1) > 0$ follows that $C_2 \geqslant 0$, $C_3 \geqslant 0$.

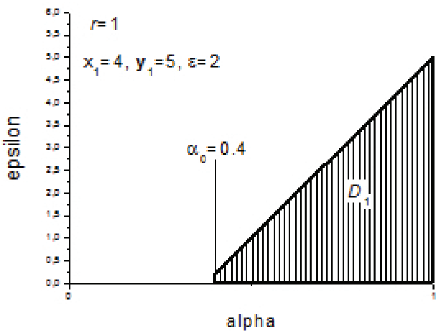

Figure 1. Domain $D_1(r)$ when $1 \leqslant r < 2$.

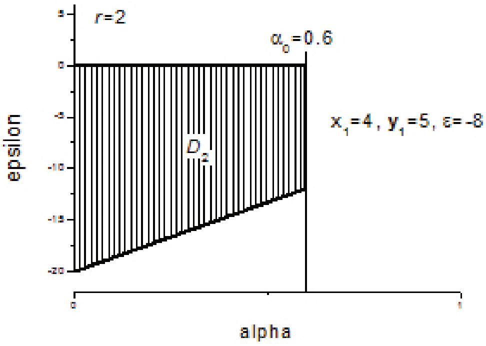

Figure 2. Domain $D_2(r)$ when $r \geqslant 2$.

Showing that $D_2 \neq \emptyset$. By integrating the function $|c(\alpha, r)|$ with respect to α, results in

$$\operatorname{mes} D_2 = \int\limits_{\alpha_0}^{1} \bigl|c(\alpha, r)\bigr| \, \mathrm{d}\alpha > 0, \tag{18}$$

because $c(\alpha, r) < 0$ and always exist $\varepsilon < 0$ that $0 < \alpha_0 < 1$. The equation of horizontal asymptote for solution (11)

$$w_a = \lim_{x \to +\infty} \frac{\varepsilon}{s(\alpha, r)} x^{1-r} = 0, \tag{19}$$

because $(1 - r) < 0$. □

At the end of the section, calculation examples of domains $D_1(r)$ and $D_2(r)$ are presented. If $x_1 = 4$, $y_1 = 5$, $\varepsilon = 2$ and $r = 1$, then domain $D_1(r)$ is trapezoid with vertexes $(\alpha_0, 0), (1, 0), (1, c(1, r)), (\alpha_0, c(\alpha_0, r))$ (see Fig. 1). If $x_1 = 4$, $y_1 = 5$, $\varepsilon = -8$ and $r = 2$, then domain $D_2(r)$ is trapezoid with vertexes $(0, c(0, r)), (1, c(\alpha_0, r)), (1, c(\alpha_0, r))$, $(0, 0)$ (see Fig. 2). Here $c(\alpha, r) = x_1^{r-1} y_1 (\alpha + 1 - r)$ and $\alpha_0 = \varepsilon(x_1^{r-1} y_1) + (r - 1)$.

4 Results and discussion

Created ALC models were tested on certain production data that was monitored at the manufacturing company. The company produces wiring harnessing products for automotive industry. Since the production is mostly manual (automated assembly is just unprofitable for low quantity orders) therefore, company is suffering heavy nonlinear assembly rates at the production processes. This company produces enormous variety of different harnesses (more than four thousand) for auto industry and the customer demand is fluctuating and changing rapidly for each product and also the demand of particular wiring harnesses sharply differs from each other: from one piece per year, to several hundred per month. As a result, several types of manufacturing layouts are applied, from assembly line to singular prototype production. The company also possesses sophisticated total productivity maintenance system (TPM) which enables to collect and analyze variety of production data and to monitor the whole production cycle: from the beginning to the phase out of the certain wiring harness. Monitored data is presented in such form:

$$x^{(i)}, y^{(i)}, x^{(i)} < x^{(i+1)}, \quad i = 1, 2, \ldots, N, \tag{20}$$

where x is number of unit, y is processing time of the $x$thunit. Let $X = (x^{(1)}, \ldots, x^{(N)})$ and $Y = (y^{(1)}, \ldots, y^{(N)})$ are experimental data (20).

Let $Y_{\mathrm{opt}} = (y_{\mathrm{opt}}(x^{(1)}), \ldots, y_{\mathrm{opt}}(x^{(N)}))$ and $W_{\mathrm{opt}} = (w_{\mathrm{opt}}(x^{(1)}), \ldots, w_{\mathrm{opt}}(x^{(N)}))$ are recovered results by using models (3), (7). The accuracy of approximation is measured by relative norm [24]

$$\delta_y = \frac{\|Y_{\mathrm{opt}} - Y\|}{\|Y_{\mathrm{opt}}\|}, \qquad \delta_w = \frac{\|W_{\mathrm{opt}} - Y\|}{\|W_{\mathrm{opt}}\|},$$

where $\|X\| = \sqrt{\sum_{i=1}^{N} (x^{(i)})^2}$ is Hilbert–Schmidt norm.

To compare the developed ALC model (8) with a traditional CLC model, first of all an optimal CLC is obtained for the experimental data (20). The optimal parameters of CLC, i.e. coefficient α_y and the number n of the data point x_n, which minimizes norm δ_y, are

$$\arg \min_{\substack{0<\alpha<1 \\ x_i,\, 1\leqslant i\leqslant N}} \delta_y = \begin{bmatrix} \alpha_y \\ n \end{bmatrix}. \tag{21}$$

Then the norm δ_y can be calculated from the solution (4) of the Cauchy problem $y(x^{(n)}, \alpha_y) = y^{(n)}$. The optimal parameters of ALC, i.e. α_w, ε_w and r_w, which minimizes norm δ_w, are

$$\arg \min_{\substack{0<\alpha<1 \\ \varepsilon\in D_i(r)}} \delta_w = \begin{bmatrix} \alpha_w \\ \varepsilon_w \\ r_w \end{bmatrix}, \tag{22}$$

where $D_i(r)$, $i = 1, 2$, is domain of constraints for α_w, ε_w, r_w. Then the norm δ_w can be calculated from the solution (8) of the Cauchy problem $w(x^{(n)}, \alpha_w, \varepsilon_w, r_w) = x^{(n)}$. The accuracy of the data approximation by models CLC and ALC is compared by $\Delta = \delta_y/\delta_w$. It is assumed that feasible relative error value is 5%.

The proposed ALC model was tested on numerous of production data sets. Three situations of the production data analyzed can be defined:

1. Repetitive orders taking place in long time and random intervals;
2. Orders with low volume, prototype production;
3. Orders with high volume.

Since many data sets were analyzed, to show the performance of the ALC model; three different examples are presented for each group of production data.

The largest group of all of the company's products is the repetitive orders arriving in random and longtime intervals. This means that operators are familiar with the product,

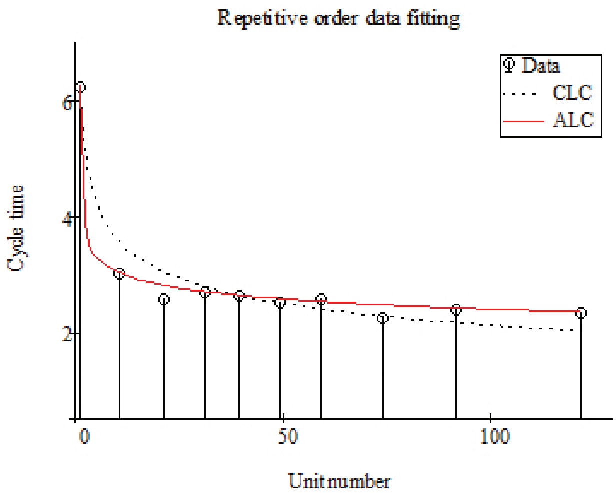

Figure 3. CLC and ALC fitting for repetitive orders taking place in long time and random intervals.

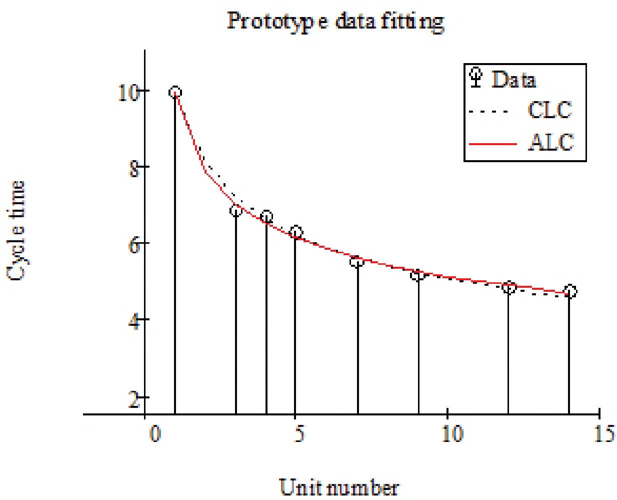

Figure 4. CLC and ALC fitting for orders with low volume (prototype production).

however they do need time to remember the assembly at the beginning of the production cycle. The typical data set representing this situation is depicted in Fig. 3.

Figure 3 shows, how CLC model results fairly poor approximation to compare with ALC (see Table 1). CLC improves gradually, however perturbation parameter in ALC enables to approximate steeper improvement of the operating time.

Other groups of production orders are small order production and prototype production. These orders are mostly singular, with quantities up to 20 pieces. The example in Fig. 4 presents such production data.

Table 1. Comparison of CLC and ALC approximation results.

Experiment	Fig. 3	Fig. 4	Fig. 5
N	10	8	21
n	5	1	10
Prod. qty.	122	14	2210
α_y	0.2277	0.2961	0.5057
α_w	0.0998	0.2657	0.0000
ε_w	-6.5885	-2.4143	-28.9483
r_w	3.7900	5.5750	2.0770
δ_y	8.80%	2.21%	14.91%
δ_w	3.47%	1.73%	3.73%
δ_y/δ_w	2.54	1.28	4.00

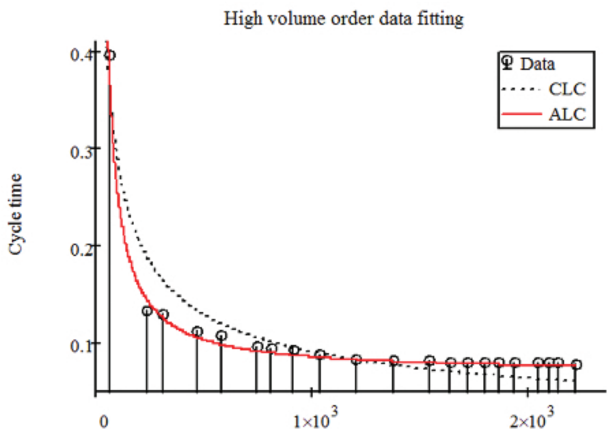

Figure 5. CLC and ALC fitting for orders with high volume.

Please note that on such data, both methods show fairly good results and provide accurate approximation, although, ALC shows slightly better result than CLC (see Table 1).

The last group of significant wiring harnesses is those with high volume orders. These harnesses possess orders up to several thousand pieces. A typical example of such production data is presented in Fig. 5.

Needed to emphasize that in this group the stabilization of operating time exists. This stabilization is known as the plateauing phenomenon and can obviously be identified in the high volume orders of this company's production. In addition, CLC is unsuitable for such data approximation (relative error is higher than 5%); however ALC shows quite accurate results and confirms the adequacy by falling within the limits of the feasible relative error value (see Table 1). CLC again improves gradually when perturbation parameter in ALC enables approximation of assembly time stabilization.

In general, ALC shows significantly better curve fitting results than CLC for all examples analyzed. In other words, ALC approximates real manufacturing data more accurately. There are several reasons for this result. First of all, the limitations of CLC model became clearly visible in the new manufacturing environment. Moreover, additional parameters introduced to ALC made this model more flexible and versatile and resulted more accurate fitting results. Also, in CLC there are two parameters with certain meaning α – learning slope, β – assembly of the first unit). In ALC, there are more parameters with open meaning, therefore, these parameters became pure curve fitting parameters. This is an advantage in curve fitting, on the other hand, it becomes complicated to understand the particular meaning of each parameter used in ALC. For instance, parameter α in ALC and CLC only for prototype data is somewhat similar (Table 1), but remaining parameters are not equal to zero, so they have impact to ALC. It means that parameter α has different meaning in ALC and CLC. The meaning of the remaining ALC parameters (ε and r) should be also identified in a view of manufacturing environment.

5 Conclusions

In this paper, an almost learning curve model is presented; model allows more accurate approximation of production data than traditional CLC model. The proposed ALC model is based on the solutions of special (with perturbation parameter) differential equations. Additional variable (perturbation parameter) improves versatility of the production data fitting. Sufficient conditions for the perturbation parameter and other parameters of ALC that enable ALC to have all necessary CLC properties are determined.

Proposed ALC model was tested on the wiring harness manufacturer production data. This company was selected due to the fact that it suffers heavy non-linear production rates. Three different groups of products were analyzed: repetitive orders taking place in long time and random intervals; orders with low volume, prototype production; orders with high volume. For repetitive orders and high volume orders with steeper operating time improvement and further stabilization, developed ALC model approximated data definitely better than classical LC. For the small order production (prototypes, small series) both models LC and ALC delivered fairly the same approximation results. This proves ALC model versatility.

Use of the ALC showed an additional insight into the analysis of learning and skill development. Data analysis performed in this research confirmed the adequacy of ALC model and its superiority over CLC, thus completing the objective of this article. Such modeling has an important managerial insight, because ALC provides more accurate data approximation than existing learning curve models. Therefore, these models have a wide application possibilities in production planning and scheduling, control, batch size optimization and other production problems.

Even the adequacy of ALC is proved, the application of it has some limitations. First of all, the perturbation parameter is not fixed. In other words, before the implementation an excessive data analysis should be performed in order to estimate parameters necessary for the application. Also, some changes and adjustment for perturbation parameter might be needed as well. Therefore, additional data analysis is required to fully understand ALC parameters and their relation with particular manufacturing environment. Moreover, all the parameters considered in ALC are deterministic, it is important to analyze ALC performance under uncertainty as well. These topics should be considered for the further research.

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