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Parameter identification based on finite-time synchronization for Cohen–Grossberg neural networks with time-varying delays*

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Abstract. In this paper, the finite-time synchronization problem for chaotic Cohen–Grossberg neural networks with unknown parameters and time-varying delays is investigated by using finite-time stability theory. Firstly, based on the parameter identification of uncertain delayed neural networks, a simple and effective feedback control scheme is proposed to tackle the unknown parameters of the addressed network. Secondly, by modifying the error dynamical system and using some inequality techniques, some novel and useful criteria for the finite-time synchronization of such a system are obtained. Finally, an example with numerical simulations is given to show the feasibility and effectiveness of the developed methods.

Keywords: Cohen–Grossberg neural network, finite-time synchronization, parameter identification, time-varying delay.

1 Introduction

Over the past decades, Cohen–Grossberg neural network (CGNN) model has been extensively studied by many researchers due to its broad application in many areas such as parallel computation, associative memory, signal processing, especially in solving some difficult optimization problems [2, 7, 33]. There are many papers concerning stability analysis and periodic oscillation of CGNNs [6, 8, 21, 24, 35, 36]. On the other hand, since neural network systems may also exhibit oscillation or chaotic behaviors, synchronization of chaotic CGNNs has attracted tremendous attention by many researchers from a wide range of disciplines. The study of the synchronization of chaotic CGNNs is also an important step for both understanding brain science and designing coupled neural networks for real world applications.

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Being a unique and very relevant nonlinear phenomenon, chaos has been intensively investigated in the context of several specific problems arising in physics, mathematics, engineering science, and secure communication, etc. Synchronization means two or more systems which are either chaotic or periodic share a common dynamical behavior and it has been shown that this common behavior can be induced by coupling or by external force. Due to this property, chaos synchronization has been successfully applied in a variety of fields, including secure communication, chemical and biological systems, human heartbeat regulation, information science, image processing, and harmonic oscillation generation, etc. [9,10,20,29]. Up to now, a wide variety of approaches have been proposed for synchronization of chaotic systems, such as adaptive control [18, 28], observer-based control [26], impulsive control [17, 25], fuzzy control [16, 37], coupling control [15], periodically intermittent control [4, 12, 13], and so on.

However, most of the above mentioned studies have assumed that the parameters of chaotic systems are known in advance. But in many practical situations, the parameters of chaotic systems are inevitably perturbed by external inartificial factors and the values of these parameters cannot be exactly known in advance, and the synchronization will be destroyed and broken by the effects of these uncertainties. Therefore, the investigation of synchronizing two chaotic systems with unknown parameters has become an important research issue [19, 22, 31]. In [14], by combining the adaptive control and linear feedback with update law, the authors investigated the synchronization of a class of chaotic Hopfield neural networks with known or unknown parameters. Based on the Lyapunov stability theory and by utilizing adaptive linear feedback control technique, the synchronization problem of chaotic CGNNs with unknown parameters and mixed time-varying delays was studied in [11]. Nevertheless, the works in [11] concerning the synchronization of CGNNs mainly focus on the Lipschitzian amplification gains and unknown parameters. There are no results for the synchronization of CGNNs with the general amplification functions, unknown parameters and time-varying delays. Therefore, it is interesting to study this problem both in theory and in applications, so there exists an open room for further improvement.

Another thing is worth to note that, all of the methods mentioned above, have been used to guarantee the asymptotic stability or exponential stability of the synchronization error dynamics. This means that the trajectories of the slave system can reach to the trajectories of the master system over the infinite horizon. In the practical engineering process, however, it is more reasonable that synchronization objective is realized in a finite horizon. To achieve faster synchronization in control systems, an effective method is using finite-time control techniques. Finite-time synchronization means the optimality in convergence time. Moreover, the finite-time control techniques have demonstrated better robustness and disturbance rejection properties [3, 5, 30]. In [1], by introducing nonsingular terminal sliding surface and designing adaptive controller, the authors studied the finite-time chaos synchronization problem between two different chaotic systems with unknown parameters. In [27], based on the finite-time stability theory and by designing feedback controller, the authors investigated the finite-time synchronization problem between two chaotic cellular neural networks with constant delays. Nevertheless, the given updated laws in [27] are highly complex and are not easily applicable. Whether it is possible to realize the finite-time synchronization and parameter identification of chaotic neural networks by designing simpler updated laws is an interesting problem to study both in theory and in applications.

Motivated by the above discussions, in this paper, we deal with the problem of finite-time topology identification and synchronization for chaotic CGNNs with time-varying delays and unknown parameters. Based on the adaptive feedback control and finite-time convergence theory, we establish some useful sufficient conditions on the finite-time synchronization of addressed model.

The rest of the paper is organized as follows. In Section 2, the master system and the slave system are introduced. In addition, some assumptions and definitions together with some useful lemmas needed in this paper are presented. Next section is devoted to investigate the finite-time topology identification and synchronization between two chaotic CGNNs with time-varying delays. In Section 4, an example with its numerical simulations is given to illustrate the effectiveness of the obtained results. Finally, some general conclusions are drawn in Section 5.

2 Preliminaries

Consider the following n-dimensional CGNNs with time-varying delays:

$$\begin{aligned}\dot{x}_i(t) = a_i\big(x_i(t)\big)\Bigg[&-b_i\big(x_i(t)\big) + \sum_{j=1}^{n} c_{ij} f_j\big(x_j(t)\big)\\ &+ \sum_{j=1}^{n} d_{ij} g_j\big(x_j\big(t-\tau_j(t)\big)\big) + I_i\Bigg],\end{aligned}\tag{1}$$

where $i \in \mathcal{I} \triangleq \{1, 2, \dots, n\}$, $n \geqslant 2$, denotes the number of neurons in the neural network; x_i corresponds to the state variable of the ith unit; a_i represents an amplification function; b_i is an appropriate behaved function; f_j and g_j are the activation functions; $\tau_j(t)$ corresponds to the transmission delay along the axon of the jth unit and satisfy $\tau_j(t) \geqslant 0$ for $t > 0$, and I_i denotes the external input. Concerning coefficients c_{ij}, d_{ij} denote respectively, the synaptic connection weight and the delayed synaptic connection weight of the unit j on the unit i.

Throughout the paper, we always use $i, j \in \mathcal{I}$, unless otherwise stated. The initial conditions associated with system (1) are given by

$$x_i(s) = \varphi_i(s), \quad s \in [-\tau, 0],\tag{2}$$

where $\tau = \max_{i\in\mathcal{I}}\{\sup_{t\in R^+} \tau_i(t)\}$, $\varphi_i(s) \in C([-\tau, 0], R)$, which denotes the Banach space of all continuous functions mapping $[-\tau, 0]$ into R with $\infty-$norm defined by

$$\|\varphi\|_\infty = \max_{i\in\mathcal{I}}\Big\{\sup_{s\in[-\tau,0]} \big|\varphi_i(s)\big|\Big\}.$$

In this paper, we refer to model (1) as the master system, the slave system is given as follows:

$$\dot{y}_i(t) = a_i\big(y_i(t)\big)\Bigg[-b_i\big(y_i(t)\big) + \sum_{j=1}^{n} \tilde{c}_{ij} f_j\big(y_j(t)\big) + \sum_{j=1}^{n} \tilde{d}_{ij} g_j\big(y_j\big(t-\tau_j(t)\big)\big) + I_i\Bigg] + u_i(t), \tag{3}$$

where $y_i(t)$ corresponds to the slave state variable of the ith neuron; $\tilde{c}_{ij}$ and $\tilde{d}_{ij}$ are estimated values of synaptic connection weights c_{ij} and d_{ij}, respectively; $u_i(t)$ indicates the control input. The initial conditions associated with system (3) are given by

$$y_i(s) = \phi_i(s), \quad s \in [-\tau, 0],\ \phi_i(s) \in C\big([-\tau, 0], R\big). \tag{4}$$

The goal of this paper is to design and implement suitable controller $u_i(t)$ for the slave system and parameters' adaptive estimation laws of $\tilde{c}_{ij}$ and $\tilde{d}_{ij}$ such that the controlled slave system (3) could be synchronous with the master system (1) in a finite time T, and all the parameters $\tilde{c}_{ij}(t) \to c_{ij}$, $\tilde{d}_{ij}(t) \to d_{ij}$ as $t \to T$, $\tilde{c}_{ij}(t) = c_{ij}$, $\tilde{d}_{ij}(t) = d_{ij}$ for $t \geqslant T$.

Let $e_i(t) = y_i(t) - x_i(t)$ for $i \in \mathcal{I}$, then from systems (1) and (3), the error dynamical system can be derived as

$$\begin{aligned}\dot{e}_i(t) = {}& a_i\big(y_i(t)\big)\Bigg[-b_i\big(y_i(t)\big) + \sum_{j=1}^{n} \tilde{c}_{ij} f_j\big(y_j(t)\big) + \sum_{j=1}^{n} \tilde{d}_{ij} g_j\big(y_j\big(t-\tau_j(t)\big)\big) \\ & + I_i + \tilde{u}_i(t)\Bigg] + a_i\big(x_i(t)\big)\Bigg[b_i\big(x_i(t)\big) - \sum_{j=1}^{n} c_{ij} f_j\big(x_j(t)\big) \\ & - \sum_{j=1}^{n} d_{ij} g_j\big(x_j\big(t-\tau_j(t)\big)\big) - I_i\Bigg],\end{aligned} \tag{5}$$

where $\tilde{u}_i(t) = u_i(t)/a_i(y_i(t))$.

It is clear that the finite-time synchronization problem between systems (1) and (3) can be transformed to the equivalent problem of the finite-time stabilization of the error system (5).

Throughout this paper, we assume that the following assumptions are satisfied.

Assumption 1. $a_i(\cdot) \in C(R, R^+)$ and there exist positive constants $\underline{a_i}$ and $\overline{a}_i$ such that

$$\underline{a_i} \leqslant a_i(u) \leqslant \overline{a}_i \quad \text{for all } u \in R.$$

Assumption 2. For each i, $b_i(u)$ is continuous and there exists a positive constant η_i such that

$$\frac{b_i(u) - b_i(v)}{u - v} \geqslant \eta_i \quad \text{for } u, v \in R \text{ and } u \neq v.$$

Assumption 3. The activation functions f_j, g_j are continuous and there exist Lipschitz constants L_j^1, L_j^2 such that, for each $j \in \mathcal{I}$,

$$\left|f_j(u) - f_j(v)\right| \leqslant L_j^1|u - v|, \quad \left|g_j(u) - g_j(v)\right| \leqslant L_j^2|u - v| \quad \text{for all } u, v \in R.$$

Definition 1. The array of systems in the neural networks are said to be finite-time synchronized and topology identified if there exists a constant $T > 0$ such that

$$\lim_{t\to T}\left(y_i(t) - x_i(t)\right) = \lim_{t\to T}\left(\tilde{c}_{ij}(t) - c_{ij}\right) = \lim_{t\to T}\left(\tilde{d}_{ij}(t) - d_{ij}\right) = 0$$

for any $i, j \in \mathcal{I}$, and $y_i(t) - x_i(t) = \tilde{c}_{ij}(t) - c_{ij} = \tilde{d}_{ij}(t) - d_{ij} = 0$ if $t > T$. The constant T called the settling time.

Lemma 1. (*See* [30].) *Assume that a continuous, positive-definite function $V(t)$ satisfies the following differential inequality*:

$$\dot{V}(t) \leqslant -\alpha V^{\eta}(t) \quad \forall t \geqslant t_0, \qquad V(t_0) \geqslant 0,$$

where $\alpha > 0$, $0 < \eta < 1$ are two constants. Then, for any given t_0, $V(t)$ satisfies the following inequality:

$$V^{1-\eta}(t) \leqslant V^{1-\eta}(t_0) - \alpha(1-\eta)(t - t_0), \quad t_0 \leqslant t \leqslant T,$$

and

$$V(t) \equiv 0 \quad \forall\, t \geqslant T$$

with T given by

$$T = t^0 + \frac{V^{1-\eta}(t_0)}{\alpha(1-\eta)}.$$

Lemma 2. (*See* [1].) *For $a_1, a_2, \ldots, a_n \in R$, the following inequality holds*:

$$|a_1| + |a_2| + \cdots + |a_n| \geqslant \sqrt{|a_1^2| + |a_2^2| + \cdots + |a_n^2|}.$$

3 Main results

In this section, we will derive some criteria to guarantee the finite-time synchronization between master system (1) and slave system (3).

Suppose that $x_i(t)$, $y_i(t)$ are the arbitrary solutions of systems (1) and (3) with initial conditions φ_i, ϕ_i, respectively, and let

$$\tilde{e}_i(t) = \int\limits_{x_i(t)}^{y_i(t)} \frac{\mathrm{d}s}{a_i(s)}, \quad i \in \mathcal{I}. \tag{6}$$

Calculating the time derivative of $\tilde{e}_i(t)$ along the trajectories of systems (1) and (3), we get

$$\begin{aligned}\dot{\tilde{e}}_i(t) &= -b_i\big(e_i(t)\big) + \sum_{j=1}^{n} \bar{c}_{ij} f_j\big(y_j(t)\big) + \sum_{j=1}^{n} c_{ij} f_j\big(e_j(t)\big) \\ &\quad + \sum_{j=1}^{n} \bar{d}_{ij} g_j\big(y_j\big(t-\tau_j(t)\big)\big) + \sum_{j=1}^{n} d_{ij} g_j\big(e_j\big(t-\tau_j(t)\big)\big) + \tilde{u}_i(t), \end{aligned} \tag{7}$$

where $\bar{c}_{ij} = \tilde{c}_{ij} - c_{ij}$, $\bar{d}_{ij} = \tilde{d}_{ij} - d_{ij}$, $b_i(e_i(t)) = b_i(y_i(t)) - b_i(x_i(t)), f_j(e_j(t)) = f_j(y_j(t)) - f_j(x_j(t))$ and $g_j(e_j(t-\tau_j(t))) = g_j(y_j(t-\tau_j(t))) - g_j(x_j(t-\tau_j(t)))$.

From Assumption 1, we have

$$\frac{|e_i(t)|}{\bar{a}_i} \leqslant \big|\tilde{e}_i(t)\big| \leqslant \frac{|e_i(t)|}{\underline{a_i}}, \quad i \in \mathcal{I}. \tag{8}$$

It is not difficult to see that $\lim_{t\to T} e_i(t) = 0$ if only if $\lim_{t\to T} \tilde{e}_i(t) = 0$. Thus, to study the finite-time stabilization of the error system (4), we only need to study the finite-time stabilization of the corresponding modified error system (7). In order to achieve this aim, we design finite-time controller $\tilde{u}_i(t)$ in system (7) as follows:

$$\tilde{u}_i(t) = b_i\big(e_i(t)\big) - \sum_{j=1}^{n} c_{ij} f_j\big(e_j(t)\big) - \sum_{j=1}^{n} d_{ij} g_j\big(e_j\big(t-\tau_j(t)\big)\big) - k\,\mathrm{sign}\big(\tilde{e}_i(t)\big), \tag{9}$$

where $k > 0$ denotes a tunable constant.

In addition, to tackle the unknown parameters, we propose the following updating laws:

$$\begin{aligned}\dot{\tilde{c}}_{ij}(t) &= -p_{ij}\left[\tilde{e}_i(t) f_j\big(y_j(t)\big) + \frac{k}{\sqrt{p_{ij}}}\,\mathrm{sign}\big(\tilde{c}_{ij} - c_{ij}\big)\right], \\ \dot{\tilde{d}}_{ij}(t) &= -q_{ij}\left[\tilde{e}_i(t) g_j\big(y_j\big(t-\tau_j(t)\big)\big) + \frac{k}{\sqrt{q_{ij}}}\,\mathrm{sign}\big(\tilde{d}_{ij} - d_{ij}\big)\right], \end{aligned} \tag{10}$$

where p_{ij} and q_{ij} are arbitrary positive constants.

Theorem 1. *Under Assumption* 1, *if the modified error system* (7) *is controlled with the control laws* (9) *and adaptive laws* (10), *then the synaptic connection weight coefficients c_{ij} and d_{ij} of network* (1) *can be identified with $\tilde{c}_{ij}$ and $\tilde{d}_{ij}$, and the slave network* (3) *can synchronize with the master network* (1) *in a finite time*

$$T_1 = \frac{V^{1/2}(0)}{\sqrt{2}k}, \tag{11}$$

where $V(0) = (1/2)\sum_{i=1}^{n} \tilde{e}_i^2(0) + (1/2)\sum_{i=1}^{n}\sum_{j=1}^{n}(1/p_{ij})(\tilde{c}_{ij}(0) - c_{ij})^2 + (1/2) \times \sum_{i=1}^{n}\sum_{j=1}^{n}(1/q_{ij})(\tilde{d}_{ij}(0) - d_{ij})^2$. *Here $\tilde{c}_{ij}(0)$ and $\tilde{d}_{ij}(0)$ are the initial values of the adaptive parameters $\tilde{c}_{ij}$ and $\tilde{d}_{ij}$, respectively.*

Proof. Consider the following Lyapunov function:

$$V(t) = \frac{1}{2}\sum_{i=1}^{n}\tilde{e}_i^2 + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{p_{ij}}\bar{c}_{ij}^2 + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{q_{ij}}\bar{d}_{ij}^2,$$

where $\bar{c}_{ij} = \tilde{c}_{ij} - c_{ij}$, $\bar{d}_{ij} = \tilde{d}_{ij} - d_{ij}$. It is obvious that $\dot{\bar{c}}_{ij} = \dot{\tilde{c}}_{ij}$ and $\dot{\bar{d}}_{ij} = \dot{\tilde{d}}_{ij}$. Taking the time derivative of $V(t)$, one has

$$\dot{V}(t) = \sum_{i=1}^{n}\tilde{e}_i\dot{\tilde{e}}_i + \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{p_{ij}}\bar{c}_{ij}\dot{\tilde{c}}_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{q_{ij}}\bar{d}_{ij}\dot{\tilde{d}}_{ij}. \tag{12}$$

Introducing $\dot{\tilde{e}}_i(t)$ from (7) and adaptive laws (10) into (12), we have

$$\begin{aligned}
\dot{V}(t) = {}& \sum_{i=1}^{n}\tilde{e}_i\Bigg[-b_i\big(e_i(t)\big) + \sum_{j=1}^{n}\bar{c}_{ij}f_j\big(y_j(t)\big) + \sum_{j=1}^{n}\bar{d}_{ij}g_j\big(y_j\big(t-\tau_j(t)\big)\big) \\
& + \tilde{u}_i(t) + \sum_{j=1}^{n}c_{ij}f_j\big(e_j(t)\big) + \sum_{j=1}^{n}d_{ij}g_j\big(e_j\big(t-\tau_j(t)\big)\big)\Bigg] \\
& - \sum_{i=1}^{n}\sum_{j=1}^{n}\bar{c}_{ij}\bigg[\tilde{e}_i(t)f_j\big(y_j(t)\big) + \frac{k}{\sqrt{p_{ij}}}\operatorname{sign}(\bar{c}_{ij})\bigg] \\
& - \sum_{i=1}^{n}\sum_{j=1}^{n}\bar{d}_{ij}\bigg[\tilde{e}_i(t)g_j\big(y_j\big(t-\tau_j(t)\big)\big) + \frac{k}{\sqrt{q_{ij}}}\operatorname{sign}(\bar{d}_{ij})\bigg].
\end{aligned}$$

Replacing $\tilde{u}_i(t)$ from (9) into the above equation and using $|u| = \operatorname{sign}(u)u$, gives

$$\begin{aligned}
\dot{V}(t) = {}& \sum_{i=1}^{n}\tilde{e}_i\Bigg[-b_i\big(e_i(t)\big) + \sum_{j=1}^{n}\bar{c}_{ij}f_j\big(y_j(t)\big) + \sum_{j=1}^{n}\bar{d}_{ij}g_j\big(y_j\big(t-\tau_j(t)\big)\big) \\
& + \sum_{j=1}^{n}c_{ij}f_j\big(e_j(t)\big) + \sum_{j=1}^{n}d_{ij}g_j\big(e_j\big(t-\tau_j(t)\big)\big) + b_i\big(e_i(t)\big) \\
& - \sum_{j=1}^{n}c_{ij}f_j\big(e_j(t)\big) - \sum_{j=1}^{n}d_{ij}g_j\big(e_j\big(t-\tau_j(t)\big)\big) - k\operatorname{sign}\big(\tilde{e}_i(t)\big)\Bigg] \\
& - \sum_{i=1}^{n}\sum_{j=1}^{n}\bar{c}_{ij}\bigg[\tilde{e}_i(t)f_j\big(y_j(t)\big) + \frac{k}{\sqrt{p_{ij}}}\operatorname{sign}(\bar{c}_{ij})\bigg] \\
& - \sum_{i=1}^{n}\sum_{j=1}^{n}\bar{d}_{ij}\bigg[\tilde{e}_i(t)g_j\big(y_j\big(t-\tau_j(t)\big)\big) + \frac{k}{\sqrt{q_{ij}}}\operatorname{sign}(\bar{d}_{ij})\bigg] \\
= {}& -k\sum_{i=1}^{n}|\tilde{e}_i| - k\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\sqrt{p_{ij}}}|\bar{c}_{ij}| - k\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\sqrt{q_{ij}}}|\bar{d}_{ij}|.
\end{aligned}$$

Using Lemma 2, we get

$$\begin{aligned}\dot{V}(t) &\leqslant -\sqrt{2}k\left(\frac{1}{2}\sum_{i=1}^{n}\tilde{e}_i^2+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\sqrt{p_{ij}}}\bar{c}_{ij}^2+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\sqrt{q_{ij}}}\bar{d}_{ij}^2\right)^{1/2}\\ &= -\sqrt{2}kV^{1/2}(t).\end{aligned}$$

Therefore, from Lemma 1, the modified error system (7) will converge to zero within T_1. That is, the error system (5) will converge to zero within T_1. Thus, under the control laws (9) and updated laws (10), the two chaotic CGNNs are synchronized and identified simultaneously in the finite-time T_1. The proof of Theorem 1 is completed. □

Remark 1. If $\lim_{t\to T}\dot{e}(t)$ exists, then we have $\lim_{t\to T}\dot{e}(t)=0$ for $\lim_{t\to T}e(t)=0$. By error system (7), we can obtain $\lim_{t\to T}\sum_{j=1}^{n}\bar{c}_{ij}f_j(y_j(t))=0$ and $\lim_{t\to T}\sum_{j=1}^{n}\bar{d}_{ij}\times g_j(y_j(t-\tau_j(t)))=0$. When $\{f_j(y_j(t))\}_{j=1}^{n}$ and $\{g_j(y_j(t-\tau_j(t)))\}_{j=1}^{n}$ are linearly independent on the orbit $\{f_j(y_j(t))\}_{j=1}^{n}$ and $\{g_j(y_j(t-\tau_j(t)))\}_{j=1}^{n}$, respectively, of synchronization manifold, then $\lim_{t\to T}\bar{c}_{ij}=0$ and $\lim_{t\to T}\bar{d}_{ij}=0$ (see [23, 34]). We can get that $\lim_{t\to T}\tilde{c}_{ij}=c_{ij}$ and $\lim_{t\to T}\tilde{d}_{ij}=d_{ij}$ for $i,j\in\mathcal{I}$; that is, the uncertain synaptic connection strengths c_{ij} and d_{ij} can be successfully identified in the finite-time.

Remark 2. According to Theorem 1, for all $i,j\in\mathcal{I}$, the following properties are guaranteed:

1. $e_i,\tilde{c}_{ij},\tilde{d}_{ij}\in L_\infty\cap L_2$;
2. $\lim_{t\to T}x_i=y_i$, $\lim_{t\to T}\tilde{c}_{ij}=c_{ij}$, $\lim_{t\to T}\tilde{d}_{ij}=d_{ij}$, $\tilde{c}_{ij}=c_{ij}$, $\tilde{d}_{ij}=d_{ij}$ for $t\geqslant T$;
3. $\lim_{t\to T}\dot{\tilde{c}}_{ij}=0$, $\lim_{t\to T}\dot{\tilde{d}}_{ij}=0$.

Assume that the synaptic connection weight coefficients c_{ij} or d_{ij} of the master system (1) are known, i.e. $\tilde{c}_{ij}=c_{ij}$ or $\tilde{d}_{ij}=d_{ij}$. Then, from Theorem 1, we can obtain the following corollaries.

Corollary 1. *Suppose Assumption* 1 *hold and $\tilde{c}_{ij}=c_{ij}$ for all $i,j\in\mathcal{I}$. If we use finite-time controller* (9) *and the following updating laws*:

$$\dot{\tilde{d}}_{ij}(t)=-q_{ij}\left[\tilde{e}_i(t)g_j\big(y_j\big(t-\tau_j(t)\big)\big)+\frac{k}{\sqrt{q_{ij}}}\operatorname{sign}(\tilde{d}_{ij}-d_{ij})\right],\tag{13}$$

where q_{ij} are arbitrary constants and k denotes a tunable constant, then the synaptic connection weight coefficients d_{ij} of network (1) *can be identified with $\tilde{d}_{ij}$, and the slave network* (3) *can synchronize with the master network* (1) *in a finite time*

$$T_2=\frac{V^{1/2}(0)}{\sqrt{2}k},\tag{14}$$

where $V(0)=(1/2)\sum_{i=1}^{n}\tilde{e}_i^2(0)+(1/2)\sum_{i=1}^{n}\sum_{j=1}^{n}(1/q_{ij})(\tilde{d}_{ij}(0)-d_{ij})^2$.

Corollary 2. *Suppose Assumption* 1 *hold and* $\tilde{d}_{ij} = d_{ij}$ *for all* $i, j \in \mathcal{I}$. *If we use finite-time controller* (9) *and the following updating laws*:

$$\dot{\tilde{c}}_{ij}(t) = -p_{ij}\left[\tilde{e}_i(t) f_j\big(y_j(t)\big) + \frac{k}{\sqrt{p_{ij}}}\,\mathrm{sign}(\tilde{c}_{ij} - c_{ij})\right], \tag{15}$$

where p_{ij} *are arbitrary constants and* k *denotes a tunable constant, then the synaptic connection weight coefficients* c_{ij} *of network* (1) *can be identified with* $\tilde{c}_{ij}$, *and the slave network* (3) *can synchronize with the master network* (1) *in a finite time*

$$T_3 = \frac{V^{1/2}(0)}{\sqrt{2}k}, \tag{16}$$

where $V(0) = (1/2)\sum_{i=1}^n \tilde{e}_i^2(0) + (1/2)\sum_{i=1}^n\sum_{j=1}^n (1/p_{ij})(\tilde{c}_{ij}(0) - c_{ij})^2$.

Corollary 3. *Suppose Assumption* 1 *hold and* $\tilde{c}_{ij} = c_{ij}$, $\tilde{d}_{ij} = c_{ij}$ *for all* $i, j \in \mathcal{I}$. *Using the finite-time controller* (9), *the slave network* (3) *can synchronize with the master network* (1) *in a finite time*

$$T_4 = \frac{V^{1/2}(0)}{\sqrt{2}k}, \tag{17}$$

where $V(0) = (1/2)\sum_{i=1}^n \tilde{e}_i^2(0)$.

In system (1), if the amplification function $a_i(u) \equiv 1$ for all $u \in R$ and $i \in \mathcal{I}$, then the mater system (1) become

$$\dot{x}_i(t) = -b_i\big(x_i(t)\big) + \sum_{j=1}^n c_{ij} f_j\big(x_j(t)\big) + \sum_{j=1}^n d_{ij} f_j\big(x_j\big(t - \tau_j(t)\big)\big) + I_i. \tag{18}$$

Definitely, the Assumption 1 is satisfied in this case. Accordingly, the slave system is reduced to the following form:

$$\dot{y}_i(t) = -b_i\big(y_i(t)\big) + \sum_{j=1}^n \tilde{c}_{ij} f_j\big(y_j(t)\big) + \sum_{j=1}^n \tilde{d}_{ij} g_j\big(y_j\big(t - \tau_j(t)\big)\big) + I_i + u_i(t). \tag{19}$$

In this case, we have $\tilde{e}_i(t) = e_i(t)$ and $\tilde{u}_i(t) = u_i(t)$. Thus, from Theorem 1, we have the following corollary.

Corollary 4. *If we use the following finite-time controller*:

$$u_i(t) = b_i\big(e_i(t)\big) - \sum_{j=1}^n c_{ij} f_j\big(e_j(t)\big) - \sum_{j=1}^n d_{ij} g_j\big(e_j\big(t - \tau_j(t)\big)\big) - k\,\mathrm{sign}\big(e_i(t)\big) \tag{20}$$

and updating laws

$$\begin{aligned}
\dot{\tilde{c}}_{ij}(t) &= -p_{ij}\left[e_i(t) f_j\big(y_j(t)\big) + \frac{k}{\sqrt{p_{ij}}}\,\mathrm{sign}(\tilde{c}_{ij} - c_{ij})\right],\\
\dot{\tilde{d}}_{ij}(t) &= -q_{ij}\left[e_i(t) g_j\big(y_j\big(t - \tau_j(t)\big)\big) + \frac{k}{\sqrt{q_{ij}}}\,\mathrm{sign}(\tilde{d}_{ij} - d_{ij})\right],
\end{aligned} \tag{21}$$

where p_{ij}, q_{ij} are arbitrary constants, k denotes a tunable constant, then the synaptic connection weight coefficients c_{ij} and d_{ij} of network (18) *can be identified with $\tilde{c}_{ij}$ and $\tilde{d}_{ij}$, and the slave network* (19) *can synchronize with the master network* (18) *in a finite time*

$$T_5 = \frac{V^{1/2}(0)}{\sqrt{2}k}, \tag{22}$$

where $V(0) = (1/2)\sum_{i=1}^n \tilde{e}_i^2(0) + (1/2)\sum_{i=1}^n \sum_{j=1}^n (1/p_{ij})(\tilde{c}_{ij}(0) - c_{ij})^2 + (1/2) \times \sum_{i=1}^n \sum_{j=1}^n (1/q_{ij})(\tilde{d}_{ij}(0) - d_{ij})^2$. *Here $\tilde{c}_{ij}(0)$ and $\tilde{d}_{ij}(0)$ are the initial values of the adaptive parameters $\tilde{c}_{ij}$ and $\tilde{d}_{ij}$, respectively.*

Remark 3. In the Theorem 1 and above four corollaries, by using special adaptive controller and updating laws, we achieved the finite-time synchronization between two chaotic CGNNs and their parameters were successfully identified. However, the used control law $u_i(t)$ is somehow expensive and not easily applicable, especially if the parameters of the master system satisfy some special condition. Below, we will modify the adaptive laws $u_i(t)$ to improve the applicability of our results.

Suppose that the activation functions g_i are bounded, and let

$$\bar{e}_i(t) = \operatorname{sign}\bigl(y_i(t) - x_i(t)\bigr) \int\limits_{x_i(t)}^{y_i(t)} \frac{\mathrm{d}s}{a_i(s)}. \tag{23}$$

Then, from master system (1) and slave system (3), we have the following modified error system:

$$\begin{aligned} \dot{\bar{e}}_i(t) = \operatorname{sign}\bigl(e_i(t)\bigr)\Biggl[&-b_i\bigl(e_i(t)\bigr) + \sum_{j=1}^n \bar{c}_{ij} f_j\bigl(y_j(t)\bigr) + \sum_{j=1}^n c_{ij} f_j\bigl(e_j(t)\bigr) \\ &+ \sum_{j=1}^n \bar{d}_{ij} g_j\bigl(y_j\bigl(t-\tau_j(t)\bigr)\bigr) + \sum_{j=1}^n d_{ij} g_j\bigl(e_j\bigl(t-\tau_j(t)\bigr)\bigr) + \frac{u_i(t)}{a_i(y_i(t))}\Biggr], \end{aligned} \tag{24}$$

where $\bar{c}_{ij}, \bar{d}_{ij}, b_i(e_i(t), f_j(e_j(t)), g_j(e_j(t-\tau_j(t)))$ are the same as in (7). From Assumption 1, it is easy to check that

$$\frac{|e_i(t)|}{\overline{a}_i} \leqslant \bar{e}_i(t) \leqslant \frac{|e_i(t)|}{\underline{a_i}}. \tag{25}$$

Thus, the finite-time stabilization of the error system (5) is equal to the finite-time stabilization of the corresponding modified error system (24).

Theorem 2. *Assume that Assumptions* 1–3 *hold and activation functions g_i satisfy the inequality*

$$\bigl|g_i(u)\bigr| \leqslant M_i \quad \textit{for all } u \in R,\ i \in \mathcal{I}. \tag{26}$$

If the modified error system (24) *is controlled with the following updative laws*:

$$\begin{aligned}\dot{\tilde{c}}_{ij}(t) &= -p_{ij}\left[\bar{e}_i(t)f_j\big(y_j(t)\big) + \frac{k}{\sqrt{p_{ij}}}\operatorname{sign}(\tilde{c}_{ij}-c_{ij})\right],\\ \dot{\tilde{d}}_{ij}(t) &= -q_{ij}\left[\bar{e}_i(t)g_j\big(y_j\big(t-\tau_j(t)\big)\big) + \frac{k}{\sqrt{q_{ij}}}\operatorname{sign}(\tilde{d}_{ij}-d_{ij})\right]\end{aligned} \tag{27}$$

and control laws

$$u_i(t) = \sum_{j=1}^{n} k_{ij}e_j(t) - a_i\big(y_i(t)\big)(\bar{k}_i + k)\operatorname{sign}\big(e_i(t)\big), \tag{28}$$

where $k_{ii} \leqslant 0$ and $\bar{k}_i \geqslant 0$, and if all the control strengths k_{ij}, $\bar{k}_i$ satisfy the inequality

$$\begin{aligned}\max\Biggl\{&\max_{i\in\mathcal{I}}\Biggl\{-\underline{a_i}\biggl(\eta_i - \frac{k_{ii}}{\overline{a_i}}\biggr) + \sum_{j=1}^{n}\hat{c}_{ij} + \sum_{j=1,j\neq i}^{n}\hat{k}_{ij}\Biggr\},\\ &\max_{i\in\mathcal{I}}\Biggl\{\sum_{j=1}^{n}M_j|d_{ij}| - \bar{k}_i\Biggr\}\Biggr\} < 0,\end{aligned} \tag{29}$$

where $\hat{c}_{ij} = (|c_{ij}|L_j^1\overline{a}_j + |c_{ji}|L_i^1\overline{a}_i)/2$, $\hat{k}_{ij} = |k_{ij}|\overline{a}_j/(2\underline{a_i}) + |k_{ji}|\overline{a}_i/(2\underline{a_j})$, then the synaptic connection weight coefficients c_{ij} and d_{ij} of network (1) *can be identified with $\tilde{c}_{ij}$ and $\tilde{d}_{ij}$, and the slave network* (3) *can synchronize with the master network* (1) *in a finite time*

$$T_6 = \frac{V^{1/2}(0)}{\sqrt{2}k}, \tag{30}$$

where $V(0) = (1/2)\sum_{i=1}^{n}\bar{e}_i^2(0) + (1/2)\sum_{i=1}^{n}\sum_{j=1}^{n}(1/p_{ij})(\tilde{c}_{ij}(0)-c_{ij})^2 + (1/2)\times\sum_{i=1}^{n}\sum_{j=1}^{n}(1/q_{ij})(\tilde{d}_{ij}(0)-d_{ij})^2$. Here $\tilde{c}_{ij}(0)$ and $\tilde{d}_{ij}(0)$ are the initial values of the adaptive parameters $\tilde{c}_{ij}$ and $\tilde{d}_{ij}$, respectively.

Proof. Consider the following Lyapunov function:

$$V(t) = \frac{1}{2}\sum_{i=1}^{n}\bar{e}_i^2 + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{p_{ij}}\bar{c}_{ij}^2 + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{q_{ij}}\bar{d}_{ij}^2.$$

Calculating the time derivative of $V(t)$ along the trajectories of modified error system (24), from the adaptive laws (27), we have

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^{n}\bar{e}_i\dot{\bar{e}}_i + \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{p_{ij}}\bar{c}_{ij}\dot{\tilde{c}}_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{q_{ij}}\bar{d}_{ij}\dot{\tilde{d}}_{ij}\\ &= \sum_{i=1}^{n}\bar{e}_i\operatorname{sign}\big(e_i(t)\big)\Biggl[-b_i\big(e_i(t)\big) + \sum_{j=1}^{n}\bar{c}_{ij}f_j\big(y_j(t)\big) + \sum_{j=1}^{n}c_{ij}f_j(e_j(t))\\ &\quad + \sum_{j=1}^{n}\bar{d}_{ij}g_j\big(y_j\big(t-\tau_j(t)\big)\big) + \sum_{j=1}^{n}d_{ij}g_j\big(e_j\big(t-\tau_j(t)\big)\big) + \frac{u_i(t)}{a_i(y_i(t))}\Biggr]\end{aligned}$$

$$
\begin{aligned}
&- \sum_{i=1}^{n}\sum_{j=1}^{n} \bar{c}_{ij}\left[\bar{e}_i(t) f_j\big(y_j(t)\big) + \frac{k}{\sqrt{p_{ij}}}\operatorname{sign}(\bar{c}_{ij})\right] \\
&- \sum_{i=1}^{n}\sum_{j=1}^{n} \bar{d}_{ij}\left[\bar{e}_i(t) g_j\big(y_j\big(t-\tau_j(t)\big)\big) + \frac{k}{\sqrt{q_{ij}}}\operatorname{sign}(\bar{d}_{ij})\right].
\end{aligned}
$$

Replacing $u_i(t)$ from (28) into the above equation, from Assumptions 2–3 and inequality (25), one has

$$
\begin{aligned}
\dot{V}(t) = {}& \sum_{i=1}^{n} \bar{e}_i \operatorname{sign}\big(e_i(t)\big)\left[-b_i\big(e_i(t)\big) + \sum_{j=1}^{n} \bar{c}_{ij} f_j\big(y_j(t)\big)\right. \\
&+ \sum_{j=1}^{n} c_{ij} f_j\big(e_j(t)\big) + \sum_{j=1}^{n} \bar{d}_{ij} g_j\big(y_j\big(t-\tau_j(t)\big)\big) \\
&\left.+ \sum_{j=1}^{n} d_{ij} g_j\big(e_j\big(t-\tau_j(t)\big)\big) + \sum_{j=1}^{n} \frac{k_{ij} e_j(t)}{a_i(y_i(t))} - (\bar{k_i} + k)\operatorname{sign}\big(e_i(t)\big)\right] \\
&- \sum_{i=1}^{n}\sum_{j=1}^{n} \bar{c}_{ij}\left[\bar{e}_i(t) f_j\big(y_j(t)\big) + \frac{k}{\sqrt{p_{ij}}}\operatorname{sign}(\bar{c}_{ij})\right] \\
&- \sum_{i=1}^{n}\sum_{j=1}^{n} \bar{d}_{ij}\left[\bar{e}_i(t) g_j\big(y_j\big(t-\tau_j(t)\big)\big) + \frac{k}{\sqrt{q_{ij}}}\operatorname{sign}(\bar{d}_{ij})\right] \\
\leqslant {}& -\sum_{i=1}^{n} \underline{a_i}\left(\eta_i - \frac{k_{ii}}{\overline{a_i}}\right)\bar{e}_i^2(t) + \sum_{i=1}^{n}\sum_{j=1}^{n} \overline{a}_j L_j^1 |c_{ij}| \bar{e}_i(t)\bar{e}_j(t) \\
&+ \sum_{i=1}^{n}\sum_{j=1}^{n} M_j |d_{ij}| \bar{e}_i(t) + \sum_{j=1, j\neq i}^{n} \frac{\overline{a}_j}{\underline{a_i}} |k_{ij}| \bar{e}_i(t)\bar{e}_j(t) \\
&- (\bar{k_i} + k)\sum_{i=1}^{n} \bar{e}_i - k\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{\sqrt{p_{ij}}}|\bar{c}_{ij}| - k\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{\sqrt{q_{ij}}}|\bar{d}_{ij}|.
\end{aligned}
$$

By inequality (29) and $a^2 + b^2 \geqslant 2ab$, we get

$$
\begin{aligned}
\dot{V}(t) \leqslant {}& \sum_{i=1}^{n}\left[-\underline{a_i}\left(\eta_i - \frac{k_{ii}}{\overline{a_i}}\right) + \sum_{j=1}^{n} \hat{c}_{ij} + \sum_{j=1, j\neq i}^{n} \hat{k}_{ij}\right]\bar{e}_i^2(t) + \sum_{i=1}^{n}\sum_{j=1}^{n} M_j |d_{ij}| \bar{e}_i \\
&- \sum_{i=1}^{n} \bar{k_i}\bar{e}_i - k\sum_{i=1}^{n} \bar{e}_i - k\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{\sqrt{p_{ij}}}|\bar{c}_{ij}| - k\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{\sqrt{q_{ij}}}|\bar{d}_{ij}| \\
\leqslant {}& -k\sum_{i=1}^{n} \bar{e}_i - k\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{\sqrt{p_{ij}}}|\bar{c}_{ij}| - k\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{\sqrt{q_{ij}}}|\bar{d}_{ij}|.
\end{aligned}
$$

Using Lemma 2, we have

$$\begin{aligned}\dot{V}(t) &\leqslant -\sqrt{2}k\left(\frac{1}{2}\sum_{i=1}^{n}\bar{e}_i^2 + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\sqrt{p_{ij}}}\bar{c}_{ij}^2 - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\sqrt{q_{ij}}}\bar{d}_{ij}^2\right)^{1/2}\\ &= -\sqrt{2}kV^{1/2}(t).\end{aligned}$$

Therefore, from Lemma 1, the modified error system (24) will converge to zero within T_6. That is, the error system (5) will converge to zero within T_6. Thus, under the control laws (28) and updated laws (27), the two chaotic CGNNs are synchronized and identified simultaneously in the finite-time T_6. The proof of Theorem 2 is completed. □

When $a_i(u) \equiv 1$, for master-slave systems (18) and (19), we have the following corollary.

Corollary 5. *Assume that Assumptions* 2–3 *hold and activation functions* g_i *satisfy inequality* (26). *Using the following adaptive laws*:

$$\begin{aligned}\dot{\tilde{c}}_{ij}(t) &= -p_{ij}\left[e_i(t)f_j\big(y_j(t)\big) + \frac{k}{\sqrt{p_{ij}}}\operatorname{sign}(\tilde{c}_{ij} - c_{ij})\right],\\ \dot{\tilde{d}}_{ij}(t) &= -q_{ij}\left[e_i(t)g_j\big(y_j\big(t-\tau_j(t)\big)\big) + \frac{k}{\sqrt{q_{ij}}}\operatorname{sign}(\tilde{d}_{ij} - d_{ij})\right]\end{aligned} \tag{31}$$

and control laws

$$u_i(t) = \sum_{j=1}^{n}k_{ij}e_j(t) - (\bar{k}_i + k)\operatorname{sign}\big(e_i(t)\big), \tag{32}$$

where control strengths k_{ij} *and* $\bar{k}_i$ *satisfy the following inequality*:

$$\max\left\{\max_{i\in\mathcal{I}}\left\{-(\eta_i-k_{ii})+\sum_{j=1}^{n}\hat{c}_{ij}+\sum_{j=1,j\neq i}^{n}\hat{k}_{ij}\right\}, \max_{i\in\mathcal{I}}\left\{\sum_{j=1}^{n}M_j|d_{ij}|-\bar{k}_i\right\}\right\} < 0, \tag{33}$$

where $\hat{c}_{ij} = (|c_{ij}|L_j^1+|c_{ji}|L_i^1)/2$, $\hat{k}_{ij} = (1/2)(|k_{ij}|+|k_{ji}|)$, *then the synaptic connection weight coefficients* c_{ij} *and* d_{ij} *of network* (18) *can be identified with* $\tilde{c}_{ij}$ *and* $\tilde{d}_{ij}$, *and the slave network* (19) *can synchronize with the master network* (18) *in a finite time*

$$T_7 = \frac{V^{1/2}(0)}{\sqrt{2}k}, \tag{34}$$

where $V(0) = (1/2)\sum_{i=1}^{n}e_i^2(0) + (1/2)\sum_{i=1}^{n}\sum_{j=1}^{n}(1/p_{ij})(\tilde{c}_{ij}(0) - c_{ij})^2 + (1/2) \times \sum_{i=1}^{n}\sum_{j=1}^{n}(1/q_{ij})(\tilde{d}_{ij}(0) - d_{ij})^2$. *Here* $\tilde{c}_{ij}(0)$ *and* $\tilde{d}_{ij}(0)$ *are the initial values of the adaptive parameters* $\tilde{c}_{ij}$ *and* $\tilde{d}_{ij}$, *respectively.*

Remark 4. In this paper, we introduce the adaptive controlling method to guarantee the finite-time synchronization and topology identification of CGNNs with uncertain parameters and time-varying delays. In this section, there are many results concerning the asymptotic or exponential synchronization and topology identification [11, 13, 14, 19, 22, 23, 31, 34]. However, to the best of our knowledge, there is no results on the finite-time topology identification between two CGNNs with uncertain parameters and time-varying delays. Obviously, our results have optimality in the convergence time of synchronization and topology identification, which are essential to a practical system. Hence, the results obtained in this paper have a better performance than those of previous works about topology identification and synchronization problem.

Remark 5. Compared to Theorem 1, the designed control laws $u_i(t)$ in Theorem 2 are simple and easy to implement. In Theorem 2, however, the condition $|g_i(u)| \leqslant M_i$ is required to derive the main results. Thus, when the activation functions g_i are bounded, Theorem 2 is more practical than Theorem 1 to some extent.

Remark 6. When $a_i(u) \equiv 1$, $b_i(u) \equiv b_i u$ and $\tau_i(t) \equiv \tau$, model (1) can be degenerated to the following cellular neural network:

$$\dot{x}_i(t) = -b_i x_i(t) + \sum_{j=1}^{n} c_{ij} f_j\bigl(x_j(t)\bigr) + \sum_{j=1}^{n} d_{ij} g_j\bigl(x_j(t-\tau)\bigr) + I_i,$$

which is studied in [27]. It is not difficult to see that Theorem 1 includes the main results in [27] as a special cases. From this point, our results are more general and have a greater applicability.

Remark 7. From the Eqs. (11), (14), (16), (17), (22), (30) and (34) we know that the convergence time T proportional to the inverse of tunable constant k. Therefore, a greater k results in shorter convergence time T. On the other hand, according to the control input in Eqs. (9), (20), (28) and (32), it is obvious that the control input $u_i(t)$ is proportional to the value of k. This means that a greater k results in a larger $u_i(t)$. Therefore, the tunable constant k should be selected in accordance with the convergence time T to be short and the control input $u_i(t)$ not to be very large, considering the designer requirements.

Remark 8. Recently, based on LaSalle's invariance principle, the topology identification problem for complex networks without delays or with constant delays has been extensively investigated by using an adaptive controlling technique [32, 38]. Nevertheless, the finite-time structure identification of complex networks with time-varying delays is not concerned to the best of our knowledge. So, it is an interesting problem concerning the finite-time synchronization and topology identification for complex networks with time-varying delays.

4 Numerical simulations

In this section, an example is given to illustrate the effectiveness of proposed finite-time synchronization schemes.

For $n = 2$, consider the following delayed CGNNs system:

$$\dot{x}_i(t) = a_i(x_i(t))\left[-b_i\bigl(x_i(t)\bigr) + \sum_{j=1}^{2} c_{ij} f_j\bigl(x_j(t)\bigr) + \sum_{j=1}^{2} d_{ij} g_j\bigl(x_j\bigl(t-\tau_j(t)\bigr)\bigr)\right], \quad (35)$$

where $f_j(u) = g_j(u) = \tanh(u)$, $b_1(u) = 1.2u$, $b_2(u) = 1.8u$, $c_{11} = 1.8$, $c_{12} = -0.1$, $c_{21} = -2$, $c_{22} = 0.4$, $d_{11} = -1.7$, $d_{12} = -0.6$, $d_{21} = 0.5$, $d_{22} = -2.5$ and

$$a_1(u) = 0.9 - \frac{0.1}{1+u^2}, \qquad a_2(u) = 1.4 + \frac{0.1}{1+u^2}, \qquad \tau_1(t) = \tau_2(t) = \frac{\mathrm{e}^t}{(1+\mathrm{e}^t)}.$$

The numerical simulation of system (35) is represented in Fig. 1, which shows that system (35) has a chaotic attractor.

It is not difficult to check that $L_1^1 = L_2^1 = L_1^2 = L_2^2 = 1$, $0.8 \leqslant a_1(u) \leqslant 0.9$, $1.4 \leqslant a_2(u) \leqslant 1.5$, $M_1 = M_2 = 1$, and

$$\frac{b_1(u) - b_1(v)}{u - v} \geqslant 1.2, \qquad \frac{b_2(u) - b_2(v)}{u - v} \geqslant 1.8 \quad \text{for } u, v \in R,$$

which shows that $\eta_1 = 1.2$, $\eta_2 = 1.8$. Therefore, Assumptions 1–3 are all satisfied.

For convenience, we assume that only the four parameters c_{11}, c_{12}, d_{12}, d_{21} will be identified. Accordingly, the slave system is given as follows:

$$\dot{y}_i(t) = a_i\bigl(y_i(t)\bigr)\left[-b_i\bigl(y_i(t)\bigr) + \sum_{j=1}^{2} \tilde{c}_{ij} f_j\bigl(y_j(t)\bigr) + \sum_{j=1}^{2} \tilde{d}_{ij} g_j\bigl(y_j\bigl(t - \tau_j(t)\bigr)\bigr)\right] + u_i(t), \tag{36}$$

where a_i, b_i, f_j, τ_j are the same as in system (35) and $\tilde{c}_{11} = 1.8$, $\tilde{c}_{12} = -0.1$, $\tilde{d}_{12} = -0.6$, $\tilde{c}_{22} = -2.5$. The initial values chosen as $\tilde{c}_{21}(0) = 1.2$, $\tilde{c}_{22}(0) = -1.4$, $\tilde{d}_{12}(0) = 3$, $\tilde{d}_{21}(0) = -3$, $y_1(s) = 0.8$, $y_2(s) = 1.3$, $s \in (-1, 0)$.

According to Theorem 2 and Eq. (28), the control inputs can be presented as follows:

$$u_i(t) = \sum_{j=1}^{n} k_{ij} \bar{e}_j(t) - (\bar{k_i} + k) \operatorname{sign}\bigl(e_i(t)\bigr), \quad i = 1, 2,$$

where $\bar{e}_j(t) = \operatorname{sign}(y_j(t) - x_j(t)) \int_{x_j(t)}^{y_j(t)} \mathrm{d}s / a_j(s)$, and the updating laws of the parameters can be obtained as

$$\begin{aligned}
\dot{\tilde{c}}_{21}(t) &= -p_{21}\left[\bar{e}_2(t) f_1\bigl(y_1(t)\bigr) + \frac{k}{\sqrt{p_{21}}} \tanh(\tilde{c}_{21} - c_{21})\right],\\
\dot{\tilde{c}}_{22}(t) &= -p_{22}\left[\bar{e}_2(t) f_2\bigl(y_2(t)\bigr) + \frac{k}{\sqrt{p_{22}}} \tanh(\tilde{c}_{22} - c_{22})\right],\\
\dot{\tilde{d}}_{12}(t) &= -q_{12}\left[\bar{e}_1(t) g_2\bigl(y_2\bigl(t - \tau_2(t)\bigr)\bigr) + \frac{k}{\sqrt{q_{12}}} \tanh(\tilde{d}_{12} - d_{12})\right],\\
\dot{\tilde{d}}_{21}(t) &= -q_{21}\left[\bar{e}_2(t) g_1\bigl(y_1\bigl(t - \tau_1(t)\bigr)\bigr) + \frac{k}{\sqrt{q_{21}}} \tanh(\tilde{d}_{21} - d_{21})\right],
\end{aligned} \tag{37}$$

where the discontinuous sign function is replaced by continuous $\tanh$ function.

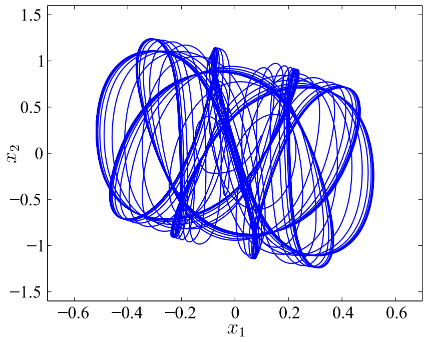

Fig. 1. The chaotic attractor of system (35).

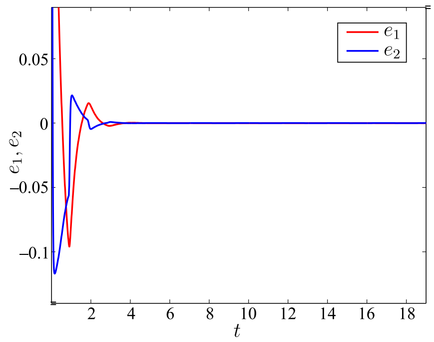

Fig. 2. The evaluation of synchronization errors.

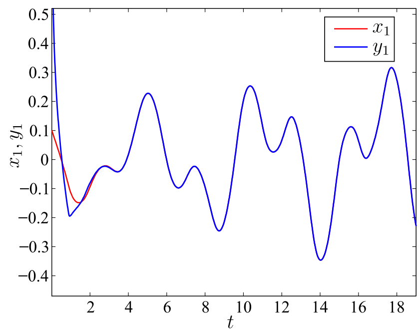

Fig. 3. Synchronization curves of x_1 and y_1.

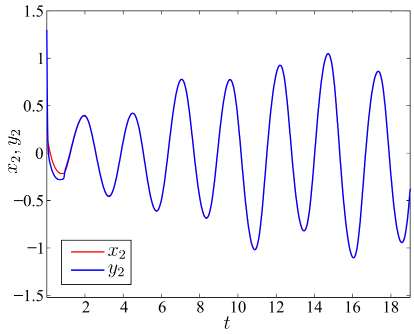

Fig. 4. Synchronization curves of x_2 and y_2.

Let

$$\chi_i = -\underline{a_i}\left(\eta_i - \frac{k_{ii}}{\overline{a_i}}\right) + \sum_{j=1}^{n} \hat{c}_{ij} + \sum_{j=1, j\neq i}^{n} \hat{k}_{ij}, \quad \xi_i = \sum_{j=1}^{n} M_j |d_{ij}| - \bar{k}_i, \quad i = 1, 2,$$

where $\hat{c}_{ij} = (|c_{ij}| L_j^1 \overline{a}_j + |c_{ji}| L_i^1 \overline{a}_i)/2$, $\hat{k}_{ij} = |k_{ij}| \overline{a}_j/(2\underline{a_i}) + |k_{ji}| \overline{a}_i/(2\underline{a_j})$. Choosing $k_{11} = -6.1$, $k_{12} = -5.6$, $k_{21} = k_{22} = 0$, $\bar{k}_1 = 2.1$, $\bar{k}_2 = 3.2$, $k = 2$ and $p_{ij} = q_{ij} = 1$, by simple computation, $\chi_1 = -3.7872$, $\chi_2 = -6.1717$, $\xi_1 = -0.1000$, $\xi_2 = -0.2000$. Thus, the inequality (29) in the Theorem 2 is satisfied. Therefore, according the Theorem 2, the controlled uncertain slave system (36) is synchronized with the master system (35) in a finite time $T_6 = 3.9250$ and its parameters satisfy

$$\begin{aligned}\lim_{t\to T}\big(\tilde{c}_{21}(t) - c_{21}\big) &= \lim_{t\to T}\big(\tilde{c}_{22}(t) - c_{22}\big) = \lim_{t\to T}\big(\tilde{d}_{12}(t) - d_{12}\big) \\ &= \lim_{t\to T}\big(\tilde{d}_{21}(t) - d_{21}\big) = 0.\end{aligned}$$

The time evolution of synchronization errors are shown in Fig. 2 and the synchronization between systems (35) and (36) is verified in Figs. 3 and 4. Figure 5 gives the identification of uncertain or unknown parameters $\tilde{c}_{21}$, $\tilde{c}_{22}$, $\tilde{d}_{12}$ and $\tilde{d}_{21}$. It is clear that the

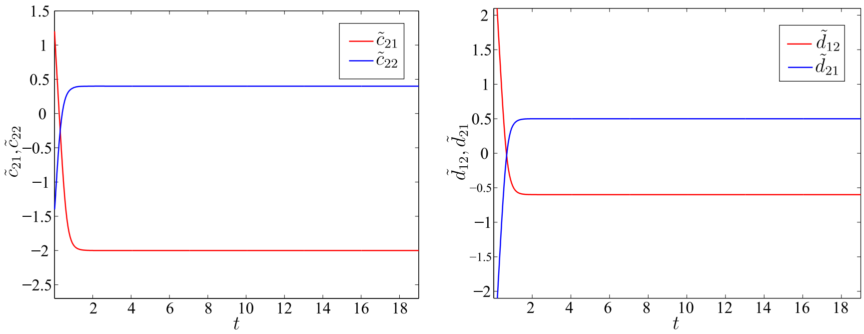

Fig. 5. Parameters identification of the neural networks (35) and (36).

unknown parameters converge to some bounded values in a finite time and the identification of system parameters is very successful.

5 Conclusion

In this paper, we investigate the finite-time topology identification and synchronization problem between two chaotic CGNNs with time-varying delays. Based on the adaptive feedback control and finite-time convergence theory, some novel and useful finite-time synchronization criteria have been obtained. Finally, an illustrative example with its numerical simulations is given to demonstrate the effectiveness and feasibility of the proposed synchronization method. Besides, a very interesting fact is revealed that the more the uncertain parameters are the longer in time to achieve finite-time synchronization of chaotic CGNNs will be.

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