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Log-linear learning model for predicting a steady-state manual assembly time

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Abstract. This paper presents the method for estimating the parameters of a two parameter learning curve (LC). Different values of parameters and different sample sizes are used for this estimation. Based on the experimental data an adequate mathematically grounded LC model is proposed for a manual assembly process of automotive wiring harness. The model enables us to determine the LC parameters α_ε (slope coefficient) and the learning rate stabilization point x_c, i.e. to completely restore LC and predict the production process. The propositions that ground the model application correctness are proved. The model adequacy is estimated, based on concrete production process monitoring data. The criterion that determines production process without stabilized learning rate is proposed.

Keywords: learning curve, data fitting, parameter estimating, mathematical modeling, manual assembly process.

1 Introduction

Learning curves (LC) are mathematical models used to estimate efficiencies gained when an activity is repeated. LC have already been considered for quite a long time, but their application is urgent up till now, as far as many enterprises are striving to apply the LC to determine their production process time, but they face various problems [1] most important of which are errors due to the wrong LC application.

When applying the LC model in practice there are three basic things that defined the accuracy of the applied model:

1. Adequacy to the specific character of production under consideration;
2. Rather exact method of LC parameters restoration;
3. Sufficient quantity of the production made according to which the producing time is predicted.

At present, very many LC models are applied that are widely reviewed in [2] and other papers. In this work, the LC model for learning phase modeling of the production process was used that meets the following requirements:

1) it is simple (minimal number of parameters);
2) it well approximates the whole length of LC;
3) it is able to define the stabilization point of the learning rate;
4) monitoring of producing process for the necessary data for restoring parameters of the model is rather cheep.

Several learning curve models have been proposed, but only two Wright‘s [3] and Crawford‘s [4, 5] models are in widespread use. By Wright‘s model the log-linear equation is the simplest and more common equation and is valid for a wide variety of processes

$$y_W(x) = \beta x^{-\alpha_W},$$

where y_W is the average time of all units produced up to the xth unit, the parameter α_W is a slope coefficient, β is the number of direct labor time required to produce the first unit. Crawford‘s model is as follows [4, 5]:

$$y_c(x) = \beta x^{-\alpha_c}, \tag{1}$$

where x is the unit number, y_c is the number of direct labor time required to produce the xth unit, the parameter α_c ($\alpha_c > 0$) is a slope coefficient, β is the number of direct labor time to produce required to produce of the first unit.

Some authors also use the Plateau model [5–8]:

$$y_p(x) = \beta x^{-\alpha_p} + \gamma,$$

where x, y_p, α_p, and β are the same as in (1), γ ($\gamma > 0$) is the constant that describes when the steady-state is reached after the learning is concluded or when machinery limitations block workers improvement. Baloff [6] studied this plateauing phenomenon and found it to be extensively present in machine intensive manufacturing.

This work investigates the LC application to the manual assembly process of automotive wiring harness. Production data for Crawford‘s model have been collected. Observing the production process, certain tendencies of assembly time dynamics have been noticed:

1) assembly time mostly decreases for the first products;
2) assembly time of produced in large amounts and constantly is stable and does not decrease.

The stability is described in the paper by the learning rate (parameter ε) that is determined by the steady-state time T_c.

By means of experiments (monitoring of the manual assembly process) LC was obtained as one or several random samples. Note that such a way, by fixing each complete cycle is most frequently used in the investigation in LC parameters restoration [9, 10]. However, it is also most expensive since it requires more expenditure for monitoring the production process: scanning equipment, its management and administration, because one group of researchers can tackle only one experiment at a time.

If learning curves are obtained experimentally by only one sample

$$x_i, y_i, x_i < x_{i+1}, \quad i = 1, 2, \ldots, n,$$

then the statistical methods, such as the least squares method maximum likelihood method and others, are not suitable here. Though a statistical method for the case with one random sample available is offered in the paper [11], in this research a deterministic method of the empirical function that approximates LC, based on the obtained monitoring data will be used, i.e. of one random sample.

Basing on the tendencies observed in most experiments done, a premise that in a assembly process there exists a Plateau phase in the sense of assembly rate stabilization is made, i.e. the assembly time is decreasing to a certain limit T_c until the steady-state assembly time is reached.

The aim of this work is to propose, to an adequate, mathematically grounded LC model, that meets the above mentioned requirements on the basis of experimental data, and which will enable us to define the LC parameters α_ε and steady-state point x_c, i.e. to restore LC completely and predict the assembly process.

2 Product and its assembly time

As typical manual assembly process wiring harness production will be studied. Since wiring harness performs electrical circuit function, its manufacturing is pure assembly process: manual, semi automatic, automatic, depending on production volume [12].

According to wiring harness definition [13], the main wiring harness components are terminated cables and wires (circuits), housings and connectors as other wrapping and protecting material (tapes, cable ties, hoses and tubes). These components constitute wiring harness layout with main trunk, branches, break-outs, legs (see Fig. 1). When protective materials such as hoses are used, additional components like adapters and manifolds are added. The main wiring harness manufacturing steps [14]:

1. Wire preparation, when wires are being cut ant terminals mounted.
2. Installation, cables and branches are being placed on assembly board according the certain layout.
3. Securing, cables and wires are wrapped together; protective hoses are pulled on branches and legs.
4. Attachment, cables and wires with mounted terminals are being assembled into housings, connectors, splices and etc.

The first step representing wiring preparation is mostly automatic process and next three steps are assembly of the wiring harness performed by operator on the assembly jig. In this research only manual assembly operations are studied, so only last tree steps will be considered.

During the assembly of wiring harness, all components must be installed and assembled into the final product. To determine assembly time of the assembly the total assembly time could be measured and also time of each operation could be measured.

Each component of the wiring harness has specific mounting operation or several operations and each operation has certain processing time i.e. to pick and place component, pull the hose, assemble terminal into housing, wrap the cable tie and etc. Even different

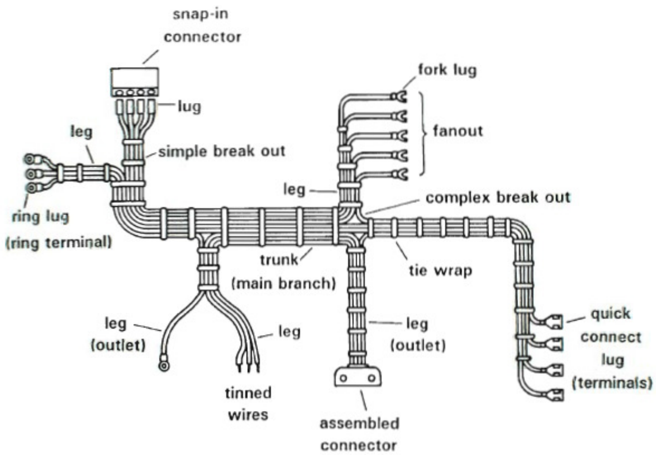

Fig. 1. The main wiring harness components and terminology [13].

wiring harness have different number of different components, the basic operation time of specific mounting procedure will be the same, for all. This basic operation time represents necessary – steady state operation time, which is reached, after operator has finished the learning phase.

There two groups of component assembly operations, simple and complex. Simple operation represents single process and depends only on number of components used i.e. the assembly of terminal, fixing of cable tie. The complex assembly operation depends on number of components and number of sub-operations or component length. For instance, to pull the hose, constant assembly time is needed, and variable assembly time depending on the length of the hose is needed as well. Analogical situation is with the sub operations: the number of wires in the splice, the number of wires in the branch and etc.

After regression analysis [14], linear relationship between number of sub-operations/ length and assembly time was determined:

$$T^v_{\mathrm{op}} = t_{\mathrm{const}} + t_{\mathrm{add}} s_{\mathrm{op}} = t_{\mathrm{const}} + t_{\mathrm{add}} l_{\mathrm{op}},$$
$$T^s_{\mathrm{op}} = t_{\mathrm{const}},$$

where t_{const} is constant operation, t_{add} is additional time for sub-operation or for additional meter, s_{op} is number of sub-operations, l_{op} is length of component.

To determine the total assembly time of the product the sum of the operating time need to be calculated. So in general the vector of all operations with certain operation times is defined:

$$\mathbf{D} = \begin{pmatrix} t_1 & t_2 & t_3 & \dots & t_m \end{pmatrix},$$

where t_i is operation time of the particular operation, m is total number of all possible operations. t_i is determined as a time measured during time study of each operation. During time study, every operation was thoroughly evaluated, not only on single, but on several different products and different type of components and performed up to the several hundred times, to warrant that pure steady state operational time is reached.

Each different wiring harness product will have different numbers and quantities of operations. This information is easily collected from product structure. So for every product operation quantity vector is determined:

$$\mathbf{W} = \begin{pmatrix} a_1 & a_2 & a_3 & \dots & a_m \end{pmatrix},$$

where a_i is the quantity of *i* operation. If particular unit does not have the certain operation the quantity is obviously 0. When fully defined vectors $\mathbf{D}$ and $\mathbf{W}$ are given, the total assembly time is calculated as a scalar product:

$$T_c = \mathbf{DW}. \tag{2}$$

After (2) is applied, the total assembly time of selected wiring harness product is determined. Since T_c is the sum of the steady state operation times, T_c as well represent the steady state assembly time of the wiring harness.

3 Learning curve model

Let one experiment is done and data

$$\{x_i, y_i\}, \quad x_i < x_{i+1}, \quad i = 1, 2, \dots, n,$$

be obtained. We shall use only "cheep" data, i.e.

$$x_0 = x_q, \quad y_0 = y_q, \quad 1 \leqslant q \leqslant n, \tag{3}$$

and the steady state time $T_c = y(x_c, \alpha)$, when the time of later operations "almost" does not change ($y(x, \alpha) \approx \text{const}$, when $x \geqslant x_c$). Parameters x_c and α are unknown. The word "almost" is treated here as a decrease in absolute value of a derivative $\partial y(x, \alpha)/\partial x$ up to an adequately chosen value $\varepsilon > 0$. We proof below that the parameter $\alpha = \alpha_\varepsilon$ is a unique solution of the equation

$$\left|\frac{\partial y}{\partial x}\right|\bigg|_{x = x_c(T_c, \alpha)} = \varepsilon, \tag{4}$$

and $x_c(T_c, \alpha_\varepsilon) = x_0(y_0/T_c)^{1/\alpha_\varepsilon}$ is an abscissa of the unique intersection point of LC $y(x, \alpha_\varepsilon) = y_0(x/x_0)^{-\alpha_\varepsilon}$ crossing the point (x_0, y_0) and the line $y = T_c$. Thus if we find α_ε and $x_c(T_c, \alpha_\varepsilon)$, we can completely restore LC:

$$Y(x, x_0, y_0, \alpha_\varepsilon) = \begin{cases} y_0(\frac{x}{x_0})^{-\alpha_\varepsilon}, & \text{when } 0 < x < x_c(T_c, \alpha_\varepsilon), \\ T_c, & \text{when } x \geqslant x_c(T_c, \alpha_\varepsilon). \end{cases} \tag{5}$$

Proposition 1. *If $x_0 \geqslant 1$, $y_0 > 0$, $\alpha > 0$, $0 < T_c < y_0$, then a bundle of curves (according of α)*

$$y(x, \alpha) = y_0\left(\frac{x}{x_0}\right)^{-\alpha} \tag{6}$$

have only one common point (x_0, y_0)*, and each curve of the bundle crosses the line* $y = T_c$ *only at one point, the abscissa of which is*

$$x_c(T_c, \alpha) = x_0 \left(\frac{y_0}{T_c}\right)^{1/\alpha} > 0, \tag{7}$$

Proof. The bundle of curves (6) has only one common point (x_0, y_0), because $y(x_0, \alpha) = y_0(x_0/x_0)^{-\alpha} = y_0$, for all $\alpha > 0$. Besides, this point is unique, because in case at least one more point appears, then $(x_{01}, y_{01}) \neq (x_0, y_0) \Rightarrow y_0(x_0/x_0, \alpha)^{-\alpha_0} = y_{01}(x_{01}/x_{01})^{-\alpha_{10}} \Rightarrow y_0 = y_{01}$.

We show that (7) is a unique solution of the equation $y(x, \alpha) - T_s = 0$. We have

$$y(x_c(T_c, \alpha), \alpha) - T_s = y_0 \left(\frac{x_0(y_0/T_c)^{1/\alpha}}{x_0}\right)^{-\alpha} - T_c = T_c - T_c = 0.$$

There exists solution (7) and it is unique if $y(1, \alpha) \geqslant T_c$, since a derivative of $y(x, \alpha)(\partial y/\partial x) = -\alpha(y_0/x)(x/x_0)^{-\alpha} < 0$, i.e. $y(x, \alpha)$ is strictly monotonously decreasing (SMD) and $\lim_{x\to+0} y(x, \alpha) = y_0$, $\lim_{x\to+\infty} y(x, \alpha) = 0$. □

Proposition 2. *If* $x_0 \geqslant 1$, $y_0 > 0$, $\alpha > 0$, $0 < T_c < y_0$*, then equation* (4) *has a unique solution* $\alpha(\varepsilon) > 0$ *for any* $\varepsilon > 0$*, besides the function* $\alpha(\varepsilon)$ *is strictly monotonously increasing* (*SMI*).

Proof. The absolute value of $\partial y/\partial x$ at points $x = x_c(T_c, \alpha)$, is equal to

$$f_\alpha(\alpha) \equiv \left|\frac{\partial y}{\partial x}\right|\Big|_{x=x_c(\alpha)} = \alpha\frac{y_0}{x_0}\left(\frac{T_c}{y_0}\right)^{(1+\alpha)/\alpha} = \alpha\frac{T_c}{x_0}\left(\frac{T_c}{y_0}\right)^{1/\alpha} > 0,$$

therefore, equation (4) becomes

$$\alpha\frac{T_c}{x_0}\left(\frac{T_c}{y_0}\right)^{1/\alpha} = \varepsilon \quad \text{or} \quad \frac{T_c}{x_0\varepsilon} = \frac{1}{\alpha}\exp\left\{\frac{1}{\alpha}\ln\left(\frac{y_0}{T_c}\right)\right\} \tag{8}$$

and after multiplying both sides of equation (8) by $c = \ln(y_0/T_c) > 0$ we have

$$cd(\varepsilon) = \left(\frac{c}{\alpha}\right)\exp\left\{\frac{c}{\alpha}\right\}, \tag{9}$$

where $d(\varepsilon) = T_c/(x_0\varepsilon)$. From (9) it follows that the function $W(cd(\varepsilon)) = c/\alpha$ is Lambert‘s function [15, 16] and

$$\alpha(\varepsilon) = \frac{c}{W(cd(\varepsilon))} = \frac{\ln(y_0/T_c)}{W[(T_c/x_0\varepsilon)\ln(y_0/T_c)]}. \tag{10}$$

From the properties of Lambert‘s function [17] and (10) it follows that the function $\alpha(\varepsilon)$ is single-valued, positive, SMI, and $\alpha(0) = 0$, because $cd(\varepsilon) > 0$ is SMD. This proves the Proposition 2. □

Proposition 3. *If $x_0 \geqslant 1$, $y_0 > 0$, $\alpha > 0$, $0 < T_c < y_0$, then the function $x_c(T_c, \alpha(\varepsilon))$ is SMD and concave.*

Proof. It follows from Proposition 1 that $x_c(T_c, \alpha(\varepsilon)) > 0$, therefore, derivatives of implicit function $x_c(T_c, \varepsilon)$ are of constant sign:

$$\frac{\partial x_c}{\partial \varepsilon} = x_c(T_c, \varepsilon)\frac{\alpha'(\varepsilon)}{\alpha^2(\varepsilon)} \ln\left(\frac{T_c}{y_0}\right) < 0$$

and

$$\frac{\partial^2 x_c}{\partial \varepsilon^2} = \left(\alpha'_\varepsilon(\varepsilon)\right)^2 \frac{\partial^2 x_c}{\partial \alpha^2} + \frac{\partial x_c}{\partial \alpha}\alpha''_\varepsilon(\varepsilon) > 0.$$

Thus after finding α_ε (there is no analytical solution), we completely restore LC (5) and the learning rate stabilization point $x_c(T_c, \alpha_\varepsilon)$, using only two experimental data (x_0, y_0) and T_c. □

4 Results and discussion

With a view to verify the adequacy of the method proposed 11 respective measurements have been performed in which only $x_q^{(k)}$, $y_q^{(k)}$ (see (1)) and $T_c^{(k)}$ have been selected from

$$x_i^{(k)},\ y_i^{(k)} \quad k = 1, 2, \ldots, 11,\ i = 1, 2, \ldots, n_k,\ 1 \leqslant q \leqslant n_k, \tag{11}$$

here k is the number of experiment, n_k is the number of the kth experiment data. In line with this parameters, the parameters $\alpha_\varepsilon^{(k)}$ and $x_c^{(k)}$ have been calculated. According to manufacturer recommendations $\varepsilon = 0.0016\,T_c^{(k)}$ is selected in all the calculations. A comparison of the LC $Y(x, x_q^{(k)}, y_q^{(k)}, \alpha_\varepsilon^{(k)})$ obtained values with the complete experimental data (11). Function (12) was chosen as adequacy criterion (average percent relative error)

$$\Delta^{(k)}(\varepsilon) = \frac{100}{n_j}\sum_{i=1}^{n_k}\left|\frac{y_i^{(k)} - Y(x_i^{(k)}, x_q^{(k)}, y_q^{(k)}, \alpha_\varepsilon^{(k)})}{y_i^{(k)}}\right|, \quad k = 1, 2, \ldots, 11. \tag{12}$$

In Table 1, the data of the first experiment are presented. The calculation results are given in Fig. 2, and, for all experiments, in Table 3. Fig. 3 shows the experimental data in log-log scale.

Table 1

i	$x_i^{(1)}$	$y_i^{(1)}$ [h]	i	$x_i^{(1)}$	$y_i^{(1)}$ [h]
1	2	6.880	8	268	2.178
2	44	3.078	9	301	2.233
3	100	2.423	10	349	2.114
4	163	2.047	11	396	2.101
5	172	2.128	12	428	2.136
6	204	2.224	13	460	2.140
7	236	2.212			

Table 2

k	$\varepsilon_m^{(k)}$	$\varphi_m^{(k)}$ [deg]	k	$\varepsilon_m^{(k)}$	$\varphi_m^{(k)}$ [deg]
1	0.0036	−0.2063	7	0.0233	−1.3348
2	0.0013	−0.7445	8	0.0082	−0.4698
3	0.0014	−0.0080	9	0.0120	−0.6875
4	0.0048	−0.2750	10	0.0033	−0.1891
5	0.0001	0.0057	11	0.0079	−0.4526
6	0.0009	−0.0516			

Table 3

k	Data								
	q	$x_q^{(k)}$	$y_q^{(k)}$	$T_c^{(k)}$ [h]	n_k	$10^2\,\varepsilon^{(k)}$	$\alpha_\varepsilon^{(k)}$	$x_c^{(k)}$	$\Delta^{(k)}$ [%]
1	2	44	3.078	2.200	13	0.308	0.2440	174	3.11
2	1	20	2.545	0.755	10	0.124	0.4601	280	5.97
3	3	3	1.230	0.900	42	0.148	0.1028	70	0.79
4	3	13	3.448	2.980	8	0.489	0.0966	58	3.40
5	2	80	1.177	0.800	21	0.131	0.3712	226	10.65
6	1	90	0.731	0.550	18	0.090	0.3404	207	1.66
7	1	10	17.061	11.000	11	1.804	0.1823	111	2.28
8	1	17	11.847	6.650	17	1.091	0.2591	157	3.96
9	3	59	14.977	6.765	20	1.109	0.4900	298	4.81
10	1	2	6.880	2.100	13	0.344	0.2692	164	2.48
11	1	19	19.630	6.750	20	1.107	0.4131	251	6.46

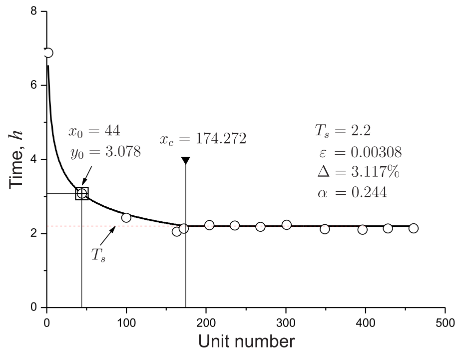

Fig. 2. Learning curve based on the proposed model (line) and a graph of the data set from Table 3, $k = 1$.

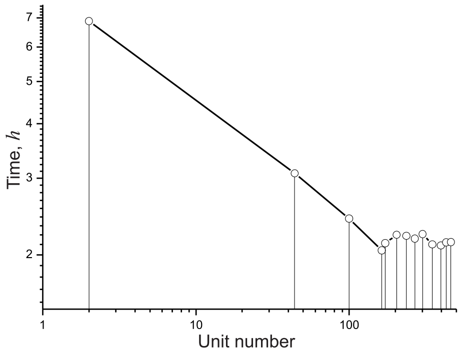

Fig. 3. Graph of the data set from Table 3, $k = 1$ (log-log scale).

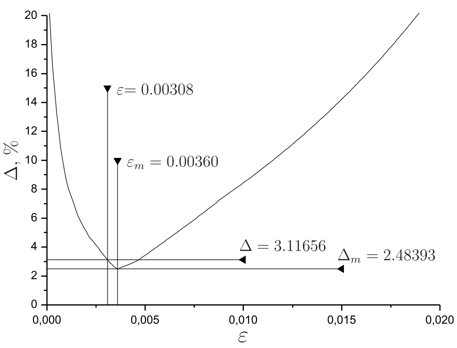

Fig. 4. Dependency of the average relative error on ε for the data set from Table 3, $k = 1$.

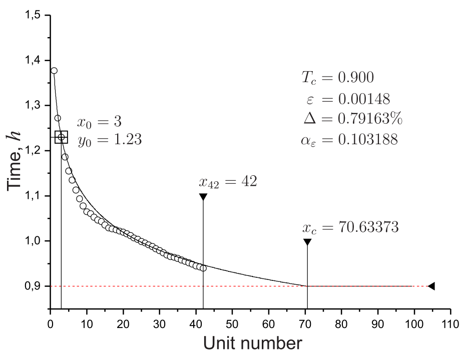

Fig. 5. Learning curve based on the proposed model (line) and a graph for the data set from Table 3, $k = 3$ (circles). Point of the stabilized learning rate x_c.

The numerical experiments have shown that function (12) is unimodal (Fig. 4) and has a minimum approximate to proposed $\varepsilon_p = 0.0016\,T_c$, i.e. $\varepsilon_m^{(k)} = \arg\min_\varepsilon \Delta^{(k)}(\varepsilon) \approx \varepsilon_p$. The values of $\varepsilon_m^{(k)}$ are illustrated in Table 2. Function (5) is smooth everywhere except for the point $x_c^{(k)}$ at which it has break of the derivative equal to $\varepsilon_m^{(k)}$, which is entirely defined the angle $\varphi_m^{(k)} = -\arctan(\varepsilon_m^{(k)})$ (if $\varphi_m^{(k)} = 0$, then function (5) is smooth everywhere). The values of angle $\varphi_m^{(k)}$ in degrees are presented in Table 2.

Note that the proposed method can be applied to predict still not stabilized learning processes. That is illustrated in Fig. 5. The condition $\max_i x_i^{(k)} < x_c^{(k)}$ can be treated here as a criterion of unsettlement and predict the steady-state point as $x_c^{(k)}$. The calculation has shown that there is a steady-state of the learning rate in the experiments $k = 1, 5, 6, 8, 9, 10, 11$; while in the rest of them stabilization is just predicted (see Table 3 and Fig. 5).

5 Conclusions

In this article an adequate mathematically grounded LC method is proposed for a manual assembly process of automotive wiring harness. All the propositions that ground the method application correctness are proved. 11 experiments were performed and several calculation results confirm premise of plateauing phenomena, i.e. that the assembly time is decreasing to a certain steady-state limit T_c. Having only one measured data point (x_0, y_0) and stabilized time T_c, learning slope α_ε and point of the stabilized learning rate x_c can be unequivocally recovered. This enables to predict stabilization point in future even the current production data has not shown stabilization yet. Moreover, this research proposed the method to fit LC model beyond statistical methods with sufficiently accuracy; small relative error values after the comparison with concrete production process monitoring data proves adequacy of the model. Such data fitting might be preferable in the manufacturing fields where production data is costly and only limited data could be provided for the analysis.

Currently presented method enables to predict sequential production development. In order to predict the initial learning phase from known stabilized time T_c LC parameters are needed to be known as well. Therefore, further research should be focused on LC parameter estimation methodology.

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