Output feedback control of nonlinear systems with uncertain ISS/iISS supply rates and noises*

Xin Yu, Guohai Liu

School of Electrical and Information Engineering, Jiangsu University
Zhenjiang, Jiangsu Province, 212013, China
xyu@ujs.edu.cn; ghliu@ujs.edu.cn

Received: 10 January 2013 / **Revised:** 31 October 2013 / **Published online:** 19 February 2014

Abstract. This paper considers the problem of global output feedback control for a class of nonlinear systems with inverse dynamics. The main contribution of paper is that: For the inverse dynamics with uncertain ISS/iISS supply rates, and the systems being disturbed by L^2 noises, we construct a reduced-order observer-based output feedback controller, which drives the output of system to zero and maintain other closed-loop signals bounded. Finally, a simulation example shows the effectiveness of the control scheme.

Keywords: nonlinear systems, ISS/iISS, uncertain supply rates, reduced-order observer, L^2 noises.

1 Introduction

Since the notion of input-to-state stability (ISS) was first introduced in [1], it has been recognized as a central concept in nonlinear control systems. [2–5] and the references therein investigated many kinds of properties of ISS. [6–9] and the references therein considered controller design and stability analysis for various classes of nonlinear systems with ISS (or ISpS) inverse dynamics. Subsequently, another important concept, integral input-to-state stability (iISS), was firstly presented in [10], and several characterizations on iISS were investigated in [11], in which iISS is proved to be strictly weaker than ISS. In [12], the authors analyzed nonlinear cascades in which the driven subsystem is iISS, and characterized the admissible iISS-gains for stability. Recently, [13–16] gave several Lyapunov-based small-gain theorems covering iISS systems.

So far, in addition to the above literatures, there are many other results on the design and analysis of controller for nonlinear systems with ISS/iISS inverse dynamics. For

*This work was supported by the National Natural Science Foundation of China (61304073, 61273154, 51077066, 61203054), Natural Science Foundation of Jiangsu Province (BK20130533, BK20130536), China Postdoctoral Science Foundation (2013M540421, 2013M541615), Specialized Research Fund for the Doctoral Program of Higher Education of China (20133227120012), Research Foundation for Advanced Talents of Jiangsu University (13JDG012).

example, Arcak et al. in [12] applied the admissible iISS-gains for stability of cascade systems to develop a new observer-based backstepping design. Jiang et al. in [17] firstly presented a unifying framework for the robust global regulation via output feedback for nonlinear systems with iISS inverse dynamics. Recently, [18] further studied output feedback regulation for a class of nonlinear systems with iISS inverse dynamics, in which the observer gain is governed by a Riccati differential equation, and Xu and Huang in [19] considered the output regulation problem for output feedback systems with relative degree one and iISS inverse dynamics. In [20], the authors considered reduced-order observer-based output feedback regulation for a class of nonlinear systems with iISS inverse dynamics. Recently, Yu et al. in [21, 22] extended the notion and some properties of iISS to stochastic nonlinear systems.

However, almost of the above papers only consider the ISS/iISS inverse dynamics with known ISS/iISS supply rates. When the inverse dynamics with uncertain ISS/iISS supply rates, how to design a feedback controller for nonlinear systems seems to be an interesting work.

The main contribution of paper is that: For the inverse dynamics with uncertain ISS/iISS supply rates, and the systems being disturbed by L^2 noises, we construct a reduced-order observer-based output feedback controller, which drives the output of system to zero and maintain other closed-loop signals bounded.

The remainder of paper is organized as follows. Section 2 is problem statements. Section 3 gives the design of output feedback controller. Section 4 is the main results. A simulation example is given in Section 5. Section 6 concludes the paper.

Notations

R_+ stands for the set of all nonnegative real numbers, R^n is the n-dimensional Euclidean space, $|x|$ is the usual Euclidean norm of a vector x. $\mathcal{K}$ denotes the set of all functions $\gamma : R_+ \to R_+$, which are continuous, strictly increasing and $\gamma(0) = 0$; $\mathcal{K}_\infty$ is the set of all functions which are of class $\mathcal{K}$ and unbounded, $\mathcal{KL}$ denotes the set of all functions $\beta(s,t) : R_+ \times R_+ \to R_+$, which are of class $\mathcal{K}$ for each fixed t, and decrease to zero as $t \to \infty$ for each fixed s. $\sigma_1(s) = \mathcal{O}(\sigma_2(s))$ as $s \to 0+$ means that $\sigma_1(s) \leqslant c_1\sigma_2(s)$ for some constant $c_1 > 0$ and all s in a small neighborhood of zero, and $\sigma_1(s) = \mathcal{O}(\sigma_2(s))$ as $s \to \infty$ means that $\sigma_1(s) \leqslant c_2\sigma_2(s)$ for some constant $c_2 > 0$ and all large enough s. $L^2(R_+; R)$ is the family of all functions $l : R_+ \to R$ such that $\int_0^\infty l^2(t)\,\mathrm{d}t < \infty$.

2 Problem statements

In this paper, we consider a class of nonlinear systems with the detailed form described as

$$\begin{aligned} \dot\eta &= q(t,\eta,y), \\ \dot x_i &= x_{i+1} + f_i(t,\bar x_i) + g_i(t,\eta,y) + d_i(t), \quad i = 1,\dots,n-1, \\ \dot x_n &= u + f_n(t,\bar x_n) + g_n(t,\eta,y) + d_n(t), \\ y &= x_1, \end{aligned} \tag{1}$$

where $x = (x_1, \dots, x_n) \in R^n$, $u \in R$, $y \in R$ are the state, the control input, and the measurable output, respectively, $\eta \in R^q$ denotes the inverse dynamics, $(x_2, \dots, x_n)$ and η are unmeasurable signals, $\bar{x}_i = (x_1, \dots, x_i) \in R^i$, $i = 1, \dots, n$. It is assumed that the modeled (or known) dynamics f_i, $i = 1, \dots, n$, are smooth, and the unmodeled (or uncertain) dynamics q and g_i, $i = 1, \dots, n$, are locally Lipschitz. $d_i(t)$, $i = 1, \dots, n$, are uncertain external noise.

The control objective is to design an output feedback controller for system (1) based on a reduced-order observer. Such controller drives the output of systems to zero asymptotically and maintains other closed-loop signals bounded.

The main results of paper are based on the following assumptions.

Assumption 1. For η-system of (1), there is a positive definite function $V_0 \in C^1$ such that

$$\underline{\alpha}_0(|\eta|) \leqslant V_0(\eta) \leqslant \overline{\alpha}_0(|\eta|), \qquad \frac{\partial V_0}{\partial \eta} q(\eta, y) \leqslant -\pi_0(|\eta|) + p_0\gamma_0(|y|), \tag{2}$$

where $\underline{\alpha}_0, \overline{\alpha}_0, \gamma_0$ are class $\mathcal{K}_\infty$ functions, π_0 is a positive-definite continuous function, and p_0 is an uncertain positive constant.

Remark 1. From [11], one knows that η-subsystem satisfying (2) is iISS, and the functions pairs $(\pi_0, p_0\gamma_0)$ are supply rates. Specially, if π_0 is class $\mathcal{K}_\infty$ function, the η-subsystem is ISS.

Since p_0 in (2) is unknown, the inverse dynamics have uncertain ISS/iISS supply rates.

Assumption 2. The modeled dynamics $f_1(t, y) \leqslant \hat{f}_1(y)$ with $\hat{f}_1(y)$ being smooth function and $\hat{f}_1(0) = 0$, $f_i(t, \bar{x}_i)$, $i = 2, \dots, n$, satisfy that

$$\left|f_i(t, \bar{x}_i) - f_i(t, \bar{\hat{x}}_i)\right| \leqslant \rho_i\left|\bar{x}_i - \bar{\hat{x}}_i\right|, \quad i = 2, \dots, n,$$

where $\bar{x}_i = (x_1, \dots, x_i)$, $\bar{\hat{x}}_i = (x_1, \hat{x}_2, \dots, \hat{x}_i) \in R^i$, and ρ_i are known positive constants with $\rho_0 = (\sum_{i=2}^n \rho_i^2)^{1/2}$ such that the linear matrix inequality

$$\begin{pmatrix} P\bar{A} + \bar{A}^{\mathrm{T}}P + SB + B^{\mathrm{T}}S^{\mathrm{T}} + \rho_0^2\delta_1 I + 2Q & P \\ P & -\delta_1 I \end{pmatrix} \leqslant 0 \tag{3}$$

hods, where $\bar{A} = \left(\begin{smallmatrix} 0 & I_{(n-2)\times(n-2)} \\ 0 & 0 \end{smallmatrix}\right)$, $B = (-1, 0, \dots, 0)_{1\times(n-1)}$, P, Q are positive definite matrices and δ_1 is a positive constant.

Remark 2. Assumption 3 shows that f_i includes not only the output, but also the unmeasured state variables. Moreover, $f_i(\bar{x}_i)$ can be any smooth function with respect to measurable variable x_1, and be Lipschitz function with respect to the unmeasurable variables $x_2, \dots, x_i$ with the Lipschitz constant satisfying LMI (3).

Assumption 3. For each $1 \leqslant i \leqslant n$, there exist unknown positive constants p_{i1}, p_{i2}, and known positive-definite smooth functions ϕ_{i1}, ϕ_{i2} such that

$$\left|g_i(t, \eta, y)\right| \leqslant p_{i1}\phi_{i1}(|y|) + p_{i2}\phi_{i2}(|\eta|).$$

Assumption 4. The external noise $d_i(t)$ satisfies $d_i(t) \in L^2(R_+; R)$, $i = 1, \dots, n$.

Remark 3. Assumption 3 is an usual condition in output feedback control of nonlinear systems (e.g., see [17, 18, 20, 23]). Assumption 4 shows that system (1) is disturbed by L^2 noises.

3 Output feedback controller design

This section gives the design procedure of global output feedback controller by using the method of adaptive backstepping.

3.1 Reduced-order observer design

Firstly, we define a new variable $v = x_2 + g_1(t, \eta, y) + d_1(t)$, which will play an important role in the following design. We construct the following $(n-1)$-dimensional state estimation:

$$\begin{aligned} \dot{\hat{x}}_i &= \hat{x}_{i+1} + f_i(t, \bar{\hat{x}}_i) + l_i(v - \hat{x}_2), \quad i = 2, \dots, n-1, \\ \dot{\hat{x}}_n &= u + f_n(t, \bar{\hat{x}}_n) + l_n(v - \hat{x}_2), \end{aligned} \tag{4}$$

where observer gain $l = (l_2, \dots, l_n)^{\mathrm{T}}$ is chosen such that

$$A^{\mathrm{T}}P + PA + \delta_1^{-1}PP + \rho_0^2\delta_1 I \leqslant -2Q, \tag{5}$$

in which $A = \left(-l \ \begin{smallmatrix} I_{n-2} \\ 0\dots0 \end{smallmatrix}\right)$, P, Q are positive definite matrices and $\delta_1 > 0$. Noting that the signal v in (4) is unmeasurable, we introduce the new observation variables

$$\xi_i = \hat{x}_i - l_i y, \quad i = 2, \dots, n, \tag{6}$$

which, together with (4), leads to

$$\begin{aligned} \dot{\xi}_i &= \hat{x}_{i+1} + f_i(t, \bar{\hat{x}}_i) - l_i\bigl(f_1(t, y) + \hat{x}_2\bigr), \quad i = 2, \dots, n-1, \\ \dot{\xi}_n &= u + f_n(t, \bar{\hat{x}}_n) - l_n\bigl(f_1(t, y) + \hat{x}_2\bigr). \end{aligned} \tag{7}$$

From (6), one obtains $f_i(t, \bar{\hat{x}}_i) = f_i(t, y, \hat{x}_2, \dots, \hat{x}_i) = f_i(t, y, \xi_2 + l_2 y, \dots, \xi_i + l_i y) := \tilde{f}_i(t, y, \bar{\xi}_i)$, where $\bar{\xi}_i = (\xi_2, \dots, \xi_i)$. Substituting (6) into (7), one obtains the reduced-order observer

$$\begin{aligned} \dot{\xi}_i &= \xi_{i+1} + l_{i+1}y + \tilde{f}_i(t, y, \bar{\xi}_i) - l_i(f_1(t, y) + \xi_2 + l_2 y), \quad i = 2, \dots, n-1, \\ \dot{\xi}_n &= u + \tilde{f}_n(t, y, \bar{\xi}_n) - l_n(f_1(t, y) + \xi_2 + l_2 y). \end{aligned} \tag{8}$$

Defining the error variables $e_i = x_i - \hat{x}_i$, $2 \leqslant i \leqslant n$, by (1) and (4), one has

$$\begin{aligned} \dot{e}_i &= -l_i e_2 + e_{i+1} + f_i(t, \bar{x}_i) - f_i(t, \bar{\hat{x}}_i) - l_i g_1(t, \eta, y) \\ &\quad - l_i d_1(t) + g_i(t, \eta, y) + d_i(t), \quad i = 2, \dots, n-1, \\ \dot{e}_n &= -l_n e_2 + f_n(t, \bar{x}_n) - f_n(t, \bar{\hat{x}}_n) - l_n g_1(t, \eta, y) \\ &\quad - l_n d_1(t) + g_n(t, \eta, y) + d_n(t), \end{aligned}$$

which, in compact notation, is rewritten as

$$\dot{e} = Ae + F(t,x) - F(t,\hat{x}) + G(t,\eta,y) + D(t), \tag{9}$$

where $e = (e_2, \dots, e_n)^{\mathrm{T}}$, $F(t,x) = (f_2(t,\bar{x}_2), \dots, f_n(t,\bar{x}_n))^{\mathrm{T}}$, $F(t,\hat{x}) = (f_2(t,\bar{\hat{x}}_2), \dots, f_n(t,\bar{\hat{x}}_n))^{\mathrm{T}}$, $G(t,\eta,y) = (g_2 - l_2 g_1, \dots, g_n - l_n g_1)^{\mathrm{T}}$, $D(t) = (d_2(t) - l_2 d_1(t), \dots, d_n(t) - l_n d_1(t))^{\mathrm{T}}$. Setting $\bar{e} = 1/p^* e$, $p^* = \max_{1 \leqslant i \leqslant n}\{1, p_{i1}, p_{i2}, p_{12}^2\}$, then (9) becomes

$$\dot{\bar{e}} = A\bar{e} + \frac{1}{p^*}\big(F(t,x) - F(t,\hat{x})\big) + \frac{1}{p^*}G(t,\eta,y) + \frac{1}{p^*}D(t), \tag{10}$$

which together with (1) and (8) consist of the following controlled system for feedback design:

$$\begin{aligned}
\dot{\eta} &= q(t,\eta,y),\\
\dot{\bar{e}} &= A\bar{e} + \frac{1}{p^*}\big(F(t,x) - F(t,\hat{x})\big) + \frac{1}{p^*}G(t,\eta,y) + \frac{1}{p^*}D(t),\\
\dot{y} &= \xi_2 + p^*\bar{e}_2 + l_2 y + f_1(t,y) + g_1(t,\eta,y) + d_1(t),\\
\dot{\xi}_2 &= \xi_3 + l_3 y + \tilde{f}_2(t,y,\bar{\xi}_2) - l_2\big(f_1(t,y) + \xi_2 + l_2 y\big),\\
&\ \ \vdots\\
\dot{\xi}_n &= u + \tilde{f}_n(t,y,\bar{\xi}_n) - l_n\big(f_1(t,y) + \xi_2 + l_2 y\big).
\end{aligned} \tag{11}$$

Remark 4. By Schur compliment lemma in [24], (5) can be solved by the linear matrix inequality (3). P, S and δ_1 in (3) can be solved by using LMI toolbox in MATLAB and the observer gain $l = P^{-1}S$.

Remark 5. In [20], there is a mistake in the choice of observer gain. Here we correct it and give a LMI algorithm of it.

3.2 Adaptive controller design

Now, we give the adaptive controller design procedure by using the backstepping method.

Step 1. Begin with the y-subsystem of (11) and consider ξ_2 as the virtual dynamic control input. We define the 1st dynamic virtual control input

$$\alpha_1 = -c\kappa\psi_1(y)y, \qquad \dot{\kappa} = \Gamma\psi_1(y)y^2, \tag{12}$$

where Γ, c are two positive parameters and ψ_1 is a smooth positive design function, introduce a new intermediate variable $v_2 = \xi_3 + l_3 y + \tilde{f}_2(y,\bar{\xi}_2) - l_2(f_1(y) + \xi_2 + l_2 y) - \partial\alpha_1/\partial\kappa\Gamma\psi_1(y)y^2$, and set $z_1 = \xi_2 - \alpha_1(\kappa,y)$, obviously,

$$\dot{z}_1 = v_2 - \frac{\partial\alpha_1}{\partial y}\big(\xi_2 + l_2 y + p^*\bar{e}_2 + f_1 + g_1 + d_1(t)\big). \tag{13}$$

Step 2. Denoting $V_1 = (1/2)y^2$, viewing ξ_2 as the virtual control input, and considering the Lyapunov function $V_2 = (1/2)y^2 + (1/2)z_1^2$, with the use of (11)–(13), one has

$$\begin{aligned}\dot{V}_2 &= z_1\left(v_2 + y - \frac{\partial\alpha_1}{\partial y}(\xi_2 + l_2 y + p^*\bar{e}_2 + f_1 + g_1 + d_1)\right)\\ &\quad - c\kappa\psi_1 y^2 + y(p^*\bar{e}_2 + l_2 y + f_1 + g_1 + d_1).\end{aligned}\tag{14}$$

By Young's inequality, one leads to

$$\begin{gathered} z_1 y \leqslant \frac{1}{2}z_1^2 + \frac{1}{2}y^2, \\ -\frac{\partial\alpha_1}{\partial y}z_1(p^*\bar{e}_2 + g_1 + d_1) \leqslant \frac{2\epsilon_2 p^* + p^{*2}}{2\epsilon_2}\left(\frac{\partial\alpha_1}{\partial y}\right)^2 z_1^2 + \epsilon_2\bar{e}_2^2 + \frac{g_1^2}{2p^*} + \frac{d_1^2}{2p^*},\end{gathered}\tag{15}$$

where ϵ_2 is a small design parameter to be determined in Appendix. We define an unknown constant θ such that $\theta \geqslant (2\epsilon_2 p^* + p^{*2})/(2\epsilon_2)$, and set $\Phi_1(t, \bar{e}_2, \eta, y) = y(p^*\bar{e}_2 + l_2 y + f_1 + g_1 + d_1) + (1/2)y^2 + \epsilon_2\bar{e}_2^2 + g_1^2/2p^* + d_1^2/2p^*$, by (14) and (15), some simple manipulations lead to

$$\dot{V}_2 \leqslant z_1\left(v_2 + \frac{1}{2}z_1 - \frac{\partial\alpha_1}{\partial y}(\xi_2 + l_2 y + f_1) + \theta\left(\frac{\partial\alpha_1}{\partial y}\right)^2 z_1\right) - c\kappa\psi_1(y)y^2 + \Phi_1.$$

Letting $\hat{\theta}$ be the estimate of the unknown parameter θ, choosing $\bar{V}_2 = V_2 + 1/(2\Gamma_\theta) \times (\hat{\theta} - \theta)^2$, where $\Gamma_\theta > 0$ is a parameter, and setting $z_2 = \xi_3 - \alpha_2(\kappa, y, \xi_2, \hat{\theta})$, $\tau_1 = \Gamma_\theta(\partial\alpha_1/\partial y)^2 z_1^2$, and $\alpha_2 = -c_1 z_1 - (1/2)z_1 - l_3 y - \tilde{f}_2 + l_2(f_1 + \xi_2 + l_2 y) + (\partial\alpha_1/\partial\kappa) \times \Gamma\psi_1 y^2 + (\partial\alpha_1/\partial y)(\xi_2 + l_2 y + f_1) - \hat{\theta}(\partial\alpha_1/\partial y)^2 z_1$, in which $c_1 > 0$ is a constant, one can verify that

$$\dot{\bar{V}}_2 \leqslant -c\kappa\psi_1(y)y^2 + \Phi_1 + z_1 z_2 - c_1 z_1^2 + \frac{1}{\Gamma_\theta}(\hat{\theta} - \theta)(\dot{\hat{\theta}} - \tau_1),\tag{16}$$

and the variable z_2 satisfies

$$\dot{z}_2 = v_3 - \frac{\partial\alpha_2}{\partial\hat{\theta}}\dot{\hat{\theta}} - \frac{\partial\alpha_2}{\partial y}(\xi_2 + p^*\bar{e}_2 + l_2 y + f_1 + g_1 + d_1),$$

where $v_3 = \xi_4 + l_4 y + \tilde{f}_3 - l_3(f_1 + \xi_2 + l_2 y) - (\partial\alpha_2/\partial\kappa)\Gamma\psi_1 y^2 - (\partial\alpha_2/\partial\xi_2)(\xi_3 + l_3 y + \tilde{f}_2 - l_2(f_1 + \xi_2 + l_2 y))$.

Step i ($3 \leqslant i \leqslant n$). At step i, one can obtain the similar property to (16). Such a result is presented by the following lemma, for notational coherence, denote $u = \xi_{n+1}$.

Lemma 1. *For each $i = 3, \dots, n$, there exist smooth functions α_i, τ_{i-1}, Φ_{i-1}, variable $z_i = \xi_{i+1} - \alpha_i$, and positive constant c_{i-1} such that $\bar{V}_i = \bar{V}_{i-1} + (1/2)z_{i-1}^2$ satisfies*

$$\begin{aligned}\dot{\bar{V}}_i &\leqslant -c\kappa\psi_1(y)y^2 + \Phi_{i-1} + z_{i-1}z_i\\ &\quad - \sum_{j=1}^{i-1} c_j z_j^2 + \frac{1}{\Gamma_\theta}\left(\hat{\theta} - \theta - \sum_{j=1}^{i-1}\Gamma_\theta z_j\frac{\partial\alpha_j}{\partial\hat{\theta}}\right)(\dot{\hat{\theta}} - \tau_{i-1}).\end{aligned}\tag{17}$$

Proof. See Appendix. □

Hence at step n, by Lemma 1, there is a smooth dynamic output feedback controller

$$u = \alpha_n(\kappa, y, \xi_2, \dots, \xi_n, \hat{\theta}), \qquad \dot{\kappa} = \Gamma\psi_1(y)y^2, \qquad \dot{\hat{\theta}} = \tau_{n-1}, \tag{18}$$

such that $\bar{V}_n = (1/2)y^2 + (1/2)\sum_{j=1}^{n-1} z_j^2 + (1/2)\Gamma_\theta(\hat{\theta} - \theta)^2$ satisfies

$$\dot{\bar{V}}_n \leqslant -c\kappa\psi_1(y)y^2 + \Phi_{n-1} - \sum_{j=1}^{n-1} c_j z_j^2. \tag{19}$$

4 Main result

Before giving the main result of paper, we need the following lemmas.

Lemma 2. *Consider the η-subsystem satisfying Assumption* 1.

(i) *If $\liminf_{s\to\infty} \pi_0(s) = \infty$, then, for any positive-definite continuous function ϕ with*

$$\phi(s) = \mathcal{O}\big(\pi_0(s)\big) \quad \textit{as } s \to 0+,$$

there always exist a positive-definite function σ and a class $\mathcal{K}_\infty$ function φ such that

$$\int_0^t \phi\big(|\eta(\tau)|\big)\,\mathrm{d}\tau \leqslant \sigma\big(|\eta_0|\big) + \tilde{p}_0 \int_0^t \varphi\big(|y(\tau)|\big)\,\mathrm{d}\tau,$$

where $\tilde{p}_0$ is unknown positive constant. Moreover, if γ_0 is such that $\gamma_0(s) = \mathcal{O}(s^2)$ as $s \to 0+$, so is φ.

(ii) *If $\liminf_{s\to\infty} \pi_0(s) < \infty$, then, for any positive-definite continuous function ϕ with*

$$\phi(s) = \mathcal{O}\big(\pi_0(s)\big) \quad \textit{as } s \to 0+ \textit{ and } s \to \infty,$$

the same conclusion of (i) *also holds.*

Proof. The proof of Lemma 2 is similar to the proof of Proposition 2 in [20]. □

Lemma 3. *There are unknown positive constant θ_0, which is dependent on ϵ_2, p^*, p_{i1}, p_{i2} $(1 \leqslant i \leqslant n)$, l_2 and relative degree n, and uncertain $L^2(R_+; R)$ functions $D_1(t)$, $D_2(t)$, $D_3(t)$ such that*

$$|\Phi_{n-1}| \leqslant n\epsilon_2\bar{e}_2^2 + \theta_0\big(y^2 + \hat{f}_1^2(y) + \phi_{11}^2\big(|y|\big) + \phi_{12}^2\big(|\eta|\big)\big) + D_1^2(t),$$

$$\frac{1}{{p^*}^2}|G|^2 \leqslant \sum_{i=2}^n 4\big(l_i^2\phi_{11}^2(|y|) + \phi_{i1}^2(|y|)\big) + \sum_{i=2}^n 4\big(l_i^2\phi_{12}^2\big(|\eta|\big) + \phi_{i2}^2\big(|\eta|\big)\big) + D_2^2(t),$$

$$\frac{1}{{p^*}^2}\big|D(t)\big|^2 \leqslant D_3^2(t).$$

Proof. With the aid of the completion of squares, it follows directly from the definitions of Φ_{n-1}, G and $D(t)$. □

Theorem 1. *Suppose that Assumptions* 1–4 *hold with the following properties*:

$$\phi_{i2}^2(s) = \begin{cases} \mathcal{O}(\pi_0(s)) \text{ as } s \to 0+ & \text{if } \liminf_{s\to\infty} \pi_0(s) = \infty, \\ \mathcal{O}(\pi_0(s)) \text{ as } s \to 0+,\ s \to \infty & \text{if } \liminf_{s\to\infty} \pi_0(s) < \infty \end{cases} \tag{20}$$

for all $i = 1, \dots, n$, *and* $\gamma_0(s) = \mathcal{O}(s^2)$ *as* $s \to 0+$. *Then by choosing the design function* ψ_1, *one has*:

(i) *The solutions of* (1), (8) *and* (18) *are well-defined and bounded over* $[0, \infty)$.

(ii) $\lim_{t\to\infty}(|y(t)| + |\eta(t)|) = 0$.

Proof. (i) Choosing the Lyapunov function $V_e = \bar{e}^{\mathrm{T}} P \bar{e}$, where $P = P^{\mathrm{T}} > 0$ is defined in (5), and $V_c = V_e + \bar{V}_n$, by (10) and Lemma 3, one has

$$\begin{aligned} \dot{V}_e &= \bar{e}^{\mathrm{T}}\bigl(A^{\mathrm{T}}P + PA\bigr)\bar{e} + \frac{2}{p^*}\bar{e}^{\mathrm{T}}P\bigl(F(t,x) - F(t,\hat{x})\bigr) + \frac{2}{p^*}\bar{e}^{\mathrm{T}}PG + \frac{2}{p^*}\bar{e}^{\mathrm{T}}PD \\ &\leqslant \bar{e}^{\mathrm{T}}\bigl(A^{\mathrm{T}}P + PA\bigr)\bar{e} + \delta_1^{-1}\bar{e}^{\mathrm{T}}PP\bar{e} + \frac{\delta_1}{p^{*2}}\bigl|F(t,x) - F(t,\hat{x})\bigr|^2 \\ &\quad + 2\delta_2^{-1}\bar{e}^{\mathrm{T}}PP\bar{e} + \frac{\delta_2}{p^{*2}}|G|^2 + \frac{\delta_2}{p^{*2}}|D|^2 \\ &\leqslant \bar{e}^{\mathrm{T}}\bigl(A^{\mathrm{T}}P + PA + \delta_1^{-1}PP + \delta_1\rho_0^2 I + 2\delta_2^{-1}PP\bigr)\bar{e} \\ &\quad + 4\delta_2\sum_{i=2}^{n}\bigl(l_i^2\phi_{11}^2\bigl(|y|\bigr) + \phi_{i1}^2\bigl(|y|\bigr)\bigr) \\ &\quad + 4\delta_2\sum_{i=2}^{n}\bigl(l_i^2\phi_{12}^2\bigl(|\eta|\bigr) + \phi_{i2}^2\bigl(|\eta|\bigr)\bigr) + \delta_2\bigl(D_2^2(t) + D_3^2(t)\bigr). \end{aligned} \tag{21}$$

From (19), (21) and Lemma 3, it follows that

$$\begin{aligned} c\dot{V}_c \leqslant{}& -c\kappa\psi_1(y)y^2 + \theta_0\bigl(y^2 + \hat{f}_1^2(y) + \phi_{11}^2\bigl(|y|\bigr) + \phi_{12}^2\bigl(|\eta|\bigr)\bigr) \\ &+ D_1^2(t) + \delta_2\bigl(D_2^2(t) + D_3^2(t)\bigr) \\ &+ \bar{e}^{\mathrm{T}}\bigl(A^{\mathrm{T}}P + PA + \delta_1^{-1}PP + \delta_1\rho_0^2 I + 2\delta_2^{-1}PP + n\epsilon_2 I\bigr)\bar{e} \\ &+ 4\delta_2\sum_{i=2}^{n}\bigl(l_i^2\phi_{11}^2\bigl(|y|\bigr) + \phi_{i1}^2\bigl(|y|\bigr)\bigr) + 4\delta_2\sum_{i=2}^{n}\bigl(l_i^2\phi_{12}^2\bigl(|\eta|\bigr) + \phi_{i2}^2\bigl(|\eta|\bigr)\bigr). \end{aligned} \tag{22}$$

One can choose sufficiently large δ_2 and sufficiently small ϵ_2 such that $2\delta_2^{-1}PP + n\epsilon_2 I \leqslant Q$, which together with (5) imply that

$$A^{\mathrm{T}}P + PA + \bigl(\delta_1^{-1} + 2\delta_2^{-1}\bigr)PP + \bigl(\rho_0^2\delta_1 + n\epsilon_2\bigr)I \leqslant -Q, \tag{23}$$

where Q is defined in (5). By Assumption 1, Lemma 2 and (20), one has

$$\int_0^t \phi_{i2}^2\bigl(|\eta(s)|\bigr)\,\mathrm{d}s \leqslant \sigma_i\bigl(|\eta(0)|\bigr) + \tilde{p}_{i0}\int_0^t \varphi_{i1}\bigl(|y(s)|\bigr)\,\mathrm{d}s, \tag{24}$$

where σ_i are positive definite functions, $\varphi_{i1} \in \mathcal{K}_\infty$ with $\varphi_{i1}(s) = \mathcal{O}(s^2)$ as $s \to 0+$, and $\tilde{p}_{i0}$ are unknown positive constants.

Choose a smooth design function ψ_1 to satisfy

$$\psi_1(y)y^2 \geqslant \max\bigl\{y^2 + \hat{f}_1^2(y) + \phi_{11}^2\bigl(|y|\bigr),\ l_i^2\phi_{11}^2\bigl(|y|\bigr) + \phi_{i1}^2\bigl(|y|\bigr),\\ \varphi_{i1}\bigl(|y|\bigr),\ \gamma_0\bigl(|y|\bigr),\ 1 \leqslant i \leqslant n\bigr\}. \tag{25}$$

Such a function ψ_1 always exists due to the fact that f_1, ϕ_{i1} are smooth near zero with $f_1(0) = \phi_{i1}(0) = 0$, and $\varphi_{i1}(s) = \mathcal{O}(s^2)$ as $s \to 0+$. Then it follows from (22) and (25) that

$$\dot{V}_c \leqslant -c\kappa\psi_1 y^2 + \bigl(\theta_0 + 4(n-1)\delta_2\bigr)\psi_1 y^2 + \theta_0\phi_{12}^2\bigl(|\eta|\bigr)\\ + 4\delta_2\sum_{i=2}^n \bigl(l_i^2\phi_{12}^2\bigl(|\eta|\bigr) + \phi_{i2}^2\bigl(|\eta|\bigr)\bigr) + D_1^2(t) + \delta_2\bigl(D_2^2(t) + D_3^2(t)\bigr). \tag{26}$$

Integrating on both sides of (26) from 0 to t, and noting $\dot{\kappa} = \mathit{\Gamma}\psi_1(y)y^2$ in (18), by (24) and (25), one gets

$$V_c(t) - V_c(0) \leqslant -\frac{c}{2\mathit{\Gamma}}\kappa^2(t) + d_1\kappa(t) + d_2 + \int_0^t D_4^2(s)\,\mathrm{d}s, \tag{27}$$

where $d_1 = (1/\mathit{\Gamma})(2\theta_0\tilde{p}_{10} + 8(n-1)\delta_2 + 4\delta_2\sum_{i=2}^n l_i^2\tilde{p}_{i0})$, $d_2 = -d_1\kappa(0) + c/(2\mathit{\Gamma}) \times \kappa^2(0) + \theta_0\sigma_1(|\eta(0)|) + 4\delta_2\sum_{i=2}^n(\sigma_i(|\eta(0)|) + l_i^2\sigma_1(|\eta(0)|))$, $D_4^2(t) = D_1^2(t) + \delta_2(D_2^2(t) + D_3^2(t))$.

Assume that the solutions of the closed-loop system are defined on a right-maximal interval $[0, T)$ with $0 < T \leqslant \infty$. Next, we will prove that $\kappa(t)$ is bounded on $[0, T)$ by contradiction. Suppose that $\kappa(t)$ is unbounded, since $\dot{\kappa} = \mathit{\Gamma}\psi_1(y)y^2 \geqslant 0$, so $\kappa(t)$ is increasing and tends to ∞ as $t \to T$. Dividing both sides of (27) by $\kappa(t)$ for sufficiently large t (where $t < T$), one gets

$$\frac{-V_c(0) - d_2 - \int_0^t D_4^2(s)\,\mathrm{d}s}{\kappa(t)} \leqslant -\frac{c}{2\mathit{\Gamma}}\kappa(t) + d_1. \tag{28}$$

Since $D_1, D_2, D_3 \in L^2(R_+; R)$, so $D_4 \in L^2(R_+; R)$. As $t \to T$, the right side of (28) converges to $-\infty$, while the left side of (28) converges to zero, which is a contradiction. Consequently, $\kappa(t)$ is bounded on $[0, T)$.

By (12), (25) and the boundedness of $\kappa(t)$, we obtain that $\int_0^t \gamma_0(|y(s)|)\,\mathrm{d}s$ is bounded on $[0, T)$, which together with (2) imply that $V_0(\eta(t))$ and $\eta(t)$ remain bounded on $[0, T)$.

Using (27) and the boundedness of $\kappa(t)$, one also concludes that $V_c(t)$ is bounded over $[0, T)$. By definition of $V_c(t)$ in (21) above, it holds that the closed-loop signals $y(t)$, $z_1(t), \ldots, z_{n-1}(t)$, $\hat{\theta}(t)$ and $\bar{e}(t)$ are all bounded over $[0, T)$. From the definition of $z_i(t)$ and $\alpha_i(t)$, it is not hard to prove that $\xi_i(t), x_i(t), u(t)$ are bounded over $[0, T)$. Therefore, $T = \infty$, and conclusion (i) holds.

(ii) By the boundedness of $y(t)$ and $\dot{y}(t)$, then $\gamma_0(|y(t)|)$ is uniformly continuous in $[0, \infty)$. Using $\int_0^\infty \gamma_0(|y(t)|)\,\mathrm{d}t < \infty$ and Barbalat's lemma in [23], one has $\lim_{t\to\infty} \gamma_0(|y(t)|) = 0$ and $\lim_{t\to\infty} y(t) = 0$. By Assumption 1, $\int_0^\infty \gamma_0(|y(t)|)\,\mathrm{d}t < \infty$ and Proposition 6 in [10], one has $\lim_{t\to\infty} \eta(t) = 0$. By (21), (23) and (24), one can obtain that $\int_0^\infty \bar{e}^{\mathrm{T}} Q \bar{e}(t)\,\mathrm{d}t < \infty$, so by Barbalat's lemma, $\lim_{t\to\infty} \bar{e}(t) = 0$. This concludes the proof. □

5 A simulation example

Consider the following nonlinear system with inverse dynamics and noises:

$$\begin{aligned}
\dot{\eta} &= -\arctan\eta + d_0 y^2,\\
\dot{x}_1 &= x_2 + f_1(x_1) + p_{11} y + p_{12}\frac{\eta}{1+|\eta|} + \frac{d_1}{1+t},\\
\dot{x}_2 &= u + f_2(\bar{x}_2) + p_{21} y^2 + p_{22}\frac{\eta^2}{1+\eta^2} + d_2 e^{-t},\\
y &= x_1,
\end{aligned} \tag{29}$$

where $f_1(x_1) = x_1^2$, $f_2(\bar{x}_2) = x_1 + \cos x_2$, and p_{11}, p_{12}, p_{21}, p_{22}, d_0, d_1 and d_2 are unknown constants. Choosing $V(\eta) = \eta\arctan\eta$, it is easy to verify that $\dot{V}(\eta) \leqslant -\arctan^2|\eta| + 3d_0 y^2$.

With the notations of Assumptions 1-4, one can take $\pi_0(|\eta|) = \arctan^2|\eta|$, $\gamma_0(|y|) = 3y^2$, $\phi_{11}(|y|) = |y|$, $\phi_{12}(|\eta|) = |\eta|/(1+|\eta|)$, $\phi_{21}(|y|) = y^2$, $\phi_{22}(|\eta|) = \eta^2/(1+\eta^2)$. Then $\phi_{i2}^2(s) = \mathcal{O}(\pi_0(s))$ as $s \to 0+$ and $s \to \infty$, $i = 1, 2$, the conditions of Theorem 1 are satisfied.

By (8), the reduced-order observer is given by

$$\dot{\xi}_2 = u + f_2(y, \xi_2 + l_2 y) - l_2\big(f_1(y) + \xi_2 + l_2 y\big). \tag{30}$$

According to Section 3, the dynamic output feedback control law can be designed as

$$\begin{aligned}
&\dot{\kappa} = \mathit{\Gamma}\psi_1(y)y^2, \qquad \dot{\hat{\theta}} = \mathit{\Gamma}_\theta\left(\frac{\partial\alpha_1}{\partial y}\right)^2 z_1^2,\\
&u = -c_1 z_1 - \frac{1}{2}z_1 - f_2(y, \xi_2 + l_2 y) + l_2\big(f_1(y) + \xi_2 + l_2 y\big)\\
&\qquad + \frac{\partial\alpha_1}{\partial\kappa}\mathit{\Gamma}\psi_1(y)y^2 + \frac{\partial\alpha_1}{\partial y}(\xi_2 + l_2 y + f_1) - \hat{\theta}\left(\frac{\partial\alpha_1}{\partial y}\right)^2 z_1,
\end{aligned} \tag{31}$$

where $\alpha_1 = -c\kappa\psi_1(y)y$, $z_1 = \xi_2 - \alpha_1$, and $\mathit{\Gamma}$, $\mathit{\Gamma}_\theta$, c, c_1 are positive parameters.

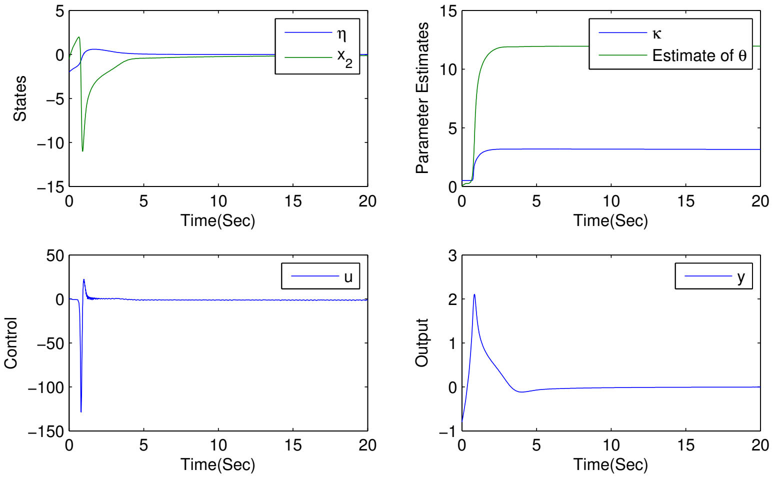

Fig. 1. The responses of closed-loop system (29)–(31).

By the proof of Proposition 2 in [20], one can take $\varphi_{11}(|y|) = \varphi_{21}(|y|) = 3y^2$ in (24). So $\psi_1(y)$ in (25) can be chosen as $\psi_1(y) = 2y^2 + l_2^2 + 3$.

In simulation, we choose the parameters $c = 0.1$, $c_1 = 0.1$, $l_2 = 1$, $\Gamma_\theta = 0.8$, $\Gamma = 0.6$, $p_{11} = 1$, $p_{12} = 1$, $p_{21} = 0.5$, $p_{22} = 0.5$, $d_0 = 1$, $d_1 = 3$, $d_2 = 5$, the initial values $\eta(0) = -2$, $x_1(0) = -0.8$, $x_2(0) = -0.5$, $\xi_2(0) = 0.1$, $\kappa(0) = 0$, $\hat{\theta}(0) = 0.5$. Fig. 1 gives the responses of closed-loop system (29)–(31).

6 Conclusions

This paper considers global output feedback control for a class of nonlinear systems with inverse dynamics and L^2 noise. For the inverse dynamics with uncertain supply rates, the reduced-order observer based output feedback controller is constructed, which drives the output of system to zero asymptotically and maintains other closed-loop signals bounded.

Appendix. The proof of Lemma 1

Assuming that $\bar{V}_{i-1}$ satisfies the similar properties to (17), noticing that

$$\begin{aligned}
\dot{z}_{i-1} &= v_i - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} - \frac{\partial \alpha_{i-1}}{\partial y}(\xi_2 + p^* \bar{e}_2 + l_2 y + f_1 + g_1 + d_1),\\
v_i &= \xi_{i+1} + l_{i+1} y + \tilde{f}_i - l_i(f_1 + \xi_2 + l_2 y) - \frac{\partial \alpha_{i-1}}{\partial \kappa}\Gamma \psi_1 y^2\\
&\quad - \sum_{j=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \xi_j}\big(\xi_{j+1} + l_{j+1} y + \tilde{f}_j - l_j(f_1 + \xi_2 + l_2 y)\big),
\end{aligned} \tag{32}$$

there holds

$$\dot{V}_i \leqslant -c\kappa\psi_1(y)y^2 + \Phi_{i-2} + z_{i-2}z_{i-1} - \sum_{j=1}^{i-2} c_j z_j^2 + \frac{1}{\Gamma_\theta}\left(\hat{\theta} - \theta - \sum_{j=1}^{i-2}\Gamma_\theta z_j \frac{\partial\alpha_j}{\partial\hat{\theta}}\right)(\dot{\hat{\theta}} - \tau_{i-2}) + z_{i-1}\left(v_i - \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\dot{\hat{\theta}} - \frac{\partial\alpha_{i-1}}{\partial y}(\xi_2 + p^*\bar{e}_2 + l_2 y + f_1 + g_1 + d_1)\right). \tag{33}$$

Using Young's inequality, it follows that

$$-\frac{\partial\alpha_{i-1}}{\partial y}z_{i-1}(p^*\bar{e}_2 + g_1 + d_1) \leqslant \epsilon_2\bar{e}_2^2 + \frac{2\epsilon_2 p^* + {p^*}^2}{2\epsilon_2}\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2 z_{i-1}^2 + \frac{g_1^2}{2p^*} + \frac{d_1^2}{2p^*}. \tag{34}$$

Define

$$\tau_{i-1} = \tau_{i-2} + \Gamma_\theta\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2 z_{i-1}^2, \qquad \Phi_{i-1} = \Phi_{i-2} + \epsilon_2\bar{e}_2^2 + \frac{g_1^2}{2p^*} + \frac{d_1^2}{2p^*},$$

$$\alpha_i = -c_{i-1}z_{i-1} - z_{i-2} - l_{i+1}y + l_i(f_1 + \xi_2 + l_2 y) - \tilde{f}_i + \frac{\partial\alpha_{i-1}}{\partial y}(\xi_2 + l_2 y + f_1) + \sum_{j=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\xi_j}\bigl(\xi_{j+1} + l_{j+1}y + \tilde{f}_j - l_j(f_1 + \xi_2 + l_2 y)\bigr) + \frac{\partial\alpha_{i-1}}{\partial\kappa}\Gamma\psi_1 y^2 + \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\tau_{i-1} - \hat{\theta}\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2 z_{i-1} + \sum_{j=1}^{i-2} z_j\frac{\partial\alpha_j}{\partial\hat{\theta}}\Gamma_\theta\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2 z_{i-1},$$

where $l_{n+1} = 0$, which together with (32)–(34) and $z_i = \xi_{i+1} - \alpha_i$ imply that

$$\begin{aligned}\dot{V}_i &\leqslant -c\kappa\psi_1(y)y^2 + \Phi_{i-1} + z_{i-1}z_{i-2} - \sum_{j=1}^{i-1} c_j z_j^2 + \frac{1}{\Gamma_\theta}\left(\hat{\theta} - \theta - \sum_{j=1}^{i-2}\Gamma_\theta z_j\frac{\partial\alpha_j}{\partial\hat{\theta}}\right)(\dot{\hat{\theta}} - \tau_{i-2}) + z_{i-1}\left((\theta - \hat{\theta})\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2 z_{i-1} + \sum_{j=1}^{i-2} z_j\frac{\partial\alpha_j}{\partial\hat{\theta}}\Gamma_\theta\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2 z_{i-1} - \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}(\dot{\hat{\theta}} - \tau_{i-1})\right)\\ &= -c\kappa\psi_1(y)y^2 + \Phi_{i-1} + z_{i-1}z_i - \sum_{j=1}^{i-1} c_j z_j^2 + \frac{1}{\Gamma_\theta}\left(\hat{\theta} - \theta - \sum_{j=1}^{i-1}\Gamma_\theta z_j\frac{\partial\alpha_j}{\partial\hat{\theta}}\right)(\dot{\hat{\theta}} - \tau_{i-1}).\end{aligned} \tag{35}$$

References

1. E.D. Sontag, Smooth stabilization implies coprime factorization, *IEEE Trans. Autom. Control*, **34**:435–443, 1989.
2. E.D. Sontag, A.R. Teel, Changing supply functions in input/state stable systems, *IEEE Trans. Autom. Control*, **40**:1476–1478, 1995.
3. E.D. Sontag, Y. Wang, On characterizations of the input-to-state stability property, *Systems Control Lett.*, **24**:351–359, 1995.
4. E.D. Sontag, Y. Wang, New characterizations of input-to-state stability, *IEEE Trans. Autom. Control*, **41**:1283–1294, 1996.
5. A.R. Teel, A nonlinear small gain theorem for the analysis of control systems with saturation, *IEEE Trans. Autom. Control*, **41**:1256–1270 1996.
6. Z.P. Jiang, A Combined backstepping and small-gain approach to adaptive output feedback control, *Automatica*, **35**:1131–1139, 1999.
7. Z.P. Jiang, I. Mareels, A small gain control method for nonlinear cascaded systems with dynamic uncertainties, *IEEE Trans. Autom. Control*, **42**:292–308, 1997.
8. Z.P. Jiang, I. Mareels, Robust nonlinear integral control, *IEEE Trans. Autom. Control*, **46**:1336–1342, 2001.
9. L. Praly, Z.P. Jiang, Stabilization by output feedback for systems with ISS inverse dynamics, *Systems Control Lett.*, **21**:19–33, 1993.
10. E.D. Sontag, Comments on integral variants of ISS, *Systems Control Lett.*, **34**:93–100, 1998.
11. D. Angeli, E.D. Sontag, Y. Wang, A characterization of integral input-to-state stability, *IEEE Trans. Autom. Control*, **45**:1082–1097, 2000.
12. M. Arcak, D. Angeli, E.D. Sontag, A unifing integral iss framework for stability of nonlinear cascades, *SIAM J. Control Optim.*, **40**:1888–1904, 2002.
13. H. Ito, State-dependent scaling problems and stability of interconnected iiss and iss systems, *IEEE Trans. Autom. Control*, **51**:1626–1643, 2006.
14. H. Ito, Z.P. Jiang, Necessary and sufficient small gain conditions for integral input-to-state stable systems: A Lyapunov perspective, *IEEE Trans. Autom. Control*, **54**:2389–2404, 2009.
15. H. Ito, A Lyapunov Approach to cascade interconnection of integral input-to-state systems, *IEEE Trans. Autom. Control*, **55**:702–708, 2010.
16. H. Ito, P. Pepe, Z.P. Jiang, A Small-gain condition for IISS of interconnected retarded systems based on Lyapunov–Krasovskii functionals, *Automatica*, **46**:1646–1656, 2010.
17. Z.P. Jiang, I. Mareels, D.J. Hill, J. Huang, A unifying framework for global regulation via nonlinear output feedback: From ISS to iISS, *IEEE Trans. Autom. Control*, **49**:549–562, 2004.
18. Y. Wu, J.B. Yu, Y. Zhao, Further results on global asymptotic regulation control for a class of nonlinear systems with iiss inverse dynamics, *IEEE Trans. Autom. Control*, **56**:941–946, 2011.
19. D. Xu, J. Huang, Output regulation for output feedback systems with iISS inverse dynamics, *J. Dyn. Syst. Meas. Control*, **133**, 044503, 4 pp., 2011.

20. X. Yu, Y. Wu, X. Xie, Reduced-order observer-based output feedback regulation for a class of nonlinear systems with iISS inverse dynamics, *Int. J. Control*, **85**(12):1942–1951, 2012.

21. X. Yu, X. Xie, Output feedback regulation of stochastic nonlinear systems with stochastic iISS inverse dynamics, *IEEE Trans. Autom. Control*, **55**:304–320, 2010.

22. X. Yu, X. Xie, N. Duan, Small-gain control method for stochastic nonlinear systems with stochastic iISS inverse dynamics, *Automatica*, **46**:1790–1798, 2010.

23. M. Krstic, I. Kanellakopoulos, P.V. Kokotovic, *Nonlinear and Adaptive Control Design*, Wiley, New York, 1995.

24. S. Boyd, L.E. Ghaoui, E. Feron, V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia, 1994.