Exponential synchronization for reaction-diffusion neural networks with mixed time-varying delays via periodically intermittent control*

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Abstract. This paper deals with the exponential synchronization problem for reaction-diffusion neural networks with mixed time-varying delays and stochastic disturbance. By using stochastic analysis approaches and constructing a novel Lyapunov–Krasovskii functional, a periodically intermittent controller is first proposed to guarantee the exponential synchronization of reaction-diffusion neural networks with mixed time-varying delays and stochastic disturbance in terms of p-norm. The obtained synchronization results are easy to check and improve upon the existing ones. Particularly, the traditional assumptions on control width and time-varying delays are removed in this paper. This paper also presents two illustrative examples and uses simulated results of these examples to show the feasibility and effectiveness of the proposed scheme.

Keywords: synchronization, neural networks, mixed time-varying delays, reaction-diffusion, periodically intermittent control.

1 Introduction

In the past decade, there has been a great interest in various types of neural networks (for example, Hopfield neural networks, cellular neural networks, Cohen–Grossberg neural networks, bidirectional associative memory neural networks, competitive neural networks, etc.) due to their wide range of applications, such as signal processing, pattern recognition, image processing, associative memory, fault diagnosis, aerospace, defense,

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telecommunications, automatic control engineering, and combinatorial optimization. However, time delays are unavoidably in the information processing of neurons due to various reasons [1–3]. For example, time delays can be caused by the finite switching speed of amplifier circuits in neural networks or deliberately introduced to achieve tasks of dealing with motion-related problems, such as moving image processing. Meanwhile, a neural network usually has a spatial nature due to the presence of an amount of parallel pathways of a variety of axon sizes and lengths, it is desired to model them by introducing distributed delays. Therefore, both discrete and distributed delays, especially both discrete and distributed time-varying delays, should be taken into account when modeling realistic neural networks [4–7].

It has been reported that if the parameters and mixed time-varying delays are appropriately chosen, the neural networks can exhibit complicated behaviors even with strange chaotic attractors. Thus, the synchronization problems of chaotic neural networks with mixed time-varying delays have received much more attention both in theory and in practice due to its potential applications in various technological fields, including chaos generators design, secure communications, chemical and biological systems, information processing, distributed computation, optics, social science, harmonic oscillation generation, human heartbeat regulation, power system protection, and so on [8–12].

In signal transmission, the signal will become weak due to diffusion, so an external control should be added until the strength of the signal reaches an upper level. Then, the external control can be removed considering the cost. Therefore, in comparison with continuous control, discontinuous controllers, which include intermittent control and impulsive control, have attracted more interest due to its wide applications in engineering fields [13]. Intermittent control, which was first introduced to control linear econometric models in [14], has been used for a variety of purposes such as manufacturing, transportation and communication in practice. In [15–17], the synchronization problems for a class of chaotic neural networks with constant delay were investigated by designing periodically intermittent controllers based on 2-norm. And then, the periodically intermittent control was applied to deal with the synchronization of chaotic neural networks without time delays in [18] based on 2-norm. In [19], Yu et al. investigated the synchronization problem of Cohen–Grossberg neural networks with time-varying delays by designing a periodically intermittent controller based on ∞-norm. Moreover, a novel intermittent impulsive synchronization scheme was proposed to realize synchronization of two chaotic delayed neural networks in [20]. As pointed out by Hu et al. [21], the most previous results presented in [15–17] were obtained by constructing Lyapunov functions and using two central differential inequalities, and the restriction that the control width is greater than the time delay was imposed in [15–17]. And the condition that the non-control width should be greater than the time delay was also required in [16]. Evidently, the applied areas of the results obtained in [15–17] are limited because of these assumptions. Therefore, in order to reduce the possible conservatism for the sake of broader applications of the intermittent control technique, under the precondition that the derivative of the time-varying delay was smaller than one, Hu et al. [21] studied the exponential stabilization and synchronization for a class of neural networks with time-varying delays for the first time using a periodically intermittent control technique based on p-norm and ∞-norm, respectively. The methods used in [21] were totally different from the corresponding previous works and the obtained conditions were less conservative. Particularly, the traditional assumptions on control width and time delay were removed.

Actually, the synaptic transmission in real neural networks can be viewed as a noisy process introduced by random fluctuations from the release of neurotransmitters and other probabilistic causes [22–25]. Hence, noise is unavoidable and should be taken into consideration in modeling. On the other hand, diffusion effects cannot be avoided in the neural networks when electrons are moving in asymmetric electromagnetic fields. So we must consider that the activations vary in space as well as in time. From the above analysis, the stochastic noise perturbation and diffusion effects on dynamic behaviors of neural networks cannot be neglected, so the theoretical results on dynamic behaviors including stochastic disturbance and diffusion parameters are more reasonable. With respect to reaction-diffusion neural networks with stochastic perturbation, a few results about the dynamic analysis have been reported in the literature [26–35].

To the best of our knowledge, there are few results, or even no results concerning the synchronization issues for neural networks with mixed time-varying delays, stochastic noise perturbation and reaction-diffusion in terms of p-norm by using periodically intermittent control. The issues of integrating mixed time-varying delays, stochastic noise perturbation and reaction-diffusion effects into the study of synchronization for neural networks require more complicated analysis. Therefore, it is interesting to study this problem both in theories and applications.

Motivated by the above discussion, this paper is concerned with the exponential synchronization for reaction-diffusion neural networks with mixed time-varying delays and stochastic perturbation in terms of p-norm by using periodically intermittent control approach. Some examples with numerical simulations are provided to show the feasibility and effectiveness of the proposed method.

The main contribution of this paper can be summarized as follows:

1. It is the first time to establish the exponential synchronization criterion for reaction-diffusion neural networks with mixed time-varying delays and stochastic noise perturbation based on periodically intermittent control.
2. Unlike the existing results of synchronization for reaction-diffusion neural networks based on 2-norm (see [36–39]), some new and useful conditions are obtained in this paper to guarantee the exponential synchronization of the proposed neural networks under the periodically intermittent control in terms of p-norm.
3. The restrictions on periodically intermittent controller that the control width is greater than the time delay and the non-control width is also greater than the time delay are removed, which is more general than those periodically intermittent controllers given in [15–17].
4. A novel Lyapunov–Krasovskii functional is proposed and the restriction in [21] that the derivative of the time-varying delay should be smaller than one is removed.
5. In [40], the authors pointed out that it is quite difficult to find a chaotic attractor for reaction-diffusion delayed neural networks. Obviously, this is an important

and interesting open problem. In this paper, by using the classical implicit format solving the partial differential equations and the method of steps for differential difference equations, we find that if the parameters are appropriately chosen, the reaction-diffusion neural networks can exhibit chaotic attractors.

The organization of this paper is as follows: in the next section, problem statement and preliminaries are presented; in Section 3, a periodically intermittent controller is proposed to ensure exponential synchronization of reaction-diffusion neural networks with mixed time-varying delays and stochastic noise perturbation in terms of p-norm; numerical simulations will be given in Section 4 to demonstrate the effectiveness and feasibility of our theoretical results. We ends this work with a conclusion in Section 5.

Notation. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively; the notation $C^{2,1}(\mathbb{R}^+ \times \mathbb{R}^n; \mathbb{R}^+)$ denotes the family of all nonnegative functions $V(t, x(t))$ on $\mathbb{R}^+ \times \mathbb{R}^n$, which are continuously twice differentiable in x and once differentiable in t; $(\Omega, \mathcal{F}, \mathcal{P})$ is a complete probability space, where Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$; $\mathbf{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure $\mathcal{P}$; "sgn" is the sign function.

2 Problem statement and preliminaries

In this paper, we are concerned with a class of reaction-diffusion neural networks with mixed time-varying delays, which can be described by the following integro-differential equations:

$$\begin{aligned}\frac{\mathrm{d}u_i(t,x)}{\mathrm{d}t} &= \sum_{k=1}^{l^*} \frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial u_i(t,x)}{\partial x_k}\right) - c_i u_i(t,x) + \sum_{j=1}^{n} a_{ij} f_j\bigl(u_j(t,x)\bigr) \\ &\quad + \sum_{j=1}^{n} b_{ij} g_j\bigl(u_j(t-\tau_{ij}(t),x)\bigr) + \sum_{j=1}^{n} d_{ij} \int\limits_{t-\tau_{ij}^*(t)}^{t} h_j\bigl(u_j(s,x)\bigr)\,\mathrm{d}s + J_i, \quad (1)\end{aligned}$$

where $i = 1, 2, \ldots, n$, n is the number of neurons in the neural networks; $x = (x_1, x_2, \ldots, x_{l^*})^{\mathrm{T}} \in \Omega \subset \mathbb{R}^{l^*}$ and $\Omega = \{x = (x_1, x_2, \ldots, x_{l^*})^{\mathrm{T}} \mid |x_k| < m_k,\ k = 1, 2, \ldots, l^*\}$ is a bound compact set with smooth boundary $\partial\Omega$ and $\mathrm{mes}\Omega > 0$ in space $\mathbb{R}^{l^*}$; $u(t,x) = (u_1(t,x), \ldots, u_n(t,x))^{\mathrm{T}}$ with $u_i(t,x)$ corresponds to the state of the ith neural unit at time t and in space x; $c_i > 0$ represents the decay rate of the ith neuron; a_{ij}, b_{ij} and d_{ij} are, respectively, the connection strength, the time-varying delay connection strength, and the distributed time-varying delay connection strength of the jth neuron on the ith neuron; $f_j(\cdot)$, $g_j(\cdot)$ and $h_j(\cdot)$ denote the activation functions; $D_{ik} \geqslant 0$ corresponds to the transmission diffusion operator along the ith neuron; $0 < \tau_{ij}(t) \leqslant \tau$ and $0 < \tau_{ij}^*(t) \leqslant \tau^*$ are the time-varying delay and the distributed time-varying delay along the axon of the jth unit from the ith unit, respectively; J_i denotes the bias of the ith neuron.

The boundary condition of system (1) is

$$u_i(t,x)|_{\partial\Omega} = 0, \quad (t,x) \in [-\bar{\tau}, +\infty) \times \partial\Omega, \tag{2}$$

and the initial value of system (1) is

$$u_i(s,x) = \phi_i(s,x), \quad (s,x) \in [-\bar{\tau}, 0] \times \Omega, \tag{3}$$

where $\bar{\tau} = \max\{\tau, \tau^*\}$, $\phi(s,x) = (\phi_1(s,x), \dots, \phi_n(s,x))^{\mathrm{T}} \in \mathcal{C}$ is bounded and continuous and $\mathcal{C} = \mathcal{C}([-\bar{\tau}, 0] \times \Omega, \mathbb{R}^n)$ be the Banach space of continuous functions, which maps $[-\bar{\tau}, 0] \times \Omega$ into $\mathbb{R}^n$ with the topology of uniform converge and p-norm (p is a positive integer) defined by

$$\|\phi\|_p = \left(\int\limits_{\Omega} \sum_{i=1}^{n} \sup_{-\bar{\tau} \leqslant s \leqslant 0} \bigl|\phi_i(s,x)\bigr|^p \,\mathrm{d}x \right)^{1/p}.$$

Chaotic systems depend extremely on initial values, and even infinitesimal changes in the initial condition will lead to an asymptotic divergence of orbits. In order to observe the synchronization behavior of system (1), we introduce another delayed neural network, which is the response system of the drive system (1). However, the initial condition of the response system is defined to be different from that of the drive system. Therefore, the controlled response system of network (1) can be described by the following equations:

$$\begin{aligned} \mathrm{d}v_i(t,x) = \Biggl\{ & \sum_{k=1}^{l^*} \frac{\partial}{\partial x_k} \left(D_{ik} \frac{\partial v_i(t,x)}{\partial x_k} \right) - c_i v_i(t,x) + \sum_{j=1}^{n} a_{ij} f_j\bigl(v_j(t,x)\bigr) \\ & + \sum_{j=1}^{n} b_{ij} g_j\bigl(v_j(t-\tau_{ij}(t),x)\bigr) + \sum_{j=1}^{n} d_{ij} \int\limits_{t-\tau_{ij}^*(t)}^{t} h_j\bigl(v_j(s,x)\bigr)\,\mathrm{d}s \\ & + J_i + w_i(t,x) \Biggr\} \mathrm{d}t + \sum_{j=1}^{n} \sigma_{ij}\bigl(e_j(t,x), e_j\bigl(t-\tau_{ij}(t),x\bigr)\bigr)\,\mathrm{d}\omega_j(t), \end{aligned} \tag{4}$$

where $i = 1, 2, \dots, n$, $v(t,x) = (v_1(t,x), \dots, v_n(t,x))^{\mathrm{T}}$ is an n-dimensional state vector of the neural networks; $e(t,x) = (e_1(t,x), \dots, e_n(t,x))^{\mathrm{T}} = v(t,x) - u(t,x)$ is the synchronization error signal; $\sigma = (\sigma_{ij})_{n\times n}$ is the diffusion coefficient matrix (or noise intensity matrix) and the stochastic disturbance $\omega(t) = [\omega_1(t), \dots, \omega_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$ is a Brownian motion defined on $(\Omega, \mathcal{F}, \mathcal{P})$, and

$$\mathbf{E}\bigl\{\mathrm{d}\omega(t)\bigr\} = 0, \qquad \mathbf{E}\bigl\{\mathrm{d}\omega^2(t)\bigr\} = \mathrm{d}t.$$

This type of stochastic perturbation can be regarded as a result from the occurrence of the internal error when the simulation circuits are constructed, such as inaccurate design of the coupling strength and some other important parameters [41], therefore, it relies on the drive system (1). $w(t,x) = (w_1(t,x),\dots,w_n(t,x))^{\mathrm{T}}$ is an intermittent controller defined by

$$w_i(t,x) = \begin{cases} \sum_{j=1}^n k_{ij}(v_j(t,x) - u_j(t,x)), & (t,x) \in [mT, mT+\delta) \times \Omega, \\ 0, & (t,x) \in [mT+\delta, (m+1)T) \times \Omega, \end{cases} \tag{5}$$

where $m = 0, 1, \dots$, k_{ij} $(i,j = 1,\dots,n)$ denotes the control strength, $T > 0$ denotes the control period and $0 < \delta < T$ is called the control width.

The boundary condition and initial condition for response system (4) are given in the forms

$$v_i(t,x)|_{\partial\Omega} = 0, \quad (t,x) \in [-\bar{\tau}, +\infty) \times \partial\Omega, \tag{6}$$

and

$$v_i(s,x) = \psi_i(s,x), \quad (s,x) \in [-\bar{\tau}, 0] \times \Omega, \tag{7}$$

where $\psi_i(s,x)$ $(i = 1,2,\dots,n)$ are bounded and continuous on $[-\bar{\tau}, 0] \times \Omega$.

Subtracting (1) from (4) yields the error system as follows:

$$\begin{aligned} \mathrm{d}e_i(t,x) = \Bigg\{ & \sum_{k=1}^{l^*} \frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial e_i(t,x)}{\partial x_k}\right) - c_i e_i(t,x) + \sum_{j=1}^n a_{ij} f_j^*\big(e_j(t,x)\big) \\ & + \sum_{j=1}^n b_{ij} g_j^*\big(e_j(t-\tau_{ij}(t),x)\big) + \sum_{j=1}^n d_{ij} \int\limits_{t-\tau_{ij}^*(t)}^{t} h_j^*\big(e_j(s,x)\big)\,\mathrm{d}s \\ & + w_i(t,x)\Bigg\}\,\mathrm{d}t + \sum_{j=1}^n \sigma_{ij}\big(e_j(t,x), e_j\big(t-\tau_{ij}(t),x\big)\big)\,\mathrm{d}\omega_j(t), \end{aligned}$$

where

$$\begin{aligned} f_j^*\big(e_j(\cdot,x)\big) &= f_j\big(v_j(\cdot,x)\big) - f_j\big(u_j(\cdot,x)\big), \\ g_j^*\big(e_j(\cdot,x)\big) &= g_j\big(v_j(\cdot,x)\big) - g_j\big(u_j(\cdot,x)\big), \\ h_j^*\big(e_j(\cdot,x)\big) &= h_j\big(v_j(\cdot,x)\big) - h_j\big(u_j(\cdot,x)\big). \end{aligned}$$

In this paper, we give the following hypotheses:

(H1) We assume that there exist positive constants L_j, M_j and N_j such that the neuron activation functions f_j, g_j and h_j satisfy the following conditions:

$$\begin{aligned} \big|f_j(\hat{v}_j) - f_i(\check{v}_j)\big| &\leqslant L_j\big|\hat{v}_j - \check{v}_j\big|, \\ \big|g_j(\hat{v}_j) - g_i(\check{v}_j)\big| &\leqslant M_j\big|\hat{v}_j - \check{v}_j\big|, \\ \big|h_j(\hat{v}_j) - h_i(\check{v}_j)\big| &\leqslant N_j\big|\hat{v}_j - \check{v}_j\big|, \end{aligned}$$

where $\hat{v}_j, \check{v}_j \in \mathbb{R}$ $(j = 1,2,\dots,n)$.

(H2) Time-varying transmission delay $\tau_{ij}(t)$ satisfies $\dot{\tau}_{ij}(t) \leqslant \varrho < 1$ or $\dot{\tau}_{ij}(t) \geqslant \varrho > 1$ for all t, where ϱ is a constant.

(H3) There exists a positive constant η_{ij} $(i, j = 1, 2, \dots, n)$ such that

$$\left|\sigma_{ij}(\hat{v}_1, \check{v}_1) - \sigma_{ij}(\hat{v}_2, \check{v}_2)\right|^2 \leqslant \eta_{ij}\left(|\hat{v}_1 - \hat{v}_2|^2 + |\check{v}_1 - \check{v}_2|^2\right)$$

for any $\hat{v}_1, \hat{v}_2, \check{v}_1, \check{v}_2 \in \mathbb{R}$, and

$$\sigma_{ij}(0, 0) = 0, \quad i, j = 1, 2, \dots, n.$$

Before ending this section, we introduce some notations, the notion of exponential synchronization for reaction-diffusion neural networks (1) and (4) under periodically intermittent controller (5) based on p-norm, and some lemmas, which will come into play later on.

For any $u(t, x) = (u_1(t, x), \dots, u_n(t, x))^{\mathrm{T}} \in \mathbb{R}^n$, define

$$\left\|u(t, x)\right\|_p = \left(\int\limits_{\Omega} \sum_{i=1}^{n} \left|u_i(t, x)\right|^p \mathrm{d}x\right)^{1/p}.$$

Let $\mathcal{PC} = \mathcal{PC}([-\bar{\tau}, 0] \times \Omega, \mathbb{R}^n)$ denote the piecewise left continuous functions ϕ: $[-\bar{\tau}, 0] \times \Omega \to \mathbb{R}^n$ with the norm

$$\|\phi\|_p = \left(\int\limits_{\Omega} \sum_{i=1}^{n} \sup_{-\bar{\tau} \leqslant s \leqslant 0} \left|\phi_i(s, x)\right|^p \mathrm{d}x\right)^{1/p}.$$

Definition 1. The reaction-diffusion neural networks (1) and (4) can be exponentially synchronized under the intermittent controller (5) based on p-norm, if there exist constants $\mu > 0$ and $M \geqslant 1$ such that

$$\mathbf{E}\left\{\left\|v(t, x) - u(t, x)\right\|_p\right\} \leqslant M\mathbf{E}\left\{\|\psi - \phi\|_p\right\}\mathrm{e}^{-\mu t}$$

for $(t, x) \in [0, +\infty) \times \Omega$.

Lemma 1. (*See* [42].) *Let* $p \geqslant 2$ *be a positive integer,* m_k $(k = 1, 2, \dots, l^*)$ *be a positive constant,* X *be a cube* $|x_k| \leqslant m_k$*, and let* $h(x)$ *be a real-valued function belonging to* $\mathcal{C}^1(\Omega)$*, which vanish on the boundary* $\partial\Omega$ *of* Ω*, i.e.,* $h(x)|_{\partial\Omega} = 0$*. Then*

$$\int\limits_{\Omega} \left|h(x)\right|^p \mathrm{d}x \leqslant \frac{p^2 m_k^2}{4} \int\limits_{\Omega} \left|h(x)\right|^{p-2} \left|\frac{\partial h}{\partial x_k}\right|^2 \mathrm{d}x.$$

Lemma 2. (*See* [43].) *Assume that there exist two continuous functions* $f(x)$, $g(x)$: $[a, b] \to \mathbb{R}$*, constants* a*,* b*,* p *and* q *satisfying*

$$b > a, \qquad p, q > 1, \qquad \frac{1}{p} + \frac{1}{q} = 1,$$

then the following inequality holds:

$$\int\limits_{a}^{b} \left|f(x)g(x)\right| \mathrm{d}x \leqslant \left[\int\limits_{a}^{b} \left|f(x)\right|^p \mathrm{d}x\right]^{1/p} \left[\int\limits_{a}^{b} \left|g(x)\right|^q \mathrm{d}x\right]^{1/q}.$$

3 Exponential synchronization criterion

In this section, suitable T, δ and k_{ij} are designed to realize exponential synchronization between reaction-diffusion neural networks (1) and (4) under the periodically intermittent controller (5) in terms of p-norm. For convenience, the following denotations are introduced.

Let

$$\begin{aligned}
\lambda_i &= p\left[c_i - a_{ii}L_i - \frac{1}{2}(p-1)\eta_{ii}\right] + \sum_{k=1}^{l^*} \frac{4(p-1)D_{ik}}{pm_k^2} \\
&\quad - \sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{l=1}^{p-1} |a_{ij}|^{p\xi_{lij}} L_j^{p\zeta_{lij}} - \frac{p-1}{2}\sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{l=1}^{p-2} \eta_{ij}^{p\xi_{lij}} - \frac{p-1}{2}\sum_{j=1}^{n}\sum_{l=1}^{p-2} \eta_{ij}^{p\epsilon^*_{lij}} \\
&\quad - \sum_{j=1}^{n}\sum_{l=1}^{p-1} \left(|b_{ij}|^{p\xi^*_{lij}} M_j^{p\zeta^*_{lij}} + |d_{ij}|^{p\xi^{**}_{pij}} N_j^{p\zeta^{**}_{pij}}\right) \\
&\quad - \sum_{\substack{j=1\\ j\neq i}}^{n} |a_{ji}|^{p\xi_{pji}} L_i^{p\zeta_{pji}} - \frac{p-1}{2}\sum_{\substack{j=1\\ j\neq i}}^{n} \left(\eta_{ji}^{p\epsilon_{(p-1)ji}} + \eta_{ji}^{p\epsilon_{pji}}\right) - \tau^* \sum_{j=1}^{n} \rho^{**}_{ij},
\end{aligned}$$

$$\omega_i = pk_{ii} + \sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{l=1}^{p-1} |k_{ij}|^{p\varpi_{lij}} + \sum_{\substack{j=1\\ j\neq i}}^{n} |k_{ji}|^{p\varpi_{pji}},$$

$$\rho_{ij} = \frac{1}{\alpha|1-\varrho|}\left[|b_{ji}|^{p\xi^*_{pji}} M_i^{p\zeta^*_{pji}} + \frac{p-1}{2}\left(\eta_{ji}^{p\epsilon^*_{(p-1)ji}} + \eta_{ji}^{p\epsilon^*_{pji}}\right)\right],$$

$$\rho^*_{ij} = \rho_{ij} - \alpha\rho_{ij}\operatorname{sgn}(1-\varrho), \qquad \rho^{**}_{ij} = (\tau^*)^{p-1}|d_{ji}|^{p\xi^{**}_{pji}} N_i^{p\zeta^{**}_{pji}},$$

where $0 < \alpha < 1$; ξ_{lij}, ζ_{lij}, ξ^*_{lij}, ζ^*_{lij}, ξ^{**}_{lij}, ζ^{**}_{lij}, ϖ_{lij}, ϵ_{lij} and ϵ^*_{lij} are nonnegative real numbers and satisfy, respectively,

$$\begin{gathered}
\sum_{l=1}^{p}\xi_{lij} = 1, \quad \sum_{l=1}^{p}\zeta_{lij} = 1, \quad \sum_{l=1}^{p}\xi^*_{lij} = 1, \quad \sum_{l=1}^{p}\zeta^*_{lij} = 1, \quad \sum_{l=1}^{p}\xi^{**}_{lij} = 1, \\
\sum_{l=1}^{p}\zeta^{**}_{lij} = 1, \quad \sum_{l=1}^{p}\varpi_{lij} = 1, \quad \sum_{l=1}^{p}\epsilon_{lij} = 1 \quad \text{and} \quad \sum_{l=1}^{p}\epsilon^*_{lij} = 1.
\end{gathered}$$

In the following, we will give an assumption

(H4) $\lambda_i - \omega_i - \sum_{j=1}^{n}\rho_{ij} > 0$ for $i = 1, 2, \ldots, n$.

Consider the function

$$F_i(\varepsilon_i) = \lambda_i - \omega_i - \varepsilon_i - \sum_{j=1}^{n}\rho_{ij}e^{\varepsilon_i\tau},$$

where $\varepsilon_i \geqslant 0$. It is easy to see that

$$F_i'(\varepsilon_i) = -1 - \sum_{j=1}^{n} \tau\rho_{ij}\mathrm{e}^{\varepsilon_i\tau} < 0, \qquad F_i(0) = \lambda_i - \omega_i - \sum_{j=1}^{n}\rho_{ij} > 0.$$

On the other hand, $F_i(\varepsilon_i)$ is continuous on $[0, +\infty)$ and $F_i(\varepsilon_i) \to -\infty$ as $\varepsilon_i \to +\infty$, then exists a positive number $\bar{\varepsilon}_i$ such that $F_i(\bar{\varepsilon}_i) \geqslant 0$ and $F_i(\varepsilon_i) > 0$ for $\varepsilon_i \in (0, \bar{\varepsilon}_i)$. Denoting $\varepsilon = \min_{i=1,\dots,n}\{\bar{\varepsilon}_i\}$, then

$$F_i(\varepsilon) = \lambda_i - \omega_i - \varepsilon - \sum_{j=1}^{n}\rho_{ij}\mathrm{e}^{\varepsilon\tau} \geqslant 0.$$

It follows from the assumption (H_4) that there exists a positive number θ_i such that

$$\lambda_i + \theta_i - \sum_{j=1}^{n}\rho_{ij} > 0$$

for all $i = 1, 2, \dots, n$. In a similar way, we have

$$G_i(\varepsilon) = \lambda_i + \theta_i - \varepsilon - \sum_{j=1}^{n}\rho_{ij}\mathrm{e}^{\varepsilon\tau} \geqslant 0, \tag{8}$$

and $G_i(\cdot)$ is decreasing.

Theorem 1. *Under assumptions* (H1)–(H4)*, the noise-perturbed response system* (4) *and the drive system* (1) *can be exponentially synchronized under the periodically intermittent controller* (5) *based on p-norm with the exponential decay rate $(\varepsilon T - (T - \delta)\theta)/pT$, if the following condition is also satisfied*:

(H5) $\varepsilon - (T - \delta)\theta/T > 0$*, where* $\theta = \max_{i=1,\dots,n}\{\theta_i\}$.

We put the proof of Theorem in Appendix.

Remark 1. Ma et al. [37] investigated the synchronization problem for a class of stochastic reaction-diffusion neural networks with time-varying delays and Dirichlet boundary conditions in terms of 2-norm by using linear feedback control under the precondition that the derivative of the time-varying delay was smaller than one. Zhao and Deng studied the exponential synchronization of reaction-diffusion neural networks with continuously distributed delays and stochastic influence in terms of 2-norm based on adaptive control in [44]. In [40], by using the Lyapunov functional method, many real parameters and inequality techniques, the global exponential synchronization for a class of delayed reaction-diffusion cellular neural networks with Dirichlet boundary conditions in terms of $2k$-norm (integer $k > 0$) was discussed. In contrast, our results are derived considering the model with both discrete and distributed time-varying delays based on p-norm ($p \geqslant 2$), which have more general application ranges. In this paper, this problem is concerned and some central criteria are derived by designing periodically intermittent controller. Our results are more general and they effectually complement or improve the previously known results in the literature where only $p = 2$ or $2k$ were considered.

Remark 2. As pointed out in [21], there are few results concerning the robust stability and robust synchronization schemes for complex networks, in particular stochastic complex networks and reaction-diffusion complex networks based on p-norm and ∞-norm using intermittent control. This motivates us to write this paper. It is the first time to establish the exponential synchronization criterion for neural networks with mixed time-varying delays, stochastic noise perturbation and reaction-diffusion effects in terms of p-norm. In this paper, the periodically intermittent control is generalized to study a more reasonable neural network model and the traditional restrictions in [15–17] that $\delta > \tau$ and $T - \delta > \tau$ are removed.

Remark 3. In Theorem 3, a novel Lyapunov–Krasovskii functional $V(t,x)$ is employed to deal with the reaction-diffusion neural networks with mixed time-varying delays and stochastic perturbation. In the novel Lyapunov–Krasovskii functional (A.1), the integral term $\int_{t-\tau}^{t} V_i(s,x)\,\mathrm{d}s$ is divided into two parts as $\rho_{ij}\int_{t-\tau_{ij}(t)}^{t} V_i(s,x)\,\mathrm{d}s$ and $\rho_{ij}^{*}\int_{t-\tau}^{t-\tau_{ij}(t)} V_i(s,x)\,\mathrm{d}s$, where ρ_{ij}^{*} is chosen as $\rho_{ij} - \alpha\rho_{ij}\operatorname{sgn}(1-\varrho)$ with $0 < \alpha < 1$, such a newly introduced variable may lead to potentially less conservative results on the upper bound of the time derivative of time-varying delay.

Remark 4. The results in this paper show that, the exponential synchronization criteria on reaction-diffusion neural networks are dependent of time-varying delays, diffusion effects and stochastic noise fluctuations. Furthermore, we can see a very interesting fact, that is, as long as diffusion coefficients D_{ik} in the system is large enough, then the assumption (H4) always can satisfy. This shows that under the boundary conditions (2) and (6), a large enough diffusion always may make the reaction-diffusion neural networks (1) and (4) globally exponentially synchronous under the intermittent controller (5) with condition (H5).

4 Numerical examples

In this section, by using the classical implicit format and the method of steps for differential difference equations, we give some examples with numerical simulations to illustrate the effectiveness of the theoretical results obtained above.

Example 1. For the sake of simplification, we consider a reaction-diffusion neural network model described by

$$\begin{aligned}\frac{\mathrm{d}u_i(t,x)}{\mathrm{d}t} &= D_i\frac{\partial^2 u_i(t,x)}{\partial x^2} - c_i u_i(t,x) + \sum_{j=1}^{2} a_{ij} f_j\bigl(u_j(t,x)\bigr)\\ &\quad + \sum_{j=1}^{2} b_{ij} g_j\bigl(u_j(t-\tau(t),x)\bigr) + \sum_{j=1}^{2} d_{ij}\int\limits_{t-\tau^*(t)}^{t} h_j\bigl(u_j(s,x)\bigr)\,\mathrm{d}s, \qquad (9)\end{aligned}$$

where $i = 1,2$, $f_i(u_i) = 0.5(|u_i+1| - |u_i-1|)$ and $g_i(u_i) = h_i(u_i) = \tanh(u_i)$. Clearly, it can be seen that the hypothesis (H1) is satisfied with $L_i = M_i = N_i = 1$

$(i = 1, 2)$. The parameters of (9) are assumed that $c_1 = c_2 = 1$, $a_{11} = 0.2$, $a_{12} = -1$, $a_{21} = -0.3$, $a_{22} = 1$, $b_{11} = -1$, $b_{12} = -1.5$, $b_{21} = -0.2$, $b_{22} = -2$, $d_{11} = 0.8$, $d_{12} = 0.1$, $d_{21} = -0.5$, $d_{22} = -1$, $D_1 = 0.1$, $D_2 = 0.2$, $\tau(t) = 0.7 + 0.1\sin(t)$ and $\tau^*(t) = 1.3 + 0.6\cos(t)$. The initial condition of drive system (9) is chosen as

$$\begin{aligned} u_1(s,x) &= 0.5\left(1 + \frac{s-\tau(s)}{\pi}\right)\sin\left(\frac{x}{\pi}\right), \\ u_2(s,x) &= 0.3\left(1 + \frac{s-\tau(s)}{\pi}\right)\sin\left(\frac{x}{\pi}\right), \end{aligned} \tag{10}$$

where $(s, x) \in [-1.9, 0] \times \Omega$. The reaction-diffusion neural network (9) with boundary condition (2) and initial condition (10) exhibits a chaotic behavior as shown in Fig. 1.

The noise-perturbed response system is described by

$$\begin{aligned} \mathrm{d}v_i(t,x) = \Bigg\{ & D_i \frac{\partial^2 v_i(t,x)}{\partial x^2} - c_i v_i(t,x) + \sum_{j=1}^{2} a_{ij} f_j\big(v_j(t,x)\big) \\ & + \sum_{j=1}^{2} b_{ij} g_j\big(v_j\big(t-\tau(t),x\big)\big) + \sum_{j=1}^{2} d_{ij} \int\limits_{t-\tau^*(t)}^{t} h_j\big(v_j(s,x)\big)\,\mathrm{d}s \\ & + w_i(t,x) \Bigg\} \mathrm{d}t + \sum_{j=1}^{2} \sigma_{ij}\big(e_j(t,x), e_j\big(t-\tau(t),x\big)\big)\,\mathrm{d}\omega_j(t), \end{aligned} \tag{11}$$

where

$$\begin{aligned} &\sigma_{11} = 0.1e_1(t,x) + 0.2e_1\big(t-\tau(t),x\big), \quad \sigma_{12} = 0, \\ &\sigma_{21} = 0, \quad \sigma_{22} = 0.2e_2(t,x) + 0.3e_2\big(t-\tau(t),x\big). \end{aligned}$$

Meanwhile, we set the initial condition for response system (11) as follows:

$$\begin{aligned} v_1(s,x) &= 0.1\left(1 + \frac{s-\tau(s)}{\pi}\right)\sin\left(\frac{x}{\pi}\right), \\ v_2(s,x) &= 0.6\left(1 + \frac{s-\tau(s)}{\pi}\right)\sin\left(\frac{x}{\pi}\right) \end{aligned}$$

for $(s, x) \in [-1.9, 0] \times \Omega$.

By simple computation, we obtain that $\dot{\tau}(t) \leqslant \varrho = 0.1 < 1$, $m_1 = 5$, $\tau = 0.8$, $\tau^* = 1.9$, $\bar{\tau} = 1.9$, $\eta_{11} = 0.04$, $\eta_{12} = \eta_{21} = 0$ and $\eta_{22} = 0.09$. In addition, for convenience, we only consider the case $p = 2$. Choosing $k_{11} = -7$, $k_{12} = 0$, $k_{21} = 0$, $k_{22} = -12$, $\alpha = 0.95$ and $\xi_{lij} = \zeta_{lij} = \xi^*_{lij} = \zeta^*_{lij} = \xi^{**}_{lij} = \zeta^{**}_{lij} = \varpi_{lij} = \epsilon_{lij} = \epsilon^*_{lij} = 1/2$ for $l, i, j = 1, 2$, then

$$\begin{aligned} &\lambda_1 = -7.865, \quad \lambda_2 = -9.135, \quad \omega_1 = -14, \quad \omega_2 = -24, \\ &\rho_{11} = 1.2163, \quad \rho_{12} = 0.2339, \quad \rho_{21} = 1.7543, \quad \rho_{22} = 2.4444, \\ &\rho^*_{11} = 0.0608, \quad \rho^*_{12} = 0.0117, \quad \rho^*_{21} = 0.0887, \quad \rho^*_{22} = 0.1222. \end{aligned}$$

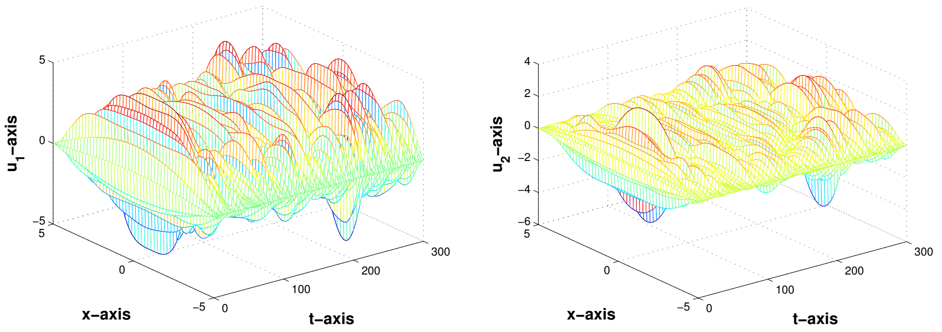

Fig. 1. Chaotic attractor of neural network model (9).

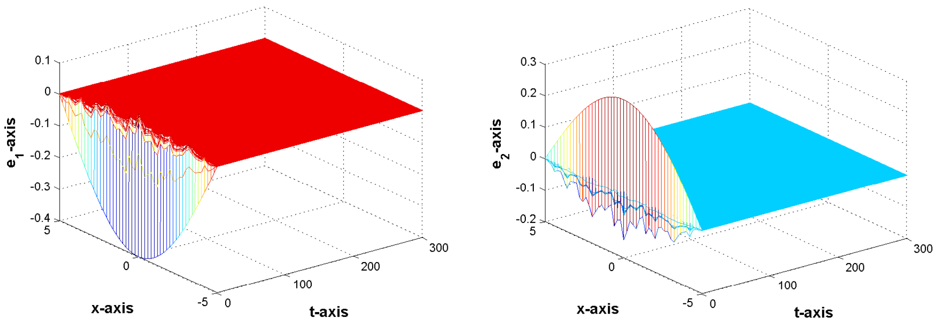

Fig. 2. Synchronization errors $e_1(t,x)$ and $e_2(t,x)$ between systems (9) and (11).

Hence, $\bar{\varepsilon}_1 = 1.4785$, $\bar{\varepsilon}_2 = 1.4518$ and $\theta \geqslant 23.9997$ by computation. Therefore, $\varepsilon = 1.4518$ and $\delta > 18.7902$ when $T = 20$ and $\theta = 24$ are taken in virtue of assumption (H_5). Select $\delta = 19$, then (H5) is also satisfied. According to Theorem 3, which implies that systems (9) and (11) are exponential synchronized in terms of p-norm as shown in Fig. 2.

Example 2. Consider the neural networks with four dynamical nodes:

$$\begin{aligned}\frac{\mathrm{d}u_i(t,x)}{\mathrm{d}t} &= D_i\frac{\partial^2 u_i(t,x)}{\partial x^2} - c_i u_i(t,x) + \sum_{j=1}^{4} a_{ij} f_j\big(u_j(t,x)\big) \\ &\quad + \sum_{j=1}^{4} b_{ij} g_j\big(u_j\big(t-\tau(t),x\big)\big) + \sum_{j=1}^{4} d_{ij} \int\limits_{t-\tau^*}^{t} h_j\big(u_j(s,x)\big)\,\mathrm{d}s + J_i, \end{aligned} \tag{12}$$

where $i = 1,2,3,4$, $f_i(u_i) = 0.5(|u_i+1| - |u_i-1|)$, $g_i(u_i) = \arctan(u_i)$ and $h_i(u_i) = \tanh(u_i)$. Similarly, we can derive that the hypothesis (H1) is satisfied with $L_i = M_i = N_i = 1$ ($i = 1,2,3,4$). The parameters of (12) are assumed that $c_i = D_i = 1$, $J_i = 0.1$ ($i = 1,2,3,4$), $a_{11} = -1$, $a_{12} = 1$, $a_{13} = 0.5$, $a_{14} = -0.5$, $a_{21} = -0.3$, $a_{22} = -1$, $a_{23} = -1$, $a_{24} = -1$, $a_{31} = 0.6$, $a_{32} = 0.1$, $a_{33} = -1$, $a_{34} = -1$, $a_{41} = -0.2$,

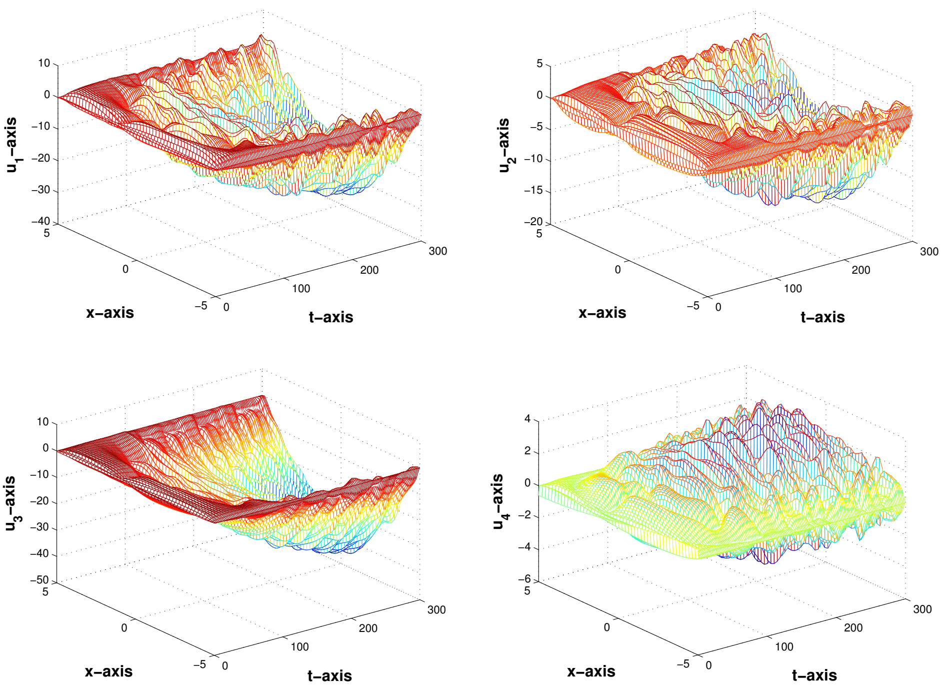

Fig. 3. Chaotic attractor of neural network model (12).

$a_{42} = 0.1$, $a_{43} = -0.1$, $a_{44} = -0.5$, $b_{11} = 1$, $b_{12} = -0.1$, $b_{13} = -1$, $b_{14} = -0.1$, $b_{21} = -0.2$, $b_{22} = -1$, $b_{23} = -0.5$, $b_{24} = 0.1$, $b_{31} = 0.1$, $b_{32} = -0.1$, $b_{33} = 0.3$, $b_{34} = -0.4$, $b_{41} = -0.3$, $b_{42} = 0.1$, $b_{43} = -0.3$, $b_{44} = -0.5$, $d_{11} = 1.2$, $d_{12} = 0.1$, $d_{13} = 0.5$, $d_{14} = -0.4$,$d_{21} = -0.5$, $d_{22} = 0.5$, $d_{23} = -0.5$, $d_{24} = -0.1$, $d_{31} = -0.1$, $d_{32} = 0.6$, $d_{33} = 0.3$, $d_{34} = -0.4$, $d_{41} = -0.1$, $d_{42} = 0.1$, $d_{43} = 0.2$, $d_{44} = -0.3$, $\tau(t) = 1.1t + 0.1$ and $\tau^* = 0.5$. The initial condition of system (12) is chosen as

$$\begin{aligned} u_1(s,x) &= 0.5\left(1 + \frac{s-\tau(s)}{\pi}\right)\sin\left(\frac{x}{\pi}\right),\\ u_2(s,x) &= 0.3\left(1 + \frac{s-\tau(s)}{\pi}\right)\sin\left(\frac{x}{\pi}\right),\\ u_3(s,x) &= 0.2\left(1 + \frac{s-\tau(s)}{\pi}\right)\cos\left(\frac{x}{\pi}\right),\\ u_4(s,x) &= 0.6\left(1 + \frac{s-\tau(s)}{\pi}\right)\cos\left(\frac{x}{\pi}\right), \end{aligned} \tag{13}$$

where $(s,x) \in (-\infty, 0] \times \Omega$. The reaction-diffusion neural network (12) with boundary condition (2) and initial condition (13) exhibits a chaotic behavior as shown in Fig. 3.

The corresponding response system can be given as

$$\begin{aligned}
\mathrm{d}v_i(t,x) = \Bigg\{ & D_i \frac{\partial^2 v_i(t,x)}{\partial x^2} - c_i v_i(t,x) + \sum_{j=1}^{4} a_{ij} f_j\big(v_j(t,x)\big) \\
& + \sum_{j=1}^{4} b_{ij} g_j\big(v_j\big(t-\tau(t),x\big)\big) + \sum_{j=1}^{4} d_{ij} \int_{t-\tau}^{t} h_j\big(v_j(s,x)\big)\,\mathrm{d}s \\
& + J_i + w_i(t,x) \Bigg\}\,\mathrm{d}t + \sum_{j=1}^{4} \sigma_{ij}\big(e_j(t,x), e_j\big(t-\tau(t),x\big)\big)\,\mathrm{d}\omega_j(t), \qquad (14)
\end{aligned}$$

where

$$\sigma_{ii} = 0.1e_i(t,x) + 0.2e_i\big(t-\tau(t),x\big), \quad \sigma_{ij} = 0, \quad i \neq j, \quad i,j = 1,2,3,4.$$

Meanwhile, the initial condition for response system (14) is given by:

$$\begin{aligned}
v_1(s,x) &= 0.1\left(1 + \frac{s-\tau(s)}{\pi}\right)\sin\left(\frac{x}{\pi}\right), \\
v_2(s,x) &= 0.6\left(1 + \frac{s-\tau(s)}{\pi}\right)\sin\left(\frac{x}{\pi}\right), \\
v_3(s,x) &= 0.8\left(1 + \frac{s-\tau(s)}{\pi}\right)\cos\left(\frac{x}{\pi}\right), \\
v_4(s,x) &= 0.2\left(1 + \frac{s-\tau(s)}{\pi}\right)\cos\left(\frac{x}{\pi}\right),
\end{aligned}$$

for $(s,x) \in (-\infty, 0] \times \Omega$.

In this case, $\dot{\tau}(t) \geqslant \varrho = 1.1 > 1$. Choosing $k_{11} = -5$, $k_{22} = -8$, $k_{33} = -10$, $k_{44} = -12$, $k_{ij} = 0$ ($i \neq j$, $i,j = 1,2,3,4$), $T = 30$ and $\delta = 29$, similar to Example 1, it follows from Fig. 4 that systems (12) and (14) are exponential synchronized in terms of p-norm.

Remark 5. In [45], the authors studied the globally exponential synchronization for a class of reaction-diffusion neural networks with discrete variable delays and finite distributed constant delays based on periodically intermittent control under Dirichlet boundary conditions. Theorem 1 in [45] cannot be used to study this example with $\dot{\tau}(t) > 1$ for all t (fast-varying delay). However, after a simple computation, the conditions of Theorem 3 hold. The numerical simulations clearly verify the effectiveness of the developed periodically intermittent controller to the exponential synchronization of reaction-diffusion neural networks with mixed time-varying delays and stochastic perturbation based on p-norm.

Remark 6. Note that the activation functions in [38, 46–50] are required to satisfy the condition

$$0 \leqslant \frac{f_j(\hat{v}_j) - f_i(\check{v}_j)}{\hat{v}_j - \check{v}_j} \leqslant L_j,$$

for any $\hat{v}_j, \check{v}_j \in \mathbb{R}$ ($j = 1,2,\dots,n$).

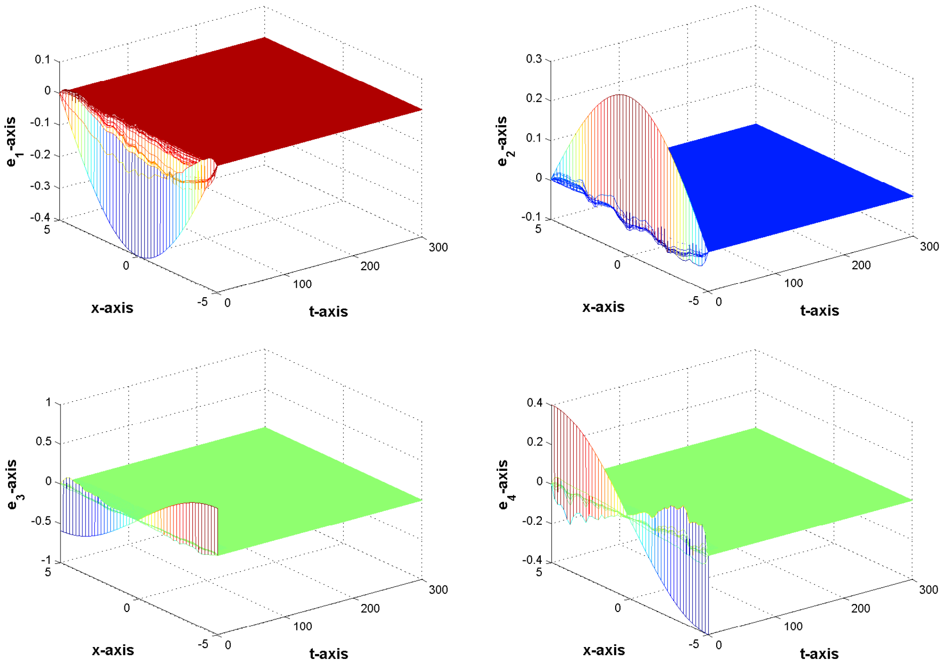

Fig. 4. Synchronization errors $e_1(t,x)$, $e_2(t,x)$, $e_3(t,x)$ and $e_4(t,x)$ between systems (12) and (14).

Obviously, this condition is stranger than the Lipstchizian condition in (H1). Hence, the results in [38, 46–50] are unavailable for this example.

5 Conclusion

In this paper, a periodically intermittent controller has been proposed to ensure the exponential synchronization for a class of reaction-diffusion neural networks with mixed time-varying delays and stochastic noise perturbation under Dirichlet boundary conditions in terms of p-norm. The problem considered in this paper is more general in many aspects and incorporates as special cases various problems, which have been studied extensively in the literature. Some remarks and numerical examples have been used to demonstrate the effectiveness of the obtained results.

It should be pointed out that there are many published papers focusing on the synchronization problems of chaotic neural networks, but mixed time-varying delays, stochastic perturbation and reaction-diffusion effects have never been taken into consideration in terms of the synchronization issue based on p-norm for a variety of neural networks. To the best knowledge of the authors, this is the first paper incorporating mixed time-varying delays, stochastic perturbation and reaction-diffusion effects into the problem of exponential synchronization for chaotic neural networks under periodically intermittent control in terms of p-norm.

From condition (H4) in Theorem 3, we see that as long as feedback strength parameter k_{ii} $(i = 1, 2, \dots, n)$ is chosen small enough, then condition (H4) always holds. Therefore, we can obtain that there always exists an appropriate periodically intermittent control input strategy in response system (4) at all time such that drive-response systems (1) and (4) with boundary conditions (2) and (6) and initial conditions (3) and (7) are global exponential synchronization. However, the decomposing way used in (A.5)–(A.10) and the inequality technique used in (A.11) maybe increase the conservatism of the criteria on the upper bound of feedback strength k_{ii}. So, this is a problem that we should study in the further.

In fact, due to the different parameters, activation functions and neural network architectures, which is unavoidable in real implementation, the master system and response system are not identical and the resulting synchronization is not exact and complex. Therefore, it is important and challenging to study the synchronization problems of non-identical chaotic neural networks. Furthermore, our analysis is carried out under the assumption $p \geqslant 2$ throughout this paper. Evidently, there is an interesting open problem concerning the exponential synchronization for non-identical reaction-diffusion neural networks with mixed time-varying delays and stochastic noise disturbance by using periodically intermittent control for $p = 1$ or based on ∞-norm. This will become our future investigative direction.

Appendix. Proof of Theorem 3

Define the Lyapunov–Krasovskii functional $V(t,x) \in \mathcal{C}^{2,1}(\mathbb{R}^+ \times \mathbb{R}^n; \mathbb{R}^+)$ as

$$
\begin{aligned}
V(t,x) = \int_{\Omega} \sum_{i=1}^{n} \Bigg[& V_i(t,x) + \mathrm{e}^{\varepsilon\tau} \sum_{j=1}^{n} \rho_{ij} \int_{t-\tau_{ij}(t)}^{t} V_i(s,x)\,\mathrm{d}s \\
& + \mathrm{e}^{\varepsilon\tau} \sum_{j=1}^{n} \rho_{ij}^{*} \int_{t-\tau}^{t-\tau_{ij}(t)} V_i(s,x)\,\mathrm{d}s + \sum_{j=1}^{n} \rho_{ij}^{**} \int_{-\tau_{ij}^{*}(t)}^{0} \int_{t+s}^{t} V_i(\eta,x)\,\mathrm{d}\eta\,\mathrm{d}s \Bigg] \mathrm{d}x, \quad \text{(A.1)}
\end{aligned}
$$

where

$$
V_i(t,x) = V_i\big(t, e(t,x)\big) = \mathrm{e}^{\varepsilon t} \big| e_i(t,x) \big|^p, \quad i = 1, 2, \dots, n.
$$

By the Itô's differential formula, we have the following stochastic differential:

$$
\mathrm{d}V\big(t, e(t,x)\big) = \mathcal{L}V\big(t, e(t,x)\big)\mathrm{d}t + V_e\big(t, e(t,x)\big)\sigma(t)\,\mathrm{d}\omega(t), \qquad \text{(A.2)}
$$

where

$$
\begin{aligned}
\mathcal{L}V\big(t, e(t,x)\big) &= V_t\big(t, e(t,x)\big) + V_e\big(t, e(t,x)\big)\varPhi \\
&\quad + \frac{1}{2}\mathrm{trace}\big[\sigma^{\mathrm{T}}(t) V_{ee}\big(t, e(t,x)\big)\sigma(t)\big], \\
V_t\big(t, e(t,x)\big) &= \frac{\partial V(t, e(t,x))}{\partial t},
\end{aligned}
$$

$$V_e\big(t, e(t,x)\big) = \left(\frac{\partial V(t,e(t,x))}{\partial e_1}, \dots, \frac{\partial V(t,e(t,x))}{\partial e_n}\right),$$
$$V_{ee}(t, e(t,x)) = \left(\frac{\partial^2 V(t,e(t,x))}{\partial e_i \partial e_j}\right)_{n\times n},$$

$$\begin{aligned}
\Phi &= (\Phi_1, \dots, \Phi_n),\\
\Phi_i &= -c_i e_i(t,x) + \sum_{j=1}^n a_{ij} f_j^*\big(e_j(t,x)\big) + \sum_{j=1}^n b_{ij} g_j^*\big(e_j\big(t-\tau_{ij}(t), x\big)\big)\\
&\quad + \sum_{j=1}^n d_{ij} \int\limits_{t-\tau_{ij}^*(t)}^{t} h_j^*\big(e_j(s,x)\big)\,\mathrm{d}s + w_i(t,x).
\end{aligned}$$

It follows from (A.2) the Dini derivation, it can be deduced that

$$\begin{aligned}
&D^+\mathbf{E}\big\{V(t,x)\big\}\\
&\quad= \int\limits_{\Omega} \sum_{i=1}^n \Bigg\{\varepsilon V_i(t,x) + p\mathrm{e}^{\varepsilon t}\big|e_i(t,x)\big|^{p-1}\\
&\qquad\times \Bigg[-c_i\big|e_i(t,x)\big| + \sum_{j=1}^n a_{ij} f_j^*\big(\big|e_j(t,x)\big|\big) + \sum_{j=1}^n b_{ij} g_j^*\big(\big|e_j\big(t-\tau_{ij}(t), x\big)\big|\big)\\
&\qquad+ \sum_{j=1}^n d_{ij} \int\limits_{t-\tau_{ij}^*(t)}^{t} h_j^*\big(\big|e_j(s,x)\big|\big)\,\mathrm{d}s + \sum_{j=1}^n k_{ij}\big|e_j(t,x)\big|\Bigg]\\
&\qquad+ \mathrm{e}^{\varepsilon\tau} \sum_{j=1}^n \rho_{ij}\big[V_i(t,x) - \big(1-\dot{\tau}_{ij}(t)\big)V_i\big(t-\tau_{ij}(t), x\big)\big]\\
&\qquad+ \mathrm{e}^{\varepsilon\tau} \sum_{j=1}^n \rho_{ij}^*\big[\big(1-\dot{\tau}_{ij}(t)\big)V_i\big(t-\tau_{ij}(t), x\big) - V_i(t-\tau, x)\big]\\
&\qquad+ \sum_{j=1}^n \rho_{ij}^{**}\Bigg[\tau_{ij}^*(t)V_i(t,x) - \int\limits_{t-\tau_{ij}^*(t)}^{t} V_i(s,x)\,\mathrm{d}s\Bigg]\\
&\qquad+ \frac{p(p-1)}{2}\mathrm{e}^{\varepsilon t}\big|e_i(t,x)\big|^{p-2} \sum_{j=1}^n \rho_{ij}^2\big(\big|e_j(t,x)\big|, \big|e_j\big(t-\tau_{ij}(t), x\big)\big|\big)\Bigg\}\,\mathrm{d}x\\
&\qquad+ \int\limits_{\Omega} \sum_{i=1}^n pe^{\varepsilon t}\big|e_i(t,x)\big|^{p-1} \sum_{k=1}^{l^*} \frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial|e_i(t,x)|}{\partial x_k}\right)\mathrm{d}x
\end{aligned} \tag{A.3}$$

for $(t,x) \in [mT, mT+\delta) \times \Omega$.

From the boundary conditions (2), (6) and Lemma 1, we can obtain [42]

$$p\int\limits_{\Omega}\left|e_i(t,x)\right|^{p-1}\sum_{k=1}^{l^*}\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial|e_i(t,x)|}{\partial x_k}\right)\mathrm{d}x \leqslant -\sum_{k=1}^{l^*}\frac{4(p-1)D_{ik}}{pm_k^2}\int\limits_{\Omega}\left|e_i(t,x)\right|^{p}\mathrm{d}x. \tag{A.4}$$

Furthermore, it follows from (H1) and the fact

$$a_1^p + a_2^p + \cdots + a_p^p \geqslant p a_1 a_2\cdots a_p, \quad a_i \geqslant 0,\ i=1,2,\dots,p,$$

that

$$\begin{aligned}
&p\left|e_i(t,x)\right|^{p-1}\sum_{\substack{j=1,\\ j\neq i}}^{n} a_{ij}f_j^*\left(\left|e_j(t,x)\right|\right)\\
&\quad\leqslant p\left|e_i(t,x)\right|^{p-1}\sum_{\substack{j=1\\ j\neq i}}^{n}|a_{ij}|L_j\left|e_j(t,x)\right|\\
&\quad=\sum_{\substack{j=1\\ j\neq i}}^{n} p\left[\prod_{l=1}^{p-1}\left(|a_{ij}|^{\xi_{lij}}L_j^{\zeta_{lij}}\left|e_i(t,x)\right|\right)\right]\left(|a_{ij}|^{\xi_{pij}}L_j^{\zeta_{pij}}\left|e_j(t,x)\right|\right)\\
&\quad\leqslant\sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{l=1}^{p-1}|a_{ij}|^{p\xi_{lij}}L_j^{p\zeta_{lij}}\left|e_i(t,x)\right|^p+\sum_{\substack{j=1\\ j\neq i}}^{n}|a_{ij}|^{p\xi_{pij}}L_j^{p\zeta_{pij}}\left|e_j(t,x)\right|^p.
\end{aligned}\tag{A.5}$$

Similarly, we have

$$\begin{aligned}
&p\left|e_i(t,x)\right|^{p-1}\sum_{j=1}^{n} b_{ij}g_j^*\left(\left|e_j\left(t-\tau_{ij}(t),x\right)\right|\right)\\
&\quad\leqslant\sum_{j=1}^{n}\sum_{l=1}^{p-1}|b_{ij}|^{p\xi_{lij}^*}M_j^{p\zeta_{lij}^*}\left|e_i(t,x)\right|^p+\sum_{j=1}^{n}|b_{ij}|^{p\xi_{pij}^*}M_j^{p\zeta_{pij}^*}\left|e_j\left(t-\tau_{ij}(t),x\right)\right|^p,
\end{aligned}\tag{A.6}$$

$$\begin{aligned}
&p\left|e_i(t,x)\right|^{p-1}\sum_{j=1}^{n} d_{ij}\int\limits_{t-\tau_{ij}^*(t)}^{t} h_j^*\left(\left|e_j(s,x)\right|\right)\mathrm{d}s\\
&\quad\leqslant\sum_{j=1}^{n}\sum_{l=1}^{p-1}|d_{ij}|^{p\xi_{lij}^{**}}N_j^{p\zeta_{lij}^{**}}\left|e_i(t,x)\right|^p\\
&\qquad+\sum_{j=1}^{n}|d_{ij}|^{p\xi_{pij}^{**}}N_j^{p\zeta_{pij}^{**}}\left[\int\limits_{t-\tau_{ij}^*(t)}^{t}\left|e_j(s,x)\right|\mathrm{d}s\right]^p,
\end{aligned}\tag{A.7}$$

$$
\begin{aligned}
&p\big|e_i(t,x)\big|^{p-1}\sum_{\substack{j=1\\ j\neq i}}^{n}k_{ij}\big|e_j(t,x)\big| \\
&\quad\leqslant \sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{l=1}^{p-1}|k_{ij}|^{p\varpi_{lij}}\big|e_i(t,x)\big|^{p}+\sum_{\substack{j=1\\ j\neq i}}^{n}|k_{ij}|^{p\varpi_{pij}}\big|e_j(t,x)\big|^{p},
\end{aligned}
\tag{A.8}
$$

$$
\begin{aligned}
&p\big|e_i(t,x)\big|^{p-2}\sum_{\substack{j=1\\ j\neq i}}^{n}\eta_{ij}\big|e_j(t,x)\big|^{2} \\
&\quad\leqslant \sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{l=1}^{p-2}\eta_{ij}^{p\epsilon_{lij}}\big|e_i(t,x)\big|^{p}+\sum_{\substack{j=1\\ j\neq i}}^{n}\big(\eta_{ij}^{p\epsilon_{(p-1)ij}}+\eta_{ij}^{p\epsilon_{pij}}\big)\big|e_j(t,x)\big|^{p}
\end{aligned}
\tag{A.9}
$$

and

$$
\begin{aligned}
&p\big|e_i(t,x)\big|^{p-2}\sum_{j=1}^{n}\eta_{ij}\big|e_j\big(t-\tau_{ij}(t),x\big)\big|^{2} \\
&\quad\leqslant \sum_{j=1}^{n}\sum_{l=1}^{p-2}\eta_{ij}^{p\epsilon^{*}_{lij}}\big|e_i(t,x)\big|^{p}+\sum_{j=1}^{n}\big(\eta_{ij}^{p\epsilon^{*}_{(p-1)ij}}+\eta_{ij}^{p\epsilon^{*}_{pij}}\big)\big|e_j\big(t-\tau_{ij}(t),x\big)\big|^{p}.
\end{aligned}
\tag{A.10}
$$

By applying (A.4)–(A.10), assumptions (H2)–(H3) and Lemmas 1–2 to (A.3), we have

$$
\begin{aligned}
&D^{+}\mathbf{E}\big\{V(t,x)\big\} \\
&\quad\leqslant \mathbf{E}\Bigg\{\int\limits_{\Omega}\sum_{i=1}^{n}\Bigg\{\Bigg[\varepsilon-p\bigg(c_i-a_{ii}L_i-k_{ii}-\frac{1}{2}(p-1)\eta_{ii}\bigg) \\
&\qquad-\sum_{k=1}^{l^{*}}\frac{4(p-1)D_{ik}}{pm_k^{2}}+\sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{l=1}^{p-1}\big(|a_{ij}|^{p\xi_{lij}}L_j^{p\zeta_{lij}}+|k_{ij}|^{p\varpi_{lij}}\big) \\
&\qquad+\sum_{j=1}^{n}\sum_{l=1}^{p-1}\big(|b_{ij}|^{p\xi^{*}_{lij}}M_j^{p\zeta^{*}_{lij}}+|d_{ij}|^{p\xi^{**}_{pij}}N_j^{p\epsilon^{**}_{pij}}\big) \\
&\qquad+\frac{p-1}{2}\sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{l=1}^{p-2}\eta_{ij}^{p\xi_{lij}}+\frac{p-1}{2}\sum_{j=1}^{n}\sum_{l=1}^{p-2}\big(\eta_{ij}^{p\epsilon^{*}_{lij}}+\eta_{ij}^{p\epsilon^{**}_{lij}}\big)\Bigg]V_i(t,x) \\
&\qquad+\Bigg[\sum_{\substack{j=1\\ j\neq i}}^{n}\big(|a_{ij}|^{p\xi_{pij}}L_j^{p\zeta_{pij}}+|k_{ij}|^{p\varpi_{pij}}\big)+\frac{p-1}{2}\sum_{\substack{j=1\\ j\neq i}}^{n}\big(\eta_{ij}^{p\epsilon_{(p-1)ij}}+\eta_{ij}^{p\epsilon_{pij}}\big)\Bigg]V_j(t,x)
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{j=1}^{n}\rho_{ij}^{**}\Bigg[\tau^{*}V_i(t,x)-\int\limits_{t-\tau_{ij}^{*}(t)}^{t}V_i(s,x)\,\mathrm{d}s\Bigg]\\
&+\sum_{j=1}^{n}|d_{ij}|^{p\xi_{pij}^{**}}N_j^{p\zeta_{pij}^{**}}\Bigg[\int\limits_{t-\tau_{ij}^{*}(t)}^{t}\big|e_j(s,x)\big|\,\mathrm{d}s\Bigg]^{p}\\
&+\sum_{j=1}^{n}\bigg[|b_{ij}|^{p\xi_{pij}^{*}}M_j^{p\zeta_{pij}^{*}}+\frac{p-1}{2}\big(\eta_{ij}^{p\epsilon_{(p-1)ij}^{*}}+\eta_{ij}^{p\epsilon_{pij}^{*}}\big)\bigg]\mathrm{e}^{\varepsilon\tau}V_j\big(t-\tau_{ij}(t),x\big)\\
&+\mathrm{e}^{\varepsilon\tau}\sum_{j=1}^{n}\rho_{ij}V_i(t,x)+\mathrm{e}^{\varepsilon\tau}\sum_{j=1}^{n}\big(-\alpha\rho_{ij}|1-\varrho|\big)V_i(t-\tau_{ij}(t),x)\bigg\}\mathrm{d}x\bigg\}\\
=&-\mathbf{E}\Bigg\{\int\limits_{\Omega}\sum_{i=1}^{n}\Bigg[\lambda_i-\omega_i-\varepsilon-\sum_{j=1}^{n}\rho_{ij}e^{\varepsilon\tau}\Bigg]V_i(t,x)\,\mathrm{d}x\Bigg\}\leqslant 0,
\end{aligned}
\tag{A.11}
$$

which implies that

$$
\mathbf{E}\big\{V(t,x)\big\}\leqslant\mathbf{E}\big\{V(mT,x)\big\}
\tag{A.12}
$$

for $(t,x)\in[mT,mT+\delta)\times\Omega$.

Similarly, for $(t,x)\in[mT+\delta,(m+1)T)\times\Omega$, we can get

$$
\begin{aligned}
&D^{+}\mathbf{E}\big\{V(t,x)\big\}\\
&\quad\leqslant-\int\limits_{\Omega}\sum_{i=1}^{n}\Bigg[\lambda_i+\theta_i-\varepsilon-\mathrm{e}^{\varepsilon\tau}\sum_{j=1}^{n}\rho_{ij}\Bigg]V_i(t,x)\,\mathrm{d}x+\int\limits_{\Omega}\sum_{i=1}^{n}\theta_iV_i(t,x)\,\mathrm{d}x\\
&\quad\leqslant\int\limits_{\Omega}\sum_{i=1}^{n}\theta V_i(t,x)\,\mathrm{d}x,
\end{aligned}
$$

which leads to

$$
\mathbf{E}\big\{V(t,x)\big\}\leqslant\mathbf{E}\big\{V(mT+\delta,x)\exp\big\{\theta(t-mT-\delta)\big\}\big\}
$$

for $(t,x)\in[mT+\delta,(m+1)T)\times\Omega$.

Combining these two cases, we summarize that:

(i) For $(t,x)\in[0,\delta)\times\Omega$, it follows from (A.12) that

$$
\mathbf{E}\big\{V(t,x)\big\}\leqslant\mathbf{E}\big\{V(0,x)\big\}.
$$

(ii) For $(t,x)\in[\delta,T)\times\Omega$, we have

$$
\mathbf{E}\big\{V(t,x)\big\}\leqslant\mathbf{E}\big\{V(\delta,x)\exp\big\{\theta(t-\delta)\big\}\big\}\leqslant\mathbf{E}\big\{V(0,x)\exp\big\{\theta(t-\delta)\big\}\big\}.
$$

(iii) For $(t,x)\in[T,T+\delta)\times\Omega$, we get

$$\mathbf{E}\big\{V(t,x)\big\}\leqslant\mathbf{E}\big\{V(T,x)\big\}\leqslant\mathbf{E}\big\{V(0,x)\exp\big\{\theta(T-\delta)\big\}\big\}.$$

(iv) For $(t,x)\in[T+\delta,2T)\times\Omega$, we know

$$\mathbf{E}\big\{V(t,x)\big\}\leqslant\mathbf{E}\big\{V(T+\delta,x)\exp\big\{\theta(t-T-\delta)\big\}\big\}\leqslant\mathbf{E}\big\{V(0,x)\exp\big\{\theta(t-2\delta)\big\}\big\}.$$

Repeating this procedure, we obtain that for $(t,x)\in[mT,mT+\delta)\times\Omega$,

$$\mathbf{E}\big\{V(t,x)\big\}\leqslant\mathbf{E}\big\{V(mT,x)\big\}\leqslant\mathbf{E}\big\{V(0,x)\exp\big\{m\theta(T-\delta)\big\}\big\}. \tag{A.13}$$

Moreover, in the case of $(t,x)\in[mT+\delta,(m+1)T)\times\Omega$, we have

$$\begin{aligned}\mathbf{E}\big\{V(t,x)\big\}&\leqslant\mathbf{E}\big\{V(mT+\delta,x)\exp\big\{\theta(t-mT-\delta)\big\}\big\}\\&\leqslant\mathbf{E}\big\{V(0,x)\exp\big\{\theta(t-(m+1)\delta)\big\}\big\}.\end{aligned} \tag{A.14}$$

If $(t,x)\in[mT,mT+\delta)\times\Omega$, we have $m\leqslant t/T$, then it follows from (A.13) that

$$\mathbf{E}\big\{V(t,x)\big\}\leqslant\mathbf{E}\bigg\{V(0,x)\exp\bigg\{\frac{(T-\delta)\theta}{T}t\bigg\}\bigg\}. \tag{A.15}$$

Similarly, if $mT+\delta\leqslant t<(m+1)T$, we have $t/T<m+1$, then it follows from (A.14) that (A.15) holds for $(t,x)\in[mT+\delta,(m+1)T)\times\Omega$. Hence, for any $(t,x)\in[0,+\infty)\times\Omega$, (A.15) always holds.

Note that

$$\begin{aligned}&\mathbf{E}\big\{V(0,x)\big\}\\&\quad=\mathbf{E}\Bigg\{\int\limits_{\Omega}\sum_{i=1}^{n}\Bigg[V_i(0,x)+\mathrm{e}^{\varepsilon\tau}\sum_{j=1}^{n}\rho_{ij}\int\limits_{-\tau_{ij}(0)}^{0}V_i(s,x)\,\mathrm{d}s\\&\qquad+\mathrm{e}^{\varepsilon\tau}\sum_{j=1}^{n}\rho_{ij}^{*}\int\limits_{-\tau}^{-\tau_{ij}(0)}V_i(s,x)\,\mathrm{d}s+\sum_{j=1}^{n}\rho_{ij}^{**}\int\limits_{-\tau_{ij}^{*}(0)}^{0}\int\limits_{s}^{0}V_i(\eta,x)\,\mathrm{d}\eta\,\mathrm{d}s\Bigg]\,\mathrm{d}x\Bigg\}\\&\quad\leqslant\mathbf{E}\Bigg\{\int\limits_{\Omega}\sum_{i=1}^{n}\Bigg[\big|\mathrm{e}_i(0,x)\big|^p+\mathrm{e}^{\varepsilon\tau}\sum_{j=1}^{n}\rho_{ij}\int\limits_{-\tau_{ij}(0)}^{0}\mathrm{e}^{\varepsilon s}\big|e_i(s,x)\big|^p\,\mathrm{d}s\\&\qquad+\mathrm{e}^{\varepsilon\tau}\sum_{j=1}^{n}\rho_{ij}^{*}\int\limits_{-\tau}^{-\tau_{ij}(0)}\mathrm{e}^{\varepsilon s}\big|e_i(s,x)\big|^p\,\mathrm{d}s+\sum_{j=1}^{n}\tau^{*}\rho_{ij}^{**}\int\limits_{-\tau_{ij}^{*}(0)}^{0}\mathrm{e}^{\varepsilon s}\big|e_i(s,x)\big|^p\,\mathrm{d}s\Bigg]\,\mathrm{d}x\Bigg\}\end{aligned}$$

$$
\begin{aligned}
&\leqslant \mathbf{E}\Biggl\{\int\limits_{\Omega}\sum_{i=1}^{n}\Biggl[\bigl|e_i(0,x)\bigr|^p + \mathrm{e}^{\varepsilon\tau}\max_{i=1,\dots,n}\Biggl\{\sum_{j=1}^{n}\rho_{ij}\Biggr\}\sum_{j=1}^{n}\int\limits_{-\tau_{ij}(0)}^{0}\mathrm{e}^{\varepsilon s}\bigl|e_i(s,x)\bigr|^p\,\mathrm{d}s \\
&\quad + \mathrm{e}^{\varepsilon\tau}\max_{i=1,\dots,n}\Biggl\{\sum_{j=1}^{n}\rho_{ij}^{*}\Biggr\}\sum_{j=1}^{n}\int\limits_{-\tau}^{-\tau_{ij}(0)}\mathrm{e}^{\varepsilon s}\bigl|e_i(s,x)\bigr|^p\,\mathrm{d}s \\
&\quad + \max_{i=1,\dots,n}\Biggl\{\tau^{*}\sum_{j=1}^{n}\rho_{ij}^{**}\Biggr\}\sum_{j=1}^{n}\int\limits_{-\tau_{ij}^{*}(0)}^{0}\mathrm{e}^{\varepsilon s}\bigl|e_i(s,x)\bigr|^p\,\mathrm{d}s\Biggr]\,\mathrm{d}x\Biggr\} \\
&\leqslant \Biggl[1 + \tau\mathrm{e}^{\varepsilon\tau}\max_{i=1,\dots,n}\Biggl\{\sum_{j=1}^{n}(\rho_{ij}+\rho_{ij}^{*})\Biggr\} + \max_{i=1,\dots,n}\Biggl\{(\tau^{*})^2\sum_{j=1}^{n}\rho_{ij}^{**}\Biggr\}\Biggr] \\
&\quad \times \mathbf{E}\bigl\{\|\psi-\phi\|_p^p\bigr\}
\end{aligned}
\tag{A.16}
$$

and

$$
\mathbf{E}\bigl\{V(t,x)\bigr\} \geqslant \Biggl\{\int\limits_{\Omega}\sum_{i=1}^{n}\mathrm{e}^{\varepsilon t}\bigl|e_i(t,x)\bigr|^p\,\mathrm{d}x\Biggr\} = \mathrm{e}^{\varepsilon t}\mathbf{E}\bigl\{\bigl\|v(t,x)-u(t,x)\bigr\|_p^p\bigr\}. \tag{A.17}
$$

Let

$$
\begin{aligned}
M &= \Biggl[1 + \tau\mathrm{e}^{\varepsilon\tau}\max_{i=1,\dots,n}\Biggl\{\sum_{j=1}^{n}(\rho_{ij}+\rho_{ij}^{*})\Biggr\} + \max_{i=1,\dots,n}\Biggl\{(\tau^{*})^2\sum_{j=1}^{n}\rho_{ij}^{**}\Biggr\}\Biggr]^{1/p} > 1, \\
\mu &= \frac{1}{p}\biggl[\varepsilon - \frac{(T-\delta)\theta}{T}\biggr] > 0.
\end{aligned}
$$

It follows from (A.15)–(A.17) that

$$
\mathbf{E}\bigl\{\bigl\|v(t,x)-u(t,x)\bigr\|_p\bigr\} \leqslant M\mathbf{E}\bigl\{\|\psi-\phi\|_p\bigr\}\mathrm{e}^{-\mu t},
$$

which implies that the noise-perturbed response system (4) and the drive system (1) can be exponentially synchronized under the intermittent controller (5) based on p-norm. This completes the proof of Theorem 3.

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