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Erratum to "Common fixed points for α-ψ-φ-contractions in generalized metric spaces" [*Nonlinear Anal. Model. Control*, 19(1):43–54, 2014]

Vincenzo La Rosa, Pascuale Vetro

Universitá degli Studi di Palermo, Dipartimento di Matematica e Informatica
Via Archirafi, 34, 90123 Palermo, Italy
vincenzo.larosa@math.unipa.it; vetro@math.unipa.it

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Abstract. In Example 1 of our paper [V. La Rosa, P. Vetro, Common fixed points for α-ψ-φ-contractions in generalized metric spaces, *Nonlinear Anal. Model. Control*, 19(1):43–54, 2014] a generalized metric has been assumed. Nevertheless some mistakes have appeared in the statement. The aim of this note is to correct this situation.

Keywords: generalized metric space, α-ψ-φ-contractive condition, fixed point.

Firstly, for the convenience of the reader, we give some notions from paper [2].

Definition 1. (See [1].) Let X be a non-empty set and $d : X \times X \to [0, +\infty[$ be a mapping such that for all $x, y \in X$ and for all distinct points $u, v \in X$ each of them different from x and y, one has

(i) $d(x, y) = 0$ if and only if $x = y$,
(ii) $d(x, y) = d(y, x)$,
(iii) $d(x, y) \leqslant d(x, u) + d(u, v) + d(v, y)$ (rectangular inequality).

Then (X, d) is called a generalized metric space (or shortly GMS).

We denote by Ψ the set of functions $\psi : [0, +\infty[\to [0, +\infty[$ satisfying the following hypotheses:

(ψ1) ψ is continuous and nondecreasing,
(ψ2) $\psi(t) = 0$ if and only if $t = 0$.

We denote by Φ the set of functions $\varphi : [0, +\infty[\to [0, +\infty[$ satisfying the following hypotheses:

(φ1) φ is lower semi-continuous,
(φ2) $\varphi(t) = 0$ if and only if $t = 0$.

Definition 2. Let $T : X \to X$ and $\alpha : X \times X \to [0, +\infty[$. The mapping T is α-admissible if for all $x, y \in X$ such that $\alpha(x, y) \geqslant 1$ we have $\alpha(Tx, Ty) \geqslant 1$.

Definition 3. Let (X, d) be a GMS and $\alpha : X \times X \to [0, +\infty[$. X is α-regular if for every sequence $\{x_n\} \subset X$ such that $\alpha(x_n, x_{n+1}) \geqslant 1$ for all $n \in \mathbb{N}$ and $x_n \to x$, then there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $\alpha(x_{n_k}, x) \geqslant 1$ for all $n \in \mathbb{N}$.

Then, the following result is given in [2].

Corollary 1. *Let (X, d) be a complete GMS, let T be a self-mapping on X and $\alpha : X \times X \to [0, +\infty[$. Assume that the following condition holds*:

$$\psi\big(\alpha(x, y)d(Tx, Ty)\big) \leqslant \psi\big(M(x, y)\big) - \varphi\big(M(x, y)\big)$$

for all $x, y \in X$, where $\psi \in \Psi$, $\varphi \in \Phi$ and

$$M(x, y) = \max\big\{d(x, y), d(x, Tx), d(y, Ty)\big\}.$$

Assume also that the following conditions hold:

(i) *T is α-admissible*;
(ii) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geqslant 1$*;
(iii) *X is α-regular and for every sequence $\{x_n\} \subset X$ such that $\alpha(x_n, x_{n+1}) \geqslant 1$, we have $\alpha(x_m, x_n) \geqslant 1$ for all $m, n \in \mathbb{N}$ with $m < n$*;
(iv) *either $\alpha(u, v) \geqslant 1$ or $\alpha(v, u) \geqslant 1$ whenever $u = Tu$ and $v = Tv$.*

Then T has a unique fixed point.

Finally, we give a correct version of Example 1 in [2].

Example 1. Let $X = [1, 2] \cup A$ with $A = \{1/5, 1/4, 1/3, 1/2\}$. Define the generalized metric d on X as follows:

$$d(x, y) = d(y, x), \quad d(x, y) = |x - y| \quad \text{if } \{x, y\} \cap [1, 2] \neq \emptyset,$$
$$d\left(\frac{1}{2}, \frac{1}{3}\right) = \frac{1}{6} + 3a, \quad d\left(\frac{1}{2}, \frac{1}{4}\right) = \frac{1}{4} + 6a, \quad d\left(\frac{1}{2}, \frac{1}{5}\right) = \frac{3}{10} + 2a,$$
$$d\left(\frac{1}{3}, \frac{1}{4}\right) = \frac{1}{12} + 2a, \quad d\left(\frac{1}{3}, \frac{1}{5}\right) = \frac{2}{15} + 6a, \quad d\left(\frac{1}{4}, \frac{1}{5}\right) = \frac{1}{20} + 3a,$$

where $a = 1/24$.

Clearly, (X, d) is a complete GMS. Let $T : X \to X$ and $\psi, \varphi : [0, +\infty[\to [0, +\infty[$ defined by

$$Tx = \begin{cases} 1/4 & \text{if } x \in A \cup \{3/2\}, \\ 3 - x & \text{if } x \in [1, 2] \setminus \{3/2\}, \end{cases} \qquad \psi(t) = t \quad \text{and} \quad \varphi(t) = \frac{t}{5}.$$

Also consider $\alpha : X \times X \to [0, +\infty[$ given by

$$\alpha(x, y) = \begin{cases} 1 & \text{if } x, y \in A \text{ or } x = y, \\ 0 & \text{otherwise.} \end{cases}$$

Then T and α satisfy all the conditions of Corollary 1 and hence T has a unique fixed point on X, that is, $x = 1/4$.

We note that if X is endowed with the standard metric $d(x, y) = |x - y|$ for all $x, y \in X$, then do not exist $\psi, \varphi : [0, +\infty[\to [0, +\infty[$, where $\psi \in \Psi$ and $\varphi \in \Phi$ such that

$$\psi\bigl(d(Tx, Ty)\bigr) \leqslant \psi\bigl(M(x, y)\bigr) - \varphi\bigl(M(x, y)\bigr)$$

for all $x, y \in X$.

References

1. A. Branciari, A fixed point theorem of Banach–Caccioppoli type on a class of generalized metric spaces, *Publ. Math.*, **57**(1–2):31–37, 2000.

2. V. La Rosa, P. Vetro, Common fixed points for α-ψ-φ-contractions in generalized metric spaces, *Nonlinear Anal. Model. Control*, **19**(1):43–54, 2014.