Common fixed point theorems on non-complete partial metric spaces*

Shaban Sedghi[a], **Nabi Shobkolaei**[b], **Ishak Altun**[c]

[a]Department of Mathematics, Qaemshahr Branch
Islamic Azad University
Qaemshahr, Iran
sedghi_gh@yahoo.com

[b]Department of Mathematics, Science and Research Branch
Islamic Azad University
14778 93855 Tehran, Iran
nabi_shobe@yahoo.com

[c]Department of Mathematics, Faculty of Science and Arts
Kirikkale University
71450 Yahsihan, Kirikkale, Turkey
ishakaltun@yahoo.com

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Abstract. In the peresent paper, we give a common fixed point theorem for four weakly compatible mappings on non-complete partial metric spaces. Some supporting examples are provided.

Keywords: fixed point, weakly compatible maps, partial metric space, semi-cyclic maps.

1 Introduction

Partial metric spaces were introduced by Matthews [1] to the study of denotational semantics of dataflow networks. In particular, he proved a partial metric version of the Banach contraction principle. Later, Valero [2] and Oltra and Valero [3] gave some generalizations of the result of Matthews. In fact, the study of fixed point theorems on partial metric metric spaces has received a lot of attention in the last three years (see, for instance, [4–17] and their references). Almost all of these papers offer fixed point or common fixed point results on complete partial metric spaces. In this paper, we present a common fixed point theorem without completeness of the space.

Now, we recall some definitions and results needed in the sequel. A partial metric on a nonempty set X is a mapping $p : X \times X \to [0, \infty)$ such that

(p1) $x = y$ if and only if $p(x, x) = p(x, y) = p(y, y)$,

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(p2) $p(x,x) \leqslant p(x,y)$,

(p3) $p(x,y) = p(y,x)$,

(p4) $p(x,y) \leqslant p(x,z) + p(z,y) - p(z,z)$

for all $x,y,z \in X$. A partial metric space is a pair (X,p) such that X is a nonempty set and p is a partial metric on X. It is clear that, if $p(x,y) = 0$, then from (p1) and (p2) $x = y$. But if $x = y$, $p(x,y)$ may not be 0. A basic example of a partial metric space is the pair (X,p), where $X = [0,\infty)$ and $p(x,y) = \max\{x,y\}$ for all $x,y \in X$.

Example 1. Let (X,d) and (X,p) be a metric space and partial metric space, respectively. Mappings $\rho_i : X \times X \to [0,\infty)$ ($i \in \{1,2,3\}$) defined by

$$\begin{aligned}\rho_1(x,y) &= d(x,y) + p(x,y),\\ \rho_2(x,y) &= d(x,y) + \max\bigl\{\omega(x),\omega(y)\bigr\},\\ \rho_3(x,y) &= d(x,y) + a\end{aligned}$$

define partial metrics on X, where $\omega : X \to [0,\infty)$ is an arbitrary function and $a \geqslant 0$.

Other examples of the partial metric spaces which are interesting from a computational point of view may be found in [1, 18, 19].

Each partial metric p on X generates a T_0 topology τ_p on X which has a family of open p-balls

$$\bigl\{B_p(x,\varepsilon)\colon\ x \in X, \varepsilon > 0\bigr\},$$

as a base, where $B_p(x,\varepsilon) = \{y \in X\colon\ p(x,y) < p(x,x) + \varepsilon\}$ for all $x \in X$ and $\varepsilon > 0$.

It is easy to see that, a sequence $\{x_n\}$ in a partial metric space (X,p) converges with respect to τ_p to a point $x \in X$ if and only if $p(x,x) = \lim_{n\to\infty} p(x,x_n)$. By $L(x_n)$, we denote the set of $x \in X$, which the sequence $\{x_n\}$ converges to x with respect to τ_p. That is, $L(x_n) = \{x \in X : x_n \to x \text{ w.r.t. } \tau_p\}$. If p is a partial metric on X, then the functions $p^s, p^m : X \times X \to [0,\infty)$ given by

$$p^s(x,y) = 2p(x,y) - p(x,x) - p(y,y)$$

and

$$\begin{aligned}p^m(x,y) &= \max\bigl\{p(x,y) - p(x,x), p(x,y) - p(y,y)\bigr\}\\ &= p(x,y) - \min\bigl\{p(x,x), p(y,y)\bigr\}\end{aligned}$$

are equivalent metrics on X.

Remark 1. Let $\{x_n\}$ be a sequence in a partial metric space (X,p) and $x \in X$, then

$$\lim_{n\to\infty} p^s(x_n,x) = 0$$

if and only if

$$p(x,x) = \lim_{n\to\infty} p(x_n,x) = \lim_{n,m\to\infty} p(x_n,x_m).$$

Definition 1. Let (X, p) be a partial metric space.

(i) A sequence $\{x_n\}$ in X is called Cauchy whenever $\lim_{n,m\to\infty} p(x_n, x_m)$ exists (and finite);

(ii) (X, p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges with respect to τ_p, to a point $x \in X$, that is, $\lim_{n,m\to\infty} p(x_n, x_m) = p(x, x)$.

The following example shows that a convergent sequence $\{x_n\}$ in a partial metric space X may not be Cauchy. In particular, it shows that the limit of a convergent sequence is not unique.

Example 2. Let $X = [0, \infty)$ and $p(x, y) = \max\{x, y\}$. Let

$$x_n = \begin{cases} 0, & n = 2k, \\ 1, & n = 2k + 1. \end{cases}$$

Then it is easy to see that $L(x_n) = [1, \infty)$. But $\lim\limits_{n,m\to\infty} p(x_n, x_m)$ does not exist.

The following Lemma shows that under certain conditions the limit is unique.

Lemma 1. (*See* [20].) *Let $\{x_n\}$ be a convergent sequence in partial metric space X such that $x_n \to x$ and $x_n \to y$. If*

$$\lim_{n\to\infty} p(x_n, x_n) = p(x, x) = p(y, y),$$

then $x = y$.

Lemma 2. (*See* [20, 21].) *Let $\{x_n\}$ and $\{y_n\}$ be two sequences in partial metric space X such that*

$$\lim_{n\to\infty} p(x_n, x) = \lim_{n\to\infty} p(x_n, x_n) = p(x, x)$$

and

$$\lim_{n\to\infty} p(y_n, y) = \lim_{n\to\infty} p(y_n, y_n) = p(y, y),$$

then $\lim_{n\to\infty} p(x_n, y_n) = p(x, y)$. In particular, $\lim_{n\to\infty} p(x_n, z) = p(x, z)$ for every $z \in X$.

Lemma 3. (*See* [1, 3].) *Let (X, p) be a partial metric space.*

(i) *$\{x_n\}$ is a Cauchy sequence in (X, p) if and only if it is a Cauchy sequence in the metric space (X, p^s).*

(ii) *A partial metric space (X, p) is complete if and only if the metric space (X, p^s) is complete.*

In the proofs of many fixed-point theorems on Partial metric space, using the metric p^s, the operations are done in the metric space (X, p^s), and then taking into account Lemma 3, again returns to the partial metric space (X, p). However, in their recent paper Haghi et al. [22] have done the proof completely on a metric space using another metric, which is obtained from the partial metric p, instead of p^s. In this paper, we do not use the technique of Haghi et al. [22], because of our contractive condition is given by implicit relation.

2 Main results

In the following we deal with the class Ψ of all functions $\psi : [0,\infty)^6 \to R$ with the property:

($\psi 1$) For $w \leqslant u$ and $v > 0$,

$$\psi(u,v,v,u,u+v,w) \leqslant 0 \quad \text{or} \quad \psi(u,v,u,v,w,u+v) \leqslant 0$$

implies that $u < v$,

($\psi 2$) $\psi(t_1,t_2,t_3,t_4,t_5,t_6)$ is non-increasing in t_5, t_6,

($\psi 3$) for every $w, w' \leqslant u$,

$$\psi(u,u,w,w',u,u) \leqslant 0, \quad \psi(u,0,0,u,u,w) \leqslant 0 \quad \text{and} \quad \psi(u,0,u,0,w,u) \leqslant 0$$

implies that $u = 0$,

($\psi 4$) ψ is continuous in any coordinates.

Two basic examples of ψ are:

1. $\psi(t_1,t_2,t_3,t_4,t_5,t_6) = t_1 - \lambda \max\{t_2,t_3,t_4,(1/2)t_5,(1/2)t_6\}$ for $0 < \lambda < 1$,
2. $\psi(t_1,t_2,t_3,t_4,t_5,t_6) = \int_0^{t_1} \phi(s)\,\mathrm{d}s - h \max\{\int_0^{t_i} \phi(s)\,\mathrm{d}s\}$ for $i = 2,3,4$, where $0 < h < 1$ and $\phi : R^+ \to R^+$ is a continuous map.

Let f and S be two self maps of a partial metric space (X,p), then we define a set $E(f,S)$ by

$$E(f,S) = \{p(fx,Sx)\colon\ x \in X\}.$$

It is clear that $\inf E(f,S)$ is exist, but may not be belong to $E(f,S)$.

It is well known that f and S are weakly compatible [23] if they are commute at their coincidence point, that is, $fx = Sx$ implies that $fSx = Sfx$.

Theorem 1. *Let (X,p) be a partial metric space and $f,g,S,T : X \to X$ are four mappings such that $f(X) \subseteq T(X)$ and $g(X) \subseteq S(X)$. Suppose for all $x,y \in X$*

$$\psi\bigl(p(fx,gy),p(Sx,Ty),p(Sx,fx),p(gy,Ty),p(Sx,gy),p(Ty,fx)\bigr) \leqslant 0, \qquad (1)$$

where $\psi \in \Psi$. If $\inf E(f,S) \in E(f,S)$, f and S as well as g and T are weakly compatible, then f, g, S and T have a unique common fixed point z in X. Moreover $p(z,z) = 0$.

Proof. Since $\inf E(f,S) \in E(f,S)$, hence if put $\alpha = \inf E(f,S)$, then there exists $u \in X$ such that $\alpha = p(fu,Su)$. Since $fu \in f(X) \subseteq T(X)$, hence there exists $v \in X$ such that $fu = Tv$. Thus

$$\alpha = p(fu,Su) = p(Tv,Su).$$

We prove that $\alpha = 0$. Let $\alpha > 0$, from (1) we get

$$\psi\big(p(fu, gv), p(Su, Tv), p(Su, fu), p(gv, Tv), p(Su, gv), p(Tv, fu)\big) \leqslant 0.$$

Since,

$$\begin{aligned} p(Su, gv) &\leqslant p(Su, fu) + p(fu, gv) - p(fu, fu) \\ &\leqslant p(Su, fu) + p(fu, gv), \end{aligned}$$

by above inequality and $(\psi 2)$ it follows that

$$\psi\big(p(fu, gv), \alpha, \alpha, p(gv, fu), \alpha + p(fu, gv), p(fu, fu)\big) \leqslant 0.$$

By $(\psi 1)$, $p(Tv, gv) = p(fu, gv) < \alpha = p(fu, Su)$. Since $gv \in g(X) \subseteq S(X)$, hence there exists $w \in X$ such that $Sw = gv$. Similarly, from (1) we get

$$\psi\big(p(fw, gv), p(Sw, Tv), p(Sw, fw), p(gv, Tv), p(Sw, gv), p(Tv, fw)\big) \leqslant 0.$$

Since,

$$\begin{aligned} p(fw, Tv) &\leqslant p(fw, Sw) + p(Sw, Tv) - p(Sw, Sw) \\ &\leqslant p(fw, Sw) + p(Sw, Tv), \end{aligned}$$

by above inequality and $(\psi 2)$ it follows that

$$\begin{aligned} &\psi\big(p(fw, Sw), p(gv, Tv), p(Sw, fw), p(gv, Tv), p(Sw, Sw), p(fw, Sw) + p(gv, Tv)\big) \\ &\quad \leqslant 0. \end{aligned}$$

If $p(gv, Tv) = 0$, then by $(\psi 1)$ we get $p(fw, Sw) = 0$. Thus, by the definition of α, we have

$$\alpha = p(fu, Su) \leqslant p(fw, Sw) = 0,$$

which is a contradiction. So, it follows that $p(gv, Tv) > 0$, hence by $(\psi 1)$, we get $p(fw, Sw) < p(gv, Tv)$. Thus,

$$\alpha = p(fu, Su) \leqslant p(fw, Sw) < p(gv, Tv) < p(fu, Su) = \alpha,$$

which is a contradiction. Hence $\alpha = 0$. This implies that $fu = Su = Tv$. Now we prove that $gv = Tv$. If $gv \neq Tv$, then by (1) and $(\psi 2)$, we get

$$\begin{aligned} &\psi\big(p(Tv, gv), p(Tv, Tv), p(Tv, Tv), p(gv, Tv), p(Tv, Tv) + p(Tv, gv), p(Tv, Tv)\big) \\ &\quad = \psi\big(p(fu, gv), p(Su, Tv), p(Su, fu), p(gv, Tv), p(Su, Tv) + p(Su, gv), p(Tv, fu)\big) \\ &\quad \leqslant \psi\big(p(fu, gv), p(Su, Tv), p(Su, fu), p(gv, Tv), p(Su, gv), p(Tv, fu)\big) \leqslant 0, \end{aligned}$$

from $(\psi 1)$ it follows that, $p(Tv, gv) = 0$ and so $Tv = gv$, because $\alpha = p(Tv, Tv) = 0$. Hence,

$$Tv = gv = fu = Su = z.$$

By weak compatibility of g and T and f and S we have $gz = Tz$ and $fz = Sz$. Now, we prove that $fz = z$. In fact by (1), we have

$$\psi\bigl(p(fz, gv), p(Sz, Tv), p(Sz, fz), p(gv, Tv), p(Sz, gv), p(Tv, fz)\bigr) \leqslant 0$$

or

$$\psi\bigl(p(fz, z), p(fz, z), p(fz, fz), p(z, z), p(fz, z), p(z, fz)\bigr) \leqslant 0.$$

By (ψ3), we have $p(fz, z) = 0$ and so $fz = z$. Therefore,

$$fz = Sz = z.$$

Similarly by (1) we have

$$\begin{aligned}&\psi\bigl(p(z, gz), p(z, gz), p(fz, fz), p(gz, gz), p(z, gz), p(z, gz)\bigr)\\ &\quad = \psi\bigl(p(fz, gz), p(Sz, Tz), p(Sz, fz), p(gz, Tz), p(Sz, gz), p(Tz, fz)\bigr) \leqslant 0.\end{aligned}$$

By (ψ3), we have $p(z, gz) = 0$ and so $gz = z$. Therefore,

$$gz = Tz = z.$$

i.e., z is a common fixed point of f, g, S and T. Moreover $p(z, z) = p(fu, Su) = \alpha = 0$.

Now we show that the common fixed point is unique. If x and y are two common fixed points of f, g, S and T, then from (1), we have

$$\begin{aligned}&\psi\bigl(p(x, y), p(x, y), p(x, x), p(y, y), p(x, y), p(y, x)\bigr)\\ &\quad = \psi\bigl(p(fx, gy), p(Sx, Ty), p(Sx, fx), p(Sy, Ty), p(Sx, gy), p(Ty, fx)\bigr) \leqslant 0.\end{aligned}$$

By (ψ3) implies that $p(x, y) = 0$ and so $x = y$. □

Remark 2. In Theorem 1, the condition $\inf E(f, S) \in E(f, S)$ can be replaced by $\inf E(g, T) \in E(g, T)$.

Corollary 1. *Let f_i, g_j, T and S $(i, j \in N)$ be self-mappings of a partial metric space (X, p) such that $f_{i_0}(X) \subseteq T(X)$, and $g_{j_0}(X) \subseteq S(X)$ for some $i_0, j_0 \in N$. Suppose for all $x, y \in X$ and $i, j \in N$*

$$\psi\bigl(p(f_i x, g_j y), p(Sx, Ty), p(Sx, f_i x), p(g_j y, Ty), p(Sx, g_j y), p(Ty, f_i x)\bigr) \leqslant 0,$$

where $\psi \in \Psi$. If $\inf E(f_{i_0}, S) \in E(f_{i_0}, S)$, f_{i_0} and S as well as g_{j_0} and T are weakly compatible, then f_i, g_j, S and T have a unique common fixed point z in X. Moreover $p(z, z) = 0$.

Proof. By Theorem 1, S, T, f_{i_0} and g_{j_0} have a unique common fixed point z in X. Moreover $p(z, z) = 0$. That is, there exists a unique $z \in X$ such that

$$Sz = Tz = f_{i_0} z = g_{j_0} z = z.$$

Now for every $j \in N$, we have from (1)

$$\begin{aligned}&\psi\big(p(z,g_jz),p(z,z),p(z,z),p(g_jz,z),p(z,g_jz)+p(z,z),p(z,z)\big)\\&\quad=\psi\big(p(z,g_jz),p(z,z),p(z,z),p(g_jz,z),p(z,g_jz),p(z,z)\big)\\&\quad=\psi\big(p(f_{i_0}z,g_jz),p(Sz,Tz),p(Sz,f_{i_0}z),p(g_jz,Tz),p(Sz,g_jz),p(Tz,f_{i_0}z)\big)\leqslant 0.\end{aligned}$$

By (ψ1), it follows that $p(g_jz,z)=0$. Hence, for every $j\in N$, we have $g_jz=z$. Similarly, for every $i\in N$, we get $f_iz=z$. Therefore, for every $i,j\in N$, we have

$$f_iz=g_jz=Sz=Tz=z. \qquad \square$$

We can obtain the following corollaries from Theorem 1, by the choosing some special function ψ.

Corollary 2. *Let (X,p) be a partial metric space and $f,g,S,T : X\to X$ are four mappings such that $f(X)\subseteq T(X)$ and $g(X)\subseteq S(X)$. Suppose for all $x,y\in X$*

$$p(fx,gy)\leqslant\lambda\max\left\{p(Sx,Ty),p(Sx,fx),p(gy,Ty),\frac{1}{2}p(Sx,gy),\frac{1}{2}p(Ty,fx)\right\},$$

where $\lambda\in(0,1)$. If $\inf E(f,S)\in E(f,S)$, f and S as well as g and T are weakly compatible, then f, g, S and T have a unique common fixed point z in X. Moreover $p(z,z)=0$.

Corollary 3. *Let (X,p) be a partial metric space and $f,g,S,T : X\to X$ are four mappings such that $f(X)\subseteq T(X)$ and $g(X)\subseteq S(X)$. Suppose for all $x,y\in X$*

$$\int_0^{p(fx,gy)}\phi(s)\,\mathrm{d}s\leqslant h\max\left\{\int_0^{p(Sx,Ty)}\phi(s)\,\mathrm{d}s,\int_0^{p(Sx,fx)}\phi(s)\,\mathrm{d}s,\int_0^{p(gy,Ty)}\phi(s)\,\mathrm{d}s\right\},$$

where $0<h<1$ and $\phi: R^+\to R^+$ is a continuous map. If $\inf E(f,S)\in E(f,S)$, f and S as well as g and T are weakly compatible, then f, g, S and T have a unique common fixed point z in X. Moreover $p(z,z)=0$.

Corollary 4. *Let (X,p) be a partial metric space and $f,g,S,T : X\to X$ are four mappings such that $f(X)\subseteq T(X)$ and $g(X)\subseteq S(X)$. Suppose for all $x,y\in X$*

$$p(fx,gy)\leqslant\lambda p(Sx,Ty),$$

where $\lambda\in(0,1)$. If $\inf E(f,S)\in E(f,S)$, f and S as well as g and T are weakly compatible, then f, g, S and T have a unique common fixed point z in X. Moreover $p(z,z)=0$.

Now we give an illustrative example.

Example 3. Let $X = [0, \infty)$ and $p(x, y) = \max\{x, y\}$, then (X, p) is a partial metric space. Define self-maps f, g, S and T on X as follows:

$$fx = x, \quad gx = \mathrm{e}^x - 1, \quad Sx = 2x \quad \text{and} \quad Tx = \mathrm{e}^{2x} - 1$$

for any $x \in X$. Hence, $\inf E(f, S) = \inf\{p(fx, Sx)\colon x \in X\} = 0 \in E(f, S)$ and

$$\begin{aligned} p(fx, gy) &= \max\bigl\{x, e^y - 1\bigr\} \leqslant \max\bigl\{x, \mathrm{e}^y \cosh y - 1\bigr\} \\ &= \frac{1}{2} \max\bigl\{2x, \mathrm{e}^{2y} - 1\bigr\} = \frac{1}{2} p(Sx, Ty) \end{aligned}$$

for every x, y in X. Also, f and S as well as g and T are weakly compatible and $f(X) = T(X)$ and $g(X) = S(X)$. Therefore, all conditions of Corollary 4 are holds, and $z = 0$ is unique common fixed point of f, g, S, T.

The following example shows that condition $\inf E(f, S) \in E(f, S)$ can not be omitted.

Example 4. Let $X = (0, \infty)$ and $p(x, y) = \max\{x, y\}$, then (X, p) is a partial metric space. Define self-maps f, g, S and T on X as follows:

$$fx = gx = \lambda x, \qquad Sx = Tx = x$$

for any $x \in X$, where $\lambda \in (0, 1)$. Hence,

$$p(fx, gy) = \max\{\lambda x, \lambda y\} = \lambda \max\{x, y\} = \lambda p(Sx, Ty)$$

for every x, y in X. Also, f and S as well as g and T are weakly compatible, $f(X) = T(X)$ and $g(X) = S(X)$. But f, g, S, T have not a common fixed point in X. Note that

$$\inf E(f, S) = \inf\{p(fx, Sx) : x \in X\} = 0 \notin E(f, S).$$

Example 5. Let $X = [0, \infty) \cap Q$, where by Q we denote the set of rational numbers and $p(x, y) = \max\{x, y\}$, then (X, p) is a non-complete partial metric space. If we define self-maps f, g, S and T on X as in Example 4 with $\lambda \in (0, 1) \cap Q$, then all conditions of Corollary 4 are holds and $z = 0$ is unique common fixed point of f, g, S, T.

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