Analytical solution of MHD free convective flow of couple stress fluid in an annulus with Hall and Ion-slip effects

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Abstract. This paper presents the Hall and Ion-slip effects on electrically conducting couple stress fluid flow between two circular cylinders in the presence of a temperature dependent heat source. The governing non-linear partial differential equations are transformed into a system of ordinary differential equations using similarity transformations and then solved using homotopy analysis method (HAM). The effects of the magnetic parameter, Hall parameter, Ion-slip parameter and couple stress fluid parameter on velocity and temperature are discussed and shown graphically.

Keywords: free convection, couple stress fluid, Hall and Ion-slip effects, circular cylinders, HAM.

1 Introduction

There has been widespread interest in the study of natural convection of a fluid in a cylindrical annulus between two vertical concentric cylinders subject to axial rotation. This motivation is primarily due to the extensive range of practical applications such as electrical machineries where heat transfer occurs in the annular gap between the rotor and stator, growth of single silicon crystals and other rotating systems. The flow in a rotating annulus can be considered as a possible analogue of the motion of a planetary atmosphere. The first numerical study of free convection in a rotating annulus was conducted by Williams [1]. He made an excellent analysis of axisymmetric flows in the annular geometry for different combinations of physical parameters. The accuracy of the numerical study was further verified with experimental observations [2] for the same geometry and are found to be in good agreement. De Vahl Davis et al. [3] made a numerical study of natural convection in a vertical annular cavity by considering two different cases. In the first case the inner and lower surfaces are heated and rotating while the other two surfaces are stationary and cooled. In the second case the inner and upper surfaces are heated, rotating and other surfaces are cooled and stationary.

In recent years, several simple flow problems associated with classical hydrodynamics have received new attention within the more general context of magnetohydrodynamics

(MHD). Several investigators have extended many of the available hydrodynamic solutions to include the effects of magnetic fields for those cases when the fluid is electrically conducting. Also, the effects of magnetic field on the non-Newtonian fluid also have great importance in engineering applications; for instance, MHD generators, accelerators, aerodynamics heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil and fluid droplets, sprays, plasma studies and geothermal energy excitations etc. Interest in rotating hydromagnetic flow in annular spaces was initiated in the late 1950's with an important analysis by Globe [4] who considered fully developed laminar MHD flow in an annular channel. Jain and Mehta [5] examined wall suction/injection effects on the Globe problem. Antimirov et al. [6] studied unsteady MHD convection in a vertical channel and Borkakati et al. [7] considered the MHD heat transfer in the flow between two coaxial cylinders. Ellahi et al. [8] have analyzed the MHD unsteady periodic flows due to non-coaxial rotations of a disk and a fluid at infinity, later they extended their work to porous disks [9] and non-newtonian fluid [10]. The analytical solutions for MHD flow of a of third order fluid in a porous medium presented by Ellahi et al. [11]. Analytic solutions for MHD flow in an annulus investigated by Hayat et al. [12]. In most of the MHD flow problems, the Hall and Ion-slip terms in Ohms law were ignored. However, in the presence of strong magnetic field, the influence of Hall current and Ion-slip are important. Tani [13] studied the Hall effects on the steady motion of electrically conducting viscous fluid in channels. Hall and Ion-slip effects on MHD Couette flow with heat transfer have been considered by Soundelgekar et al. [14]. Hazem Ali et al. [15] examined the MHD flow and heat transfer in a rectangular duct with temperature dependent viscosity and Hall Effect. Emmanuel Osalusi et al. [16] investigated on the effectiveness of viscous dissipation and Joule heating on steady MHD flow and heat transfer of a Bingham fluid over a porous rotating disk in the presence of Hall and Ion-slip currents. Hayat et al. [17] studied the Hall effect on the unsteady flow due to non coaxially rotating disk and fluid at infinity. Many of the problems in the literature deal with MHD flow between parallel plates/flow through circular pipes with Hall and Ion-slip effects, but not much attention has been given to the flow through a closed rectangular channel and concentric cylinders.

During recent years the study of convection heat and mass transfer in non-Newtonian fluids has received much attention and this is because the traditional Newtonian fluids cannot precisely describe the characteristics of the real fluids. In addition, progress has been considerably made in the study heat and mass transfer in magneto hydrodynamic flow of non-Newtonian fluids due to its application in many devices, like the MHD power generator, aerodynamics heating, electrostatic precipitation and Hall accelerator etc. Different models have been proposed to explain the behavior of non-Newtonian fluids. Among these, couple stress fluids introduced by Stokes [18] have distinct features, such as the presence of couple stresses, body couples and non-symmetric stress tensor. The couple stress fluid theory presents models for fluids whose microstructure is mechanically significant. The effect of very small microstructure in a fluid can be felt if the characteristic geometric dimension of the problem considered is of the same order of magnitude as the size of the microstructure. The main feature of couple stresses is to introduce a size dependent effect. Classical continuum mechanics neglects the size effect of material particles within the continua. This is consistent with ignoring the rotational interaction among particles, which results in symmetry of the force-stress tensor. However, in some important cases such as fluid flow with suspended particles, this cannot be true and a size dependent couple-stress theory is needed. The spin field due to microrotation of freely suspended particles set up an antisymetric stress, known as couple-stress, and thus forming couple-stress fluid. These fluids are capable of describing various types of lubricants, blood, suspension fluids etc. The study of couple stress fluids has applications in a number of processes that occur in industry such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plate in a bath, and colloidal solutions etc. A review of couple stress (polar) fluid dynamics was reported by Stokes [19]. Basic concepts and methods on steady and unsteady flow problems for Newtonian and non-Newtonian fluids given by Ellahi [20]. Srinivasacharya and Srikanth [21] studied the effect of couple stresses on the flow in a constricted annulus. Recently, Ghosh et al. [22] studied theoretically the transient rotating hydromagnetic flow of a viscous, incompressible, electrically-conducting, couple stress fluid in a channel with non-conducting walls.

In this paper, we have investigated the Hall and Ion-slip effects on steady free convective heat transfer flow between two concentric cylinders in couple stress fluid. The homotopy analysis method is employed to solve the nonlinear problem. Homotopy analysis method (HAM), introduced by Liao [23], is one of the most efficient methods in solving different types of nonlinear equations such as coupled, decoupled, homogeneous and non-homogeneous. Also, HAM provides us a great freedom to choose different base functions to express solutions of a nonlinear problem [24].

2 Mathematical formulation

Consider an incompressible electrically conducting couple stress fluid between two coaxial concentric circular cylinders of radii a and b ($a < b$). Choose the cylindrical polar coordinate system (r, φ, z) with z-axis as the common axis for both cylinders. The inner is at rest and the outer cylinder is rotating with constant angular velocity Ω. The flow being generated due to the rotation of the outer cylinder. A uniform magnetic field (B_0) is applied in the axial direction. Assume that the flow is steady and magnetic Reynolds number is very small so that the induced magnetic field can be neglected in comparison with the applied magnetic field. The electron-atom collision frequency is assumed to be relatively high, so that the Hall effect and the Ion-slip cannot be neglected. Further, assume that all the fluid properties are constant except the density in the buoyancy term of the balance of momentum equation. The flow is a free convection caused by buoyancy forces. With the above assumptions, the equations governing the steady flow of an incompressible couple stress fluid, under usual MHD approximations are

$$\frac{\partial u}{\partial \varphi} = 0, \tag{1}$$

$$\frac{\partial p}{\partial r} = \frac{u}{r^2} - \frac{\sigma B_0^2 \beta_h}{\alpha_e^2 + \beta_h^2} u, \tag{2}$$

$$\eta_1 \nabla_1^4 u - \mu \nabla_1^2 u - \rho g \beta_T (T - T_a) + \frac{\sigma B_0^2 \alpha_e}{\alpha_e^2 + \beta_h^2} u = 0, \tag{3}$$

$$K_f \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right] + \mu \left[\left(\frac{\partial u}{\partial r} \right)^2 - 2 \frac{u}{r} \frac{\partial u}{\partial r} + \left(\frac{u}{r} \right)^2 \right] + \eta_1 (\nabla_1^2 u)^2 + \gamma_0 \Omega (T - T_a) = 0, \tag{4}$$

where u is the velocity component of the fluid in the direction of φ, p is the pressure, ρ is the density, μ is the coefficient of viscosity, σ is the electrical conductivity, β_h is the Hall parameter, β_i is the Ion-slip parameter, $\alpha_e = 1 + \beta_i \beta_h$, β_T is the coefficient of thermal expansion, K_f is the coefficient of thermal conductivity, η_1 is the couple stress fluid parameter, γ_0 is the constant of proportionality, $\gamma_0 \Omega (T - T_a)$ is the amount of heat generated per unit volume in unit time, which is assumed to be a linear function of temperature, $\nabla_1^2 u = \frac{\partial}{\partial r}[\frac{1}{r}\frac{\partial}{\partial r}(ru)]$.

The boundary conditions are given by

$$u = 0 \quad \text{at } r = a, \quad u = b\Omega \text{ at } r = b, \tag{5a}$$

$$\nabla_1^2 u = 0 \quad \text{at } r = a \quad \text{and} \quad r = b, \tag{5b}$$

$$T = T_a \text{ at } r = a, \quad T = T_b \text{ at } r = b. \tag{5c}$$

The boundary condition (5a) corresponds to the classical no-slip condition from viscous fluid dynamics. The boundary condition (5b) imply that the couple stresses are zero at the surfaces.

Introducing the following similarity transformations

$$r = b\sqrt{\lambda}, \quad u = \frac{\Omega}{\sqrt{\lambda}} f(\lambda), \quad T - T_a = (T_b - T_a)\theta \tag{6}$$

in Eqs. (3)–(4), we get the following nonlinear system of differential equations

$$\alpha^2 \left[\lambda^2 f^{(iv)} + 2\lambda f^{'''} \right] - \frac{1}{4} \lambda f'' - \frac{1}{16} \frac{Gr}{Re} \sqrt{\lambda} \theta + \frac{1}{16} \frac{Ha^2 \alpha_e}{\alpha_e^2 + \beta_h^2} f = 0, \tag{7}$$

$$\left[\lambda^3 \theta'' + \lambda^2 \theta' \right] + Br \left[\lambda^2 (f')^2 - \frac{3}{2} \lambda f f' + \frac{3}{4} f^2 \right] + 4Br\alpha^2 \lambda^3 (f'')^2 + \frac{1}{4} \gamma_1 Re Pr \lambda^2 \theta = 0, \tag{8}$$

where primes denote differentiation with respect to λ, $Re = \frac{\rho \Omega b}{\mu}$ is the Reynolds number, $Pr = \frac{\mu C_p}{K_f}$ is the Prandtl number, $Ha = B_0 b \sqrt{\frac{\sigma}{\mu}}$ is the Hartmann number, $Gr = \frac{\rho^2 g \beta_T b^3}{\mu^2}(T_b - T_a)$ is the Grashof number, $\gamma_1 = \frac{\gamma_0 b}{\rho C_p}$ is the dimensionless vertical distance and $Br = \frac{\mu \Omega^2}{(T_b - T_a) K_f}$ is the Brinkman number, $\alpha = \frac{1}{b}\sqrt{\frac{\eta_1}{\mu}}$ is the couple stress parameter. The effects of couple-stress are significant for large values of $\alpha (= l/b)$, where $l = \sqrt{\frac{\eta_1}{\mu}}$ is the material constant. If l is a function of the molecular dimensions of the liquid, it will vary greatly for different liquids. For example, the length of a polymer chain may be a million times the diameter of water molecule [18].

Boundary conditions (5) in terms of f, θ become

$$\begin{aligned} &f = 0, \quad f'' = 0, \quad \theta = 0 \quad \text{at } \lambda = \lambda_0, \\ &f = b, \quad f'' = 0, \quad \theta = 1 \quad \text{at } \lambda = 1, \end{aligned} \tag{9}$$

where $\lambda_0 = (\frac{a}{b})^2$.

3 The HAM solution of the problem

For HAM solutions, we choose the initial approximations of $f(\lambda)$ and $\theta(\lambda)$ as follows:

$$f_0(\lambda) = \frac{b}{1-\lambda_0}(\lambda - \lambda_0), \quad \theta_0(\lambda) = \frac{\lambda - \lambda_0}{1-\lambda_0} \tag{10}$$

and choose the auxiliary linear operators:

$$L_1(f) = f^{(iv)}, \quad L_2(\theta) = \theta'' \tag{11}$$

such that

$$L_1\big(c_1 + c_2\lambda + c_3\lambda^2 + c_4\lambda^3\big) = 0, \quad L_2(c_5 + c_6\lambda) = 0, \tag{12}$$

where c_i $(i = 1, 2, \ldots, 6)$ are constants. Introducing non-zero auxiliary parameters h_1 and h_2, we developed the zeroth-order deformation problems as follow:

$$(1-p)L_1\big[f(\lambda;p) - f_0(\lambda)\big] = ph_1N_1\big[f(\lambda;p)\big], \tag{13}$$

$$(1-p)L_2\big[\theta(\lambda;p) - \theta_0(\lambda)\big] = ph_2N_2\big[\theta(\lambda;p)\big] \tag{14}$$

subject to the boundary conditions

$$\begin{aligned} &f(\lambda_0;p) = 0, \quad f''(\lambda_0;p) = 0, \quad f(1;p) = b, \\ &f''(1;p) = 0, \quad \theta(\lambda_0;p) = 0, \quad \theta(1;p) = 1, \end{aligned} \tag{15}$$

where $p \in [0,1]$ is the embedding parameter and the non-linear operators N_1 and N_2 are defined as:

$$\begin{aligned} N_1\big[f(\lambda,p), \theta(\lambda,p)\big] &= \alpha^2\big[\lambda^2 f^{(iv)} + 2\lambda f^{'''}\big] - \frac{1}{4}\lambda f'' - \frac{1}{16}\frac{Gr}{Re}\sqrt{\lambda}\theta \\ &\quad + \frac{1}{16}\frac{Ha^2\alpha_e}{\alpha_e^2 + \beta_h^2} f, \end{aligned} \tag{16}$$

$$\begin{aligned} N_2\big[f(\lambda,p), \theta(\lambda,p)\big] &= \big[\lambda^3\theta'' + \lambda^2\theta'\big] + Br\left[\lambda^2(f')^2 - \frac{3}{2}\lambda f f' + \frac{3}{4}f^2\right] \\ &\quad + 4Br\alpha^2\lambda^3(f'')^2 + \frac{1}{4}\gamma_1 RePr\lambda^2\theta. \end{aligned} \tag{17}$$

For $p = 0$ we have the initial guess approximations

$$f(\lambda;0) = f_0(\lambda), \quad \theta(\lambda;0) = \theta_0(\lambda). \tag{18}$$

When $p = 1$, Eqs. (13)–(14) are same as (7)–(8) respectively, therefore at $p = 1$ we get the final solutions

$$f(\lambda;1) = f(\lambda), \quad \theta(\lambda;1) = \theta(\lambda). \tag{19}$$

The initial guess approximations $f_0(\lambda)$ and $\theta_0(\lambda)$, the linear operators L_1, L_2 and the auxiliary parameters h_1 and h_2 are assumed to be selected such that Eqs. (13)–(15) have solution at each point $p \in [0,1]$ and also with the help of Maclaurin's series and due to Eq. (18) $f(\lambda;p)$ and $\theta(\lambda;p)$ can be expressed as

$$f(\lambda;p) = f_0(\lambda) + \sum_{m=1}^{\infty} f_m(\lambda)p^m, \tag{20}$$

$$\theta(\lambda;p) = \theta_0(\lambda) + \sum_{m=1}^{\infty} \theta_m(\lambda)p^m, \tag{21}$$

where

$$f_m(\lambda) = \frac{1}{m!}\frac{\partial^m f(\lambda;p)}{\partial p^m}, \quad \theta_m(\lambda) = \frac{1}{m!}\frac{\partial^m \theta(\lambda;p)}{\partial p^m}. \tag{22}$$

In which h_1 and h_2 are choosen in such a way that the series (20)–(21)are convergent at $p = 1$, therefore we have from (19) that

$$f(\lambda) = f_0(\lambda) + \sum_{m=1}^{\infty} f_m(\lambda), \tag{23}$$

$$\theta(\lambda) = \theta_0(\lambda) + \sum_{m=1}^{\infty} \theta_m(\lambda). \tag{24}$$

Differentiating the 0th-order deformation Eqs. (13)–(14), m-times with respect to p and then dividing them by $m!$ and finally setting $p = 0$, we obtain the following mth-order deformation problem.

$$L_1\big[f_m(\lambda) - \chi_m f_{m-1}(\lambda)\big] = h_1 R_m^f(\lambda), \tag{25}$$

$$L_2\big[\theta_m(\lambda) - \chi_m \theta_{m-1}(\lambda)\big] = h_2 R_m^\theta(\lambda) \tag{26}$$

with the boundary conditions

$$\begin{aligned} f_m(\lambda_0) &= 0, \quad f_m(1) = 0, \quad f_m''(\lambda_0) = 0, \\ f_m''(1) &= 0, \quad \theta_m(\lambda_0) = 0, \quad \theta_m(1) = 0, \end{aligned} \tag{27}$$

where

$$R_m^f(\eta) = \alpha^2\left[\lambda^2 f^{(iv)} + 2\lambda f^{'''}\right] - \frac{1}{4}\lambda f'' - \frac{1}{16}\frac{Gr}{Re}\sqrt{\lambda}\theta + \frac{1}{16}\frac{Ha^2\alpha_e}{\alpha_e^2+\beta_h^2}f, \tag{28}$$

$$R_m^\theta(\eta) = \left[\lambda^3\theta'' + \lambda^2\theta'\right] + Br\lambda^2\sum_{n=0}^{m-1} f'_{m-1-n}f'_n - \frac{3}{2}Br\lambda\sum_{n=0}^{m-1} f_{m-1-n}f'_n + \frac{3}{4}Br\sum_{n=0}^{m-1} f_{m-1-n}f_n + 4Br\alpha^2\lambda^3\sum_{n=0}^{m-1} f''_{m-1-n}f''{}_n + \frac{1}{4}\gamma_1 PrRe\lambda^2\theta \tag{29}$$

and, for m being integer

$$\chi_m = \begin{cases} 0 & \text{for } m \le 1, \\ 1 & \text{for } m > 1. \end{cases} \tag{30}$$

We emphasize here that Eqs. (25)–(26) are linear for all $m \ge 1$. These linear equations are solved using MATHEMATICA for the first 15 values of m and the expressions for $f(\lambda)$ and $\theta(\lambda)$ are calculated. As the expressions for $f(\lambda)$ and $\theta(\lambda)$ are too long, they have not presented here.

4 Results and discussion

The expressions for f and θ contains the auxiliary parameters h_1 and h_2. As pointed out by Liao [23], the convergence and the rate of approximation for the HAM solution strongly depend on the values of auxiliary parameter h. For this purpose, h-curves are plotted by choosing h_1 and h_2 in such a manner that the solutions (20)–(21) ensure convergence [23]. Here to see the admissible values of h_1 and h_2, the h-curves are plotted for 15th-order of approximation in Figs. (1)–(2) by taking the values of the parameters $Br = 0.5$, $Pr = 0.71$, $\gamma_1 = 1$, $Re = 10$, $Gr/Re = 5$, $a = 0.5$, $b = 1$, $\beta_h = 2$, $\beta_i = 2$, $\alpha = 0.5$. It is clearly noted from Fig. 1 that the range for the admissible values of h_1 is $-0.65 < h_1 < 0$. From Fig. 2, it can be seen that the h-curve has a parallel line segment that corresponds to a region $-0.3 < h_2 < -0.1$. It is found from computation that the series given by (20)–(21) converge in the whole region of λ when $h_1 = -0.5$ and $h_2 = -0.2$.

The solutions for $f(\lambda)$ and $\theta(\lambda)$ have been computed and shown graphically in Figs. 3 to 10. The effects of magnetic parameter (Ha), Hall parameter (β_h), Ion-slip parameter (β_i) and couple stress fluid parameter (α) have been discussed. To study the effect of Ha, β_h, β_i and α, computations were carried out by taking $Br = 0.5$, $Pr = 0.71$, $\gamma_1 = 1$, $Re = 20$, $Gr/Re = 5$, $a = 0.5$, $b = 1$.

Figures 3 to 4 display the effect of the magnetic parameter Ha on $f(\lambda)$ and $\theta(\lambda)$. It can be observed from these figures that the velocity $f(\lambda)$ decreases and the temperature $\theta(\eta)$ increases with an increase in the parameter Ha. This happens because of the imposing of a magnetic field normal to the flow direction. This magnetic field gives rise to a resistive force and slows down the movement of the fluid.

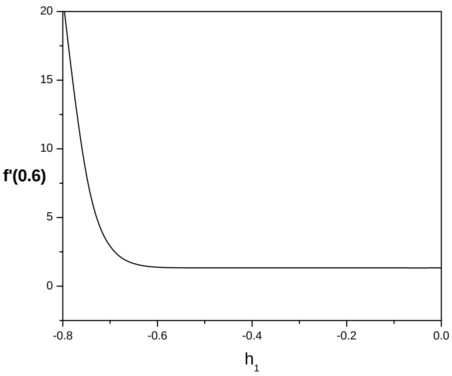

Fig. 1. h curve for $f(\eta)$ at $\beta_h = 2$, $\beta_i = 2$, $\alpha = 0.5$, $Ha = 20$.

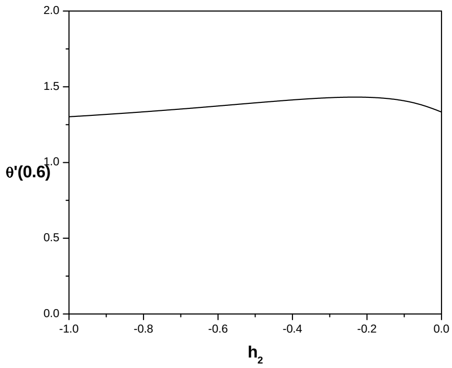

Fig. 2. h curve for $\theta(\eta)$ at $\beta_h = 2$, $\beta_i = 2$, $\alpha = 0.5$, $Ha = 20$.

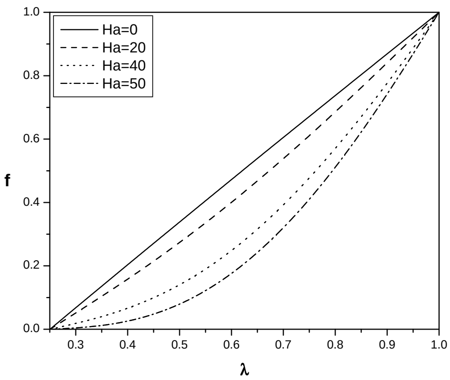

Fig. 3. Effect of Ha on f at $\beta_i = 2.0$, $\beta_h = 2.0$, $\alpha = 0.5$, $h_1 = -0.5$, $h_2 = -0.2$.

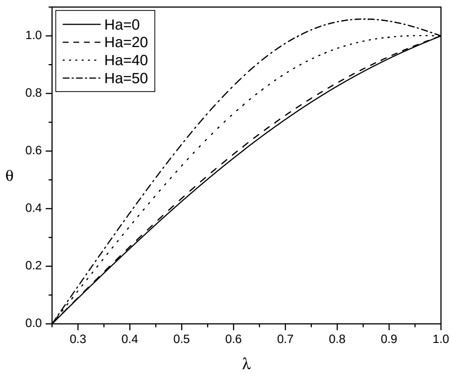

Fig. 4. Effect of Ha on $\beta_i = 2.0$, $\beta_h = 2.0$, $\alpha = 0.5$, $h_1 = -0.5$, $h_2 = -0.2$.

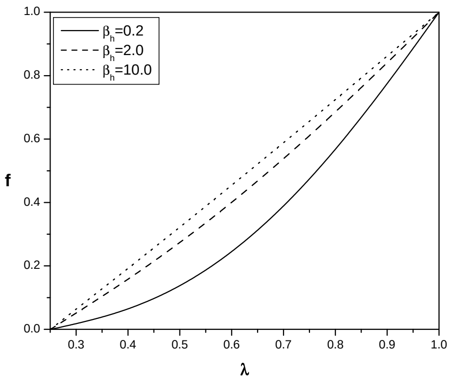

Fig. 5. Effect of β_h on f at $\beta_i = 0.2$, $\alpha = 0.5$, $Ha = 20$, $h_1 = -0.5$, $h_2 = -0.2$.

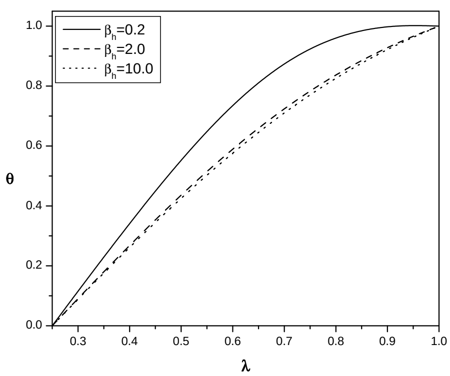

Fig. 6. Effect of β_h on θ at $\beta_i = 0.2$, $\alpha = 0.5$, $Ha = 20$, $h_1 = -0.5$, $h_2 = -0.2$.

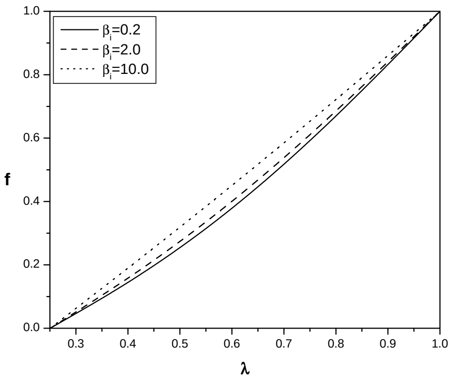

Fig. 7. Effect of β_i on f at $\beta_h = 0.2$, $\alpha = 0.5$, $Ha = 20$, $h_1 = -0.5$, $h_2 = -0.2$.

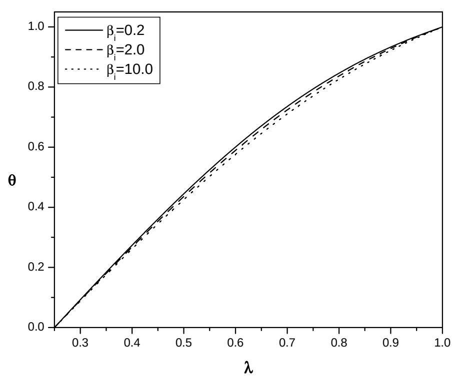

Fig. 8. Effect of β_i on θ at $\beta_h = 0.2$, $\alpha = 0.5$, $Ha = 20$, $h_1 = -0.5$, $h_2 = -0.2$.

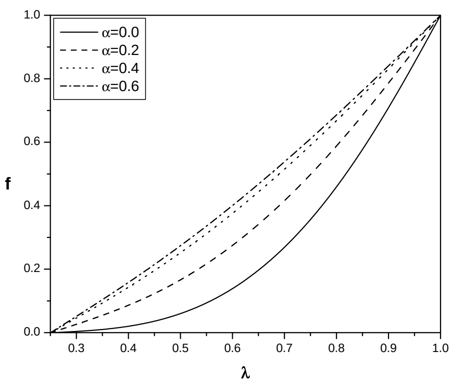

Fig. 9. Effect of (α) on f at $\beta_h = 2.0$, $\beta_i = 2.0$, $\alpha = 0.5$, $Ha = 20$, $h_1 = -0.5$, $h_2 = -0.2$.

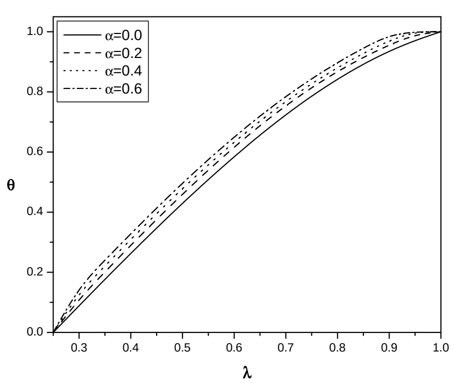

Fig. 10. Effect of (α) on θ at $\beta_h = 2.0$, $\beta_i = 2.0$, $\alpha = 0.5$, $Ha = 20$, $h_1 = -0.5$, $h_2 = -0.2$.

It is seen from Figs. 5 to 6 that, the variation of velocity $f(\lambda)$ and temperature $\theta(\lambda)$ for several values of β_h. We see that the dimensionless velocity $f(\lambda)$ increase by increasing β_h. Figure 4 shows that the temperature $\theta(\lambda)$ decreases as β_h increases. The inclusion of Hall parameter decreases the resistive force imposed by the magnetic field due to its effect in reducing the effective conductivity. Hence, the velocity component $f(\lambda)$ increases as the Hall parameter increases and the temperature $\theta(\lambda)$ decreases as β_h increases.

Figures 7 to 8 represent the effect of the Ion-slip parameter β_i on $f(\lambda)$ and $\theta(\lambda)$. It can be seen from these figures that the velocity $f(\lambda)$ increase with an increase in the parameter β_i. The temperature $\theta(\lambda)$ decreases as β_i increases. As β_i increases the effective conductivity also increases, in turn, decreases the damping force on the velocity component in the direction of the flow and hence the velocity component in the flow direction increases.

Figures 9 to 10 indicate the effect of the couple stress fluid parameter α on $f(\lambda)$ and $\theta(\lambda)$. As the couple stress fluid parameter α increases, the velocity increases. It is also clear that the temperature $\theta(\lambda)$ increases with an increase in α. Thus, the presence of couple stresses in the fluid increases the velocity and temperature.

5 Conclusions

In this paper, the Hall and Ion-slip effects on fully developed electrically conducting couple stress fluid flow between two concentric cylinders has been studied. Using similarity transformations, the governing equations have been transformed into non-linear ordinary differential equations. The similarity solutions are obtained numerically applying HAM [23]. From the present study wee see that the the flow fields are appreciably influenced by Magnetic parameter, Hall and Ion-slip effects. Also, it is noticed that the presence of couple stresses in the fluid increases the velocity and temperature.

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