# Analysis of the flow and heat transfer characteristics for MHD free convection in an enclosure with a heated obstacle

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**Abstract.** Finite element method based on Galerkin weighted Residual approach is used to solve two-dimensional governing mass, momentum and energy equations for steady state, natural convection flow in presence of magnetic field inside a square enclosure. The cavity consists of three adiabatic walls and one constantly heated wall. A uniformly heated circular solid body is located at the centre of the enclosure. The aim of this study is to describe the effects of MHD on the flow and thermal fields in presence of such heated obstacle. The investigations are conducted for different values of Rayleigh number ($Ra$) and Hartmann number ($Ha$). Various characteristics of streamlines, isotherms and heat transfer rate in terms of the average Nusselt number ($Nu$) are presented for different parameters. The effect of physical parameter ($D$) is also shown here. The results indicate that the flow pattern and temperature field are significantly dependent on the above mentioned parameters.

**Keywords:** MHD, free convection, heated obstacle, enclosure, finite element method.

## Nomenclature

| | | | |
|---|---|---|---|
| $B_0$ | applied magnetic field strength | $Pr$ | Prandtl number |
| $C_p$ | specific heat at constant pressure | $Ra$ | Rayleigh number |
| $d$ | diameter of the heated obstacle | $T$ | temperature of the fluid |
| $D$ | non dimensional diameter | $T_c$ | constant temperature of the left wall |
| $g$ | acceleration due to gravity | $T_{heat}$ | temperature of the solid obstacle |
| $Ha$ | Hartmann number | $u, v$ | velocity component along $x, y$-direction |
| $k$ | thermal conductivity of fluid | $U, V$ | dimensionless velocity component along $X, Y$-direction |
| $L$ | length of the enclosure | $x, y$ | Cartesian coordinates |
| $Nu$ | Nusselt number | $X, Y$ | dimensionless Cartesian coordinates |
| $p$ | pressure of the fluid | | |
| $P$ | non dimensional pressure of the fluid | | |

**Greek symbols**

$\alpha$ thermal diffusivity
$\beta$ coefficient of thermal expansion
$\theta$ dimensionless temperature
$\mu$ dynamic viscosity of the fluid
$\nu$ kinematic viscosity of the fluid
$\rho$ density of the fluid
$\sigma$ electrical conductivity of the fluid

## 1 Introduction

The influence of the magnetic field on the convective heat transfer and the natural convection flow of the fluid are of paramount importance in engineering. Several numerical and experimental methods have been developed to investigate flow characteristics inside cavities with and without obstacle because these geometries have practical engineering and industrial applications, such as in the design of solar collectors, thermal design of building, air conditioning, cooling of electronic devices, furnaces, lubrication technologies, chemical processing equipment, drying technologies etc. Many authors have recently studied heat transfer in enclosures with partitions, fins and block which influence the convection flow phenomenon. One of the novelties of magnetohydrodynamic (MHD) is that a gas can have a free surface, not constrained by a rigid wall and prone to waves. A related application is the use of MHD acceleration to shoot plasma into fusion devices or to produce high energy wind tunnels for simulating hypersonic flight.

House et al. [1] studied the effect of a centered, square, heat conducting body on natural convection in a vertical enclosure. They showed that heat transfer across the cavity enhanced or reduced by a body with a thermal conductivity ratio less or greater than unity. Garandet et al. [2] analyzed the buoyancy driven convection in a rectangular enclosure with a transverse magnetic field. The geometry considered in the numerical study of Oh et al. [3] was that the conducting body generated heat within the cavity. Under these situations, it was shown that the flow was driven by a temperature difference across the cavity and a temperature difference caused by the heat-generating source. Roychowdhury et al. [4] analyzed the natural convective flow and heat transfer features for a heated cylinder placed in a square enclosure with different thermal boundary conditions. Natural convection in a horizontal layer of fluid with a periodic array of square cylinder in the interior were conducted by Ha et al. [5], in which they concluded that the transition of the flow from quasi-steady up to unsteady convection depended on the presence of bodies and aspect ratio effect of the cell. Lee et al. [6] considered the problem of natural convection in a horizontal enclosure with a square body. Braga and de Lemos [7] investigated steady laminar natural convection within a square cavity filled with a fixed volume of conducting solid material consisting of either circular or square obstacles. They used finite volume method with a collocated grid to solve governing equations. They found that the average Nusselt number for cylindrical rods was slightly lower than those for square rods. Lee and Ha [8] considered a numerical study of natural convection in a horizontal enclosure with a conducting body. Natural convective heat transfer in square enclosures heated from below was investigated by Calcagni et al. [9]. Sarris et al. [10] studied MHD natural convection in a laterally and volumetrically heated square cavity. They concluded that the usual damping effect of increasing Hartmann number was not found to be straightforward connected with the resulting flow patterns. Roy and Basak [11] analyzed finite element method of natural convection flows in a square cavity with non-uniformly heated wall(s).

Comparatively little work has been reported on MHD free convection flow in an enclosure. In the light of the above literature, it has been pointed out that there is no significant information about MHD free convection processes when a heated circular body exists at the centre of the cavity. The present study addresses the effects of magnetic field which may increase or decrease the heat transfer on natural convection in a square cavity with a heated circular solid block. Numerical solutions are obtained over a wide range of Rayleigh number, Hartmann number and diameter of the heated body. The numerical results are presented graphically in terms of streamlines and isothermal lines. Finally the average Nusselt number of the fluid in the cavity is calculated.

## 2 Physical configuration

A schematic diagram of the system considered in the present study is shown in Fig. 1. The system consists of a square cavity with sides of length $L$ and a heated circular solid obstacle of diameter $d$ is located at the centre of the enclosure. A Cartesian co-ordinate system is used with origin at the lower left corner of the computational domain. The left wall of the cavity is considered at a constant temperature $T_c$ while the other three walls are kept adiabatic. The uniform temperature of the block is assumed to be $T_{heat}$. Here $T_c$ is much less than $T_{heat}$. A magnetic field of strength $B_0$ is applied horizontally normal to the side walls.

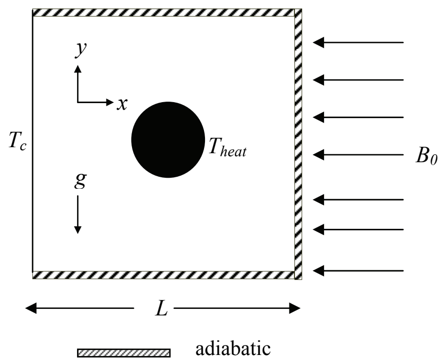



Fig. 1. Schematic diagram of the problem.

## 3 Mathematical formulation

In the present problem, it can be considered that the flow is steady, two-dimensional, laminar, incompressible and there is no viscous dissipation. The gravitational force acts in the vertically downward direction and radiation effect is neglected. The governing equations under Boussinesq approximation are as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + g\rho\beta(T - T_c) - \sigma B_0^2 v, \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right), \tag{4}$$

where $\alpha = k/\rho C_p$ is the thermal diffusivity of the fluid. The boundary conditions are

$$\begin{aligned}
&p = 0 \quad \text{at all boundaries},\\
&u(x,0) = u(x,L) = u(0,y) = u(L,y) = 0,\\
&v(x,0) = v(x,L) = v(0,y) = v(L,y) = 0,\\
&T(0,y) = T_c,\\
&\frac{\partial T(x,0)}{\partial y} = \frac{\partial T(x,L)}{\partial y} = \frac{\partial T(L,y)}{\partial x} = 0.
\end{aligned}$$

At the circular body surface $u(x,y) = v(x,y) = 0,\ T(x,y) = T_{heat}$.

The local Nusselt number at the heated circular body of the square enclosure is evaluated by the following expression in dimensional form as

$$Nu_{local} = hd/k$$

such local values have been further averaged over the entire heated surface to obtain the mean Nusselt number as follows

$$Nu = \frac{1}{L_s}\int_0^{L_s} Nu_{local}\,\mathrm{d}s,$$

where $L_s$, $h$ and $s$ are the length, the local convective heat transfer coefficient and coordinate along the circular surface respectively.

The above equations are non-dimensionalized by using the following dimensionless dependent and independent variables

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad D = \frac{d}{L},$$
$$U = \frac{uL}{\alpha}, \quad V = \frac{vL}{\alpha}, \quad P = \frac{pL^2}{\rho\alpha^2}, \quad \theta = \frac{T - T_c}{T_{heat} - T_c}.$$

After substitution of the above variables into the equations (1) to (4), we get the following non-dimensional equations as

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{5}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right), \tag{6}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + RaPr\theta - Ha^2 PrV, \tag{7}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right), \tag{8}$$

where $Pr = \frac{\nu}{\alpha}$ is Prandtl number, $Ra = \frac{g\beta(T_{heat}-T_c)L^3}{\nu\alpha}$ is Rayleigh number and $Ha$ is Hartmann number which is defined as $Ha^2 = \frac{\sigma B_0^2 L^3}{\mu}$.

The dimensionless boundary conditions under consideration are as follows:

$$\begin{aligned}
&P = 0 \quad \text{at all boundaries},\\
&U(X,0) = U(X,1) = U(0,Y) = U(1,Y) = 0,\\
&V(X,0) = V(X,1) = V(0,Y) = V(1,Y) = 0,\\
&\theta(0,Y) = 0,\\
&\frac{\partial\theta(X,0)}{\partial Y} = \frac{\partial\theta(X,1)}{\partial Y} = \frac{\partial\theta(1,Y)}{\partial X} = 0.
\end{aligned}$$

At the circular body surface $U(X,Y) = V(X,Y) = 0,\ \theta(X,Y) = 1$.

The average Nusselt number at the body of the enclosure may be expressed as

$$Nu = -\frac{1}{L_s}\int_0^{L_s/L} \frac{\partial\theta}{\partial n}\,\mathrm{d}S,$$

where $\frac{\partial\theta}{\partial n} = \sqrt{(\frac{\partial\theta}{\partial X})^2 + (\frac{\partial\theta}{\partial Y})^2}$ and $S$ is the non-dimensional coordinate along the circular surface.

## 4 Numerical technique

The numerical procedure used in this work is based on the Galerkin weighted residual method of finite element formulation. The application of this technique is well described by Taylor and Hood [12] and Dechaumphai [13]. In this method, the solution domain is discretized into finite element meshes, which are composed of non-uniform triangular elements. Then the nonlinear governing partial differential equations (i.e., mass, momentum and energy equations) are transferred into a system of integral equations by applying

Galerkin weighted residual method. The integration involved in each term of these equations is performed by using Gauss's quadrature method. The nonlinear algebraic equations so obtained are modified by imposition of boundary conditions. These modified nonlinear equations are transferred into linear algebraic equations by using Newton's method. Finally, these linear equations are solved by using triangular factorization method.

## 5 Results and discussion

Finite element simulation is applied to perform the analysis of laminar free convection heat transfer and fluid flow in a square cavity with a horizontal circular heated obstacle. Effects of the parameters such as Rayleigh number ($Ra$), Hartmann number ($Ha$) and diameter ($D$) on heat transfer and fluid flow inside the cavity have been studied. We have presented the results in two sections. The first section has focused on flow and temperature fields, which contains streamlines and isotherms for the different cases. Heat transfer including average Nusselt number at the heated body has been discussed in the following section. The ranges of $Ra$, $Ha$ and $D$ for this investigation vary from $10^3$ to $10^6$, $0$ to $70$ and $0.15$ to $0.5$ respectively while the Prandtl number is kept fixed at $Pr = 0.7$.

The influence of Rayleigh number *Ra* (from $10^3$ to $10^6$) on streamlines as well as isotherms for the present configuration at $Ha = 50$, $Pr = 0.7$, $D = 0.25$ has been demonstrated in Fig. 2. The flow with $Ra = 10^3$ has been affected by the buoyancy force, thus creating a vortex near the left side of the heated body. This region increases with increasing Rayleigh number as shown in Fig. 2(a). For $Ra = 10^6$, the size of the existing recirculation region becomes larger and another vortex is developed at the right top corner of the cavity. Fig. 2(b) illustrates the temperature field in the flow region. The high temperature region remains near the left side of the circular block and the isothermal lines are linear and parallel to the left vertical wall for $Ra = 10^3$. These lines become more curved from the left top corner because of growing $Ra$. The isothermal lines concentrate near the bottom of the heated body and the top corner of the left wall for larger values of $Ra$. Consequently they occupy almost the whole region of the enclosure.

Fig. 3 shows the effect of Hartmann number $Ha$ (from 0 to 70) on flow and thermal field at $Ra = 10^5$, $Pr = 0.7$, $D = 0.25$. In the absence of magnetic field, the streamlines consist of a primary recirculation cell including the heated body and a secondary eddy at the top right corner. The smaller vortex losses its strength and finally is disappeared with rising $Ha$. The corresponding temperature field shows that a thermal plume rising from the heated obstacle changes its direction from left to right, the concentrated region becomes less compressed and the isothermal lines are less bend from the left top corner due to the elevating Hartmann number.

The effect of diameter $D$ on the flow field is depicted in Fig. 4(a) where $Ra = 10^5$, $Pr = 0.7$ and $Ha = 50$. The streamlines contain two rotating cells at $D = 0.15$. The right top one is small which is vanished, the left one covers almost the whole cavity including the block and it contains a small vortex which is increased by size due to the rising values of $D$. For $D = 0.50$, the existing recirculation region becomes larger and two inner vortices are developed. Fig. 4(b) illustrates the temperature field in the flow region. The thermal field becomes less compressed at the left side of the cavity for decreasing $D$.

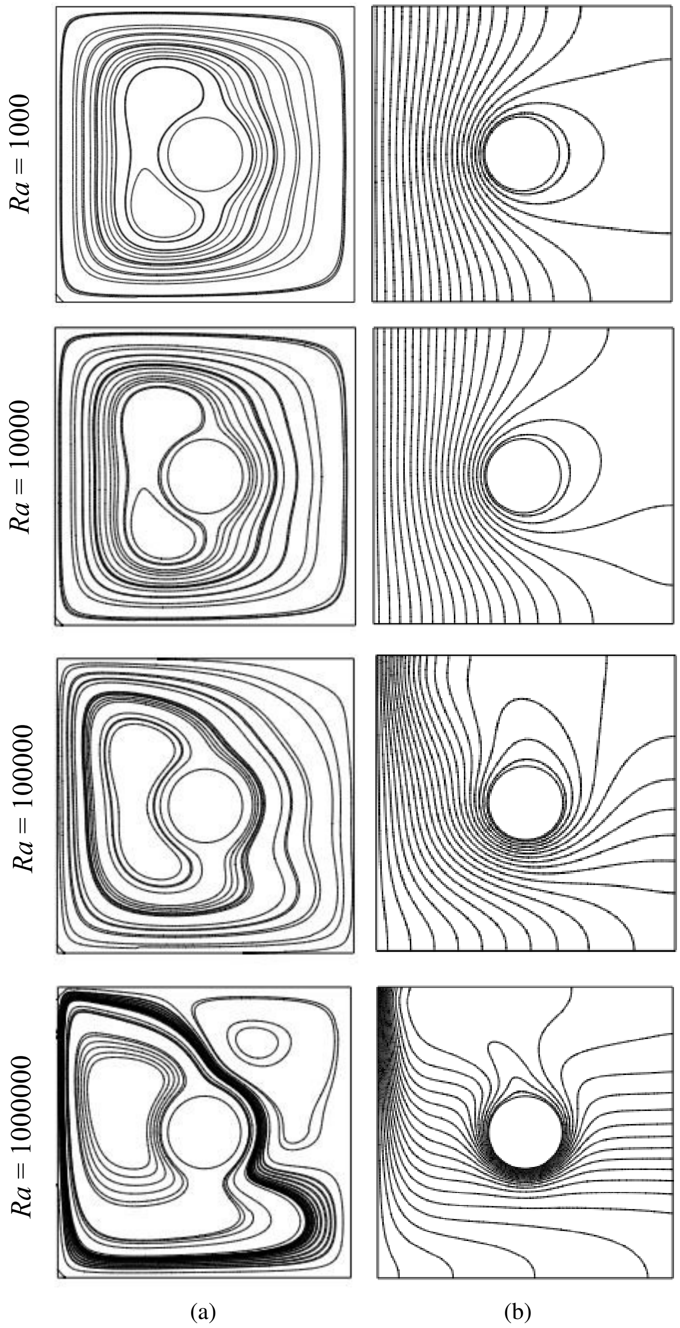



Fig. 2. (a) Streamlines and (b) isotherms for various $Ra$ with $Ha = 50$, $D = 0.25$ and $Pr = 0.7$.

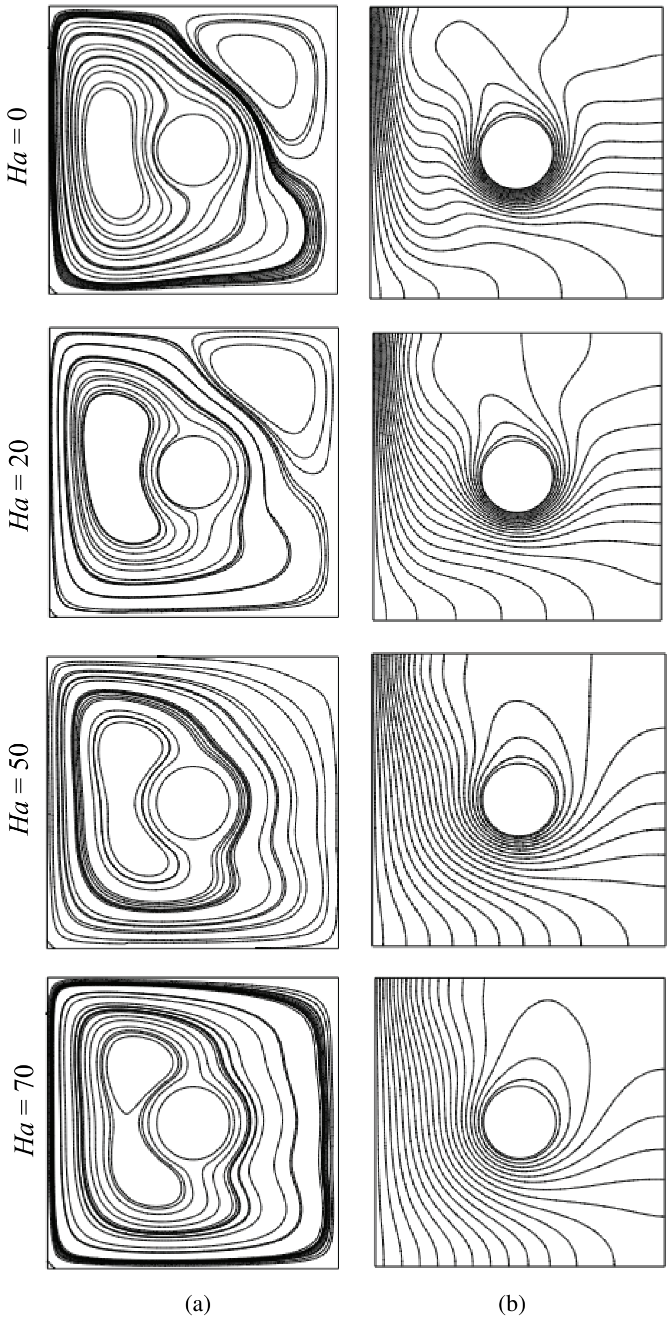



Fig. 3. (a) Streamlines and (b) isotherms for various $Ha$ with $Ra = 10^5$, $D = 0.25$ and $Pr = 0.7$.

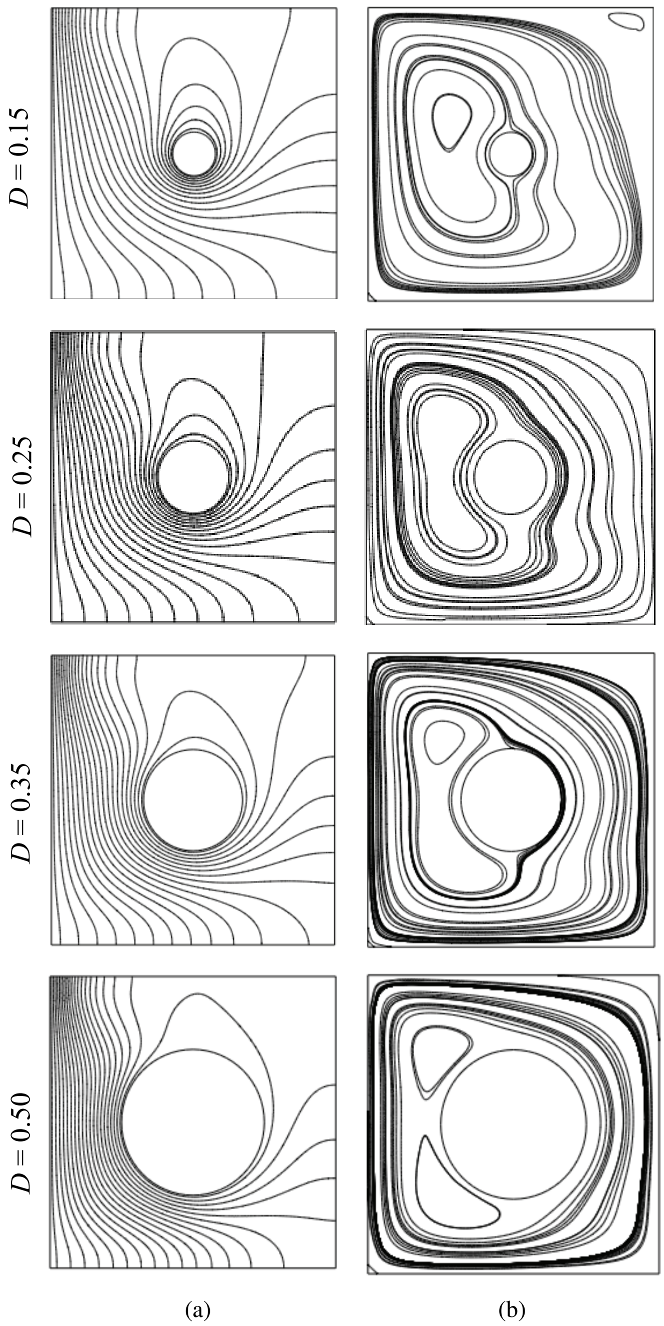
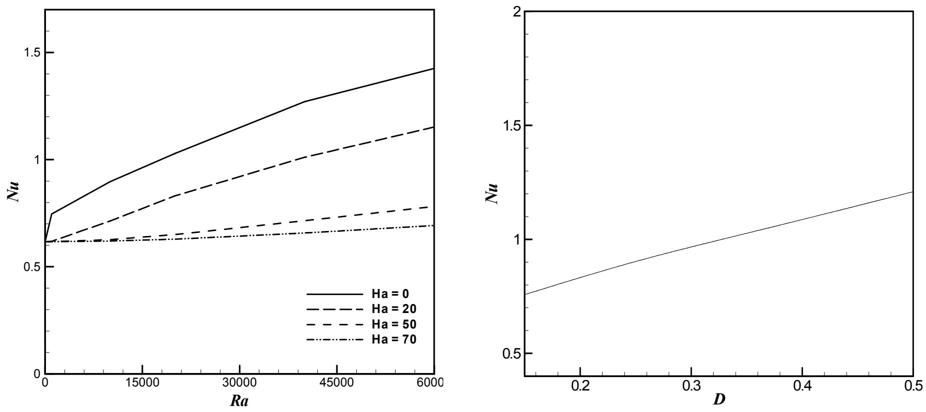



Fig. 4. (a) Streamlines and (b) isotherms for various $D$ with $Ra = 10^5$, $Ha = 50$ and $Pr = 0.7$.

In order to evaluate how the presence of magnetic field affects the heat transfer rate along the heated surface, the average Nusselt number is plotted as a function of Rayleigh number as shown in Fig. 5(a) for four different Hartmann number ($Ha = 0, 20, 50$ and $70$) while $Pr = 0.7$ and $D = 0.25$. It is observed that $Nu$ rises with increasing $Ra$ and decreasing $Ha$. The maximum heat transfer rate is obtained for the lowest $Ha$ and the highest $Ra$, because the magnetic field tends to retard the motion.

Also Fig. 5(b) represents the average Nusselt number as a function of diameter $D$ while $Pr = 0.7$, $Ha = 50$ and $Ra = 10^5$. Elevation in $D$ rises the rate of heat transfer. $Nu$ is found to be maximum at the largest value of $D$ due to the fact that larger surface is capable to transfer more heat.




(a) (b)

Fig. 5. (a) Effect of $Ra$ on average Nusselt number $Nu$ for different $Ha$ while $Pr = 0.7$ and $D = 0.25$; (b) effect of $D$ on average Nusselt number $Nu$ with $Ra = 10^5$ and $Pr = 0.7$.

## 6 Conclusion

A finite element method is used to make the present investigation for steady-state, incompressible, laminar and MHD free convection flow in a cavity with a heated body. The major conclusions have been drawn as follows:

- The free convection parameter $Ra$ has a significant effect on the flow and temperature fields. Buoyancy-induced vortex in the streamlines is increased and thermal layer near the heated surface becomes thick with increasing $Ra$.

- The influence of Magnetic parameter $Ha$ on streamlines and isotherms are remarkable. The eddies in the streamlines are reduced and the thermal current surrounding the hot body is thin with elevating $Ha$.

- The diameter of the body has a significant effect on the flow and temperature fields. The recirculation cell in the streamlines is enhanced and the isothermal lines near the heated surface become denser for higher values of $D$.
- The average Nusselt number $Nu$ at the circular body surface is enhanced for larger $Ra$ and $D$ where as devalued with growing $Ha$.

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