Soret and Dufour effects on mixed convection in a non-Darcy porous medium saturated with micropolar fluid

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Abstract. In this paper, the Soret and Dufour effects on the steady, laminar mixed convection heat and mass transfer along a semi-infinite vertical plate embedded in a non-Darcy porous medium saturated with micropolar fluid are studied. The governing partial differential equations are transformed into ordinary differential equations. The local similarity solutions of the transformed dimensionless equations for the flow, microrotation, heat and mass transfer characteristics are evaluated using Keller-box method. Numerical results are presented in the form of velocity, microrotation, temperature and concentration profiles within the boundary layer for different parameters entering into the analysis. Also the effects of the pertinent parameters on the local skin friction coefficient and rates of heat and mass transfer in terms of the local Nusselt and Sherwood numbers are also discussed.

Keywords: mixed convection, non-Darcy porous medium, micropolar fluid, Soret and Dufour effects.

1 Introduction

A situation where both the forced and free convection effects are of comparable order is called mixed or combined convection. The analysis of mixed convection boundary layer flow along a vertical plate embedded in a fluid saturated porous media has received considerable theoretical and practical interest. The phenomenon of mixed convection occurs in many technical and industrial problems such as electronic devices cooled by fans, nuclear reactors cooled during an emergency shutdown, a heat exchanger placed in a low-velocity environment, solar collectors and so on. Several authors have studied the problem of mixed convection about different surface geometries. The analysis of convective transport in a porous medium with the inclusion of non-Darcian effects has also been a matter of study in recent years. The inertia effect is expected to be important at a higher flow rate and it can be accounted for through the addition of a velocity squared term in the momentum equation, which is known as the Forchheimer's extension of the

Darcy's law. A detailed review of convective heat transfer in Darcy and non-Darcy porous medium can be found in the book by Nield and Bejan [1].

When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of a more intricate nature. It has been observed that an energy flux can be generated not only by temperature gradients but also by concentration gradients. The energy flux caused by a concentration gradient is termed the diffusion-thermo (Dufour) effect. On the other hand, mass fluxes can also be created by temperature gradients and this embodies the thermal-diffusion (Soret) effect. In most of the studies related to heat and mass transfer process, Soret and Dufour effects are neglected on the basis that they are of a smaller order of magnitude than the effects described by Fourier's and Fick's laws. But these effects are considered as second order phenomena and may become significant in areas such as hydrology, petrology, geosciences, etc. The Soret effect, for instance, has been utilized for isotope separation and in mixture between gases with very light molecular weight and of medium molecular weight. The Dufour effect was recently found to be of order of considerable magnitude so that it cannot be neglected (Eckert and Drake [2]). Dursunkaya and Worek [3] studied diffusion-thermo and thermal-diffusion effects in transient and steady natural convection from a vertical surface, whereas Kafoussias and Williams [4] presented the same effects on mixed convective and mass transfer steady laminar boundary layer flow over a vertical flat plate with temperature dependent viscosity. Postelnicu [5] studied numerically the influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Both free and forced convection boundary layer flows with Soret and Dufour effects have been addressed by Abreu et al. [6]. Alam and Rahman [7] have investigated the Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction. Recently, the effect of Soret and Dufour parameters on free convection heat and mass transfers from a vertical surface in a doubly stratified Darcian porous medium has been reported by Lakshmi Narayana and Murthy [8].

The study of non-Newtonian fluid flows has gained much attention from the researchers because of its applications in biology, physiology, technology and industry. In addition, the effects of heat and mass transfer in non-Newtonian fluid also have great importance in engineering applications like the thermal design of industrial equipment dealing with molten plastics, polymeric liquids, foodstuffs, or slurries. Several investigators have extended many of the available convection heat and mass transfer problems to include the non Newtonian effects. Many of the non-Newtonian fluid models describe the nonlinear relationship between stress and the rate of strain. But the micropolar fluid model introduced by Eringen [9] exhibits some microscopic effects arising from the local structure and micro motion of the fluid elements. Further, the micropolar fluid can sustain couple stresses and include classical Newtonian fluid as a special case. The model of micropolar fluid represents fluids consisting of rigid, randomly oriented (or spherical) particles suspended in a viscous medium where the deformation of the particles is ignored. Micropolar fluids have been shown to accurately simulate the flow characteristics of polymeric additives, geomorphological sediments, colloidal suspensions, haematological suspensions, liquid crystals, lubricants etc. The mathematical theory of equations of micropolar fluids and applications of these fluids in the theory of lubrication and porous media are presented by Lukaszewicz [10]. The heat and mass transfer in micropolar fluids is also important in the context of chemical engineering, aerospace engineering and also industrial manufacturing processes. The problem of mixed convection heat and mass transfer in the boundary layer flow along a vertical surface submerged in a micropolar fluid has been studied by a number of investigators. Ahmadi [11] studied the boundary layer flow of a micropolar fluid over a semi-infinite plate. Laminar mixed convection boundary layer flow of a micropolar fluid from an isothermal vertical flat plate has been considered by Jena and Mathur [12]. Asymptotic boundary layer solutions are presented to study the combined convection from a vertical semi-infinite plate to a micropolar fluid by Gorla et al. [13]. Tian-Yih Wang [14] examined the effect of wall conduction on laminar mixed convection heat transfer of micropolar fluids along a vertical flat plate. Although the Soret and Dufour effects of the medium on the heat and mass transfer in a micropolar fluid are important, very little work has been reported in the literature. Beg et al. [15] discussed the steady double-diffusive free convective heat and mass transfer of a chemically-reacting micropolar fluid flowing through a Darcian porous regime adjacent to a vertical stretching plane. Beg et al. [16] analyzed the two dimensional coupled heat and mass transfer of an incompressible micropolar fluid past a moving vertical surface embedded in a Darcy–Forchheimer porous medium in the presence of Soret and Dufour effects. A mathematical model for the steady thermal convection heat and mass transfer in a micropolar fluid saturated Darcian porous medium in the presence of Dufour and Soret effects and viscous heating is presented by Rawat and Bhargava [17].

The present paper deals with Soret and Dufour effects on the mixed convection from a semi-infinite vertical plate embedded in a stable, non-Darcy micropolar fluid with uniform wall temperature and concentration. The Keller-box method given in Cebeci and Bradshaw [18] is employed to solve the nonlinear system in the problem. The effects of micropolar parameter, non-Darcy parameter, X-location, Prandtl number, Soret and Dufour numbers are examined and are displayed through graphs. The results are compared with relevant results in the existing literature and are found to be in good agreement.

2 Mathematical formulation

Consider a steady, laminar, incompressible, two-dimensional mixed convective heat and mass transfer along a semi-infinite vertical plate embedded in a free stream of micropolar fluid saturated non-Darcy porous medium. The free stream velocity which is parallel to the vertical plate is u_∞, temperature is T_∞ and concentration is C_∞. Assume that the fluid and the porous medium have constant physical properties. The fluid flow is moderate and the permeability of the medium is low so that the Forchheimer flow model is applicable and the boundary effect is neglected. The fluid and the porous medium are in local thermodynamical equilibrium. Choose the coordinate system such that x-axis is along the vertical plate and y-axis normal to the plate. The physical model and coordinate system are shown in Fig. 1. The plate is maintained at uniform wall temperature and concentration T_w and C_w respectively. These values are assumed to be greater than the ambient temperature T_∞ and concentration C_∞ at any arbitrary reference point in the medium (inside the boundary layer). In addition, the Soret and Dufour effects are considered.

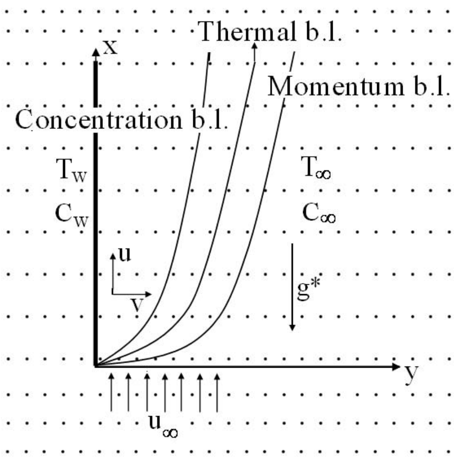


Fig. 1. Physical model and coordinate system.

Assuming that the Boussinesq and boundary-layer approximations hold and using the Darcy–Forchheimer model and Dupuit–Forchheimer relationship [1], the governing equations for the micropolar fluid are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\begin{aligned}&\frac{\rho}{\epsilon^2}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)\\ &\quad = \frac{\mu+\kappa}{\epsilon}\frac{\partial^2 u}{\partial y^2} + \kappa\frac{\partial \omega}{\partial y} + \rho g^*\big(\beta_T(T-T_\infty) + \beta_C(C-C_\infty)\big)\\ &\qquad + \frac{\mu}{K_p}(u_\infty - u) + \frac{\rho b}{K_p}\big(u_\infty^2 - u^2\big),\end{aligned} \tag{2}$$

$$\frac{\rho j}{\epsilon}\left(u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y}\right) = \gamma\frac{\partial^2 \omega}{\partial y^2} - \kappa\left(2\omega + \frac{1}{\epsilon}\frac{\partial u}{\partial y}\right), \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{DK_T}{C_s C_p}\frac{\partial^2 C}{\partial y^2}, \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + \frac{DK_T}{T_m}\frac{\partial^2 T}{\partial y^2}, \tag{5}$$

where u and v are Darcy velocity components in x and y directions respectively, ω is the component of microrotation whose direction of rotation lies in the xy-plane, T is the temperature, C is the concentration, g^* is the acceleration due to gravity, ρ is the density, b is the Forchheimer constant, K_p is the permeability, ϵ is the porosity, μ is the dynamic coefficient of viscosity, β_T is the coefficient of thermal expansion, β_C is the coefficient of solutal expansions, κ is the vortex viscosity, j is the micro-inertia density, γ is the spin-gradient viscosity, α is the effective thermal diffusivity, D is the effective solutal diffusivity of the medium, C_p is the specific heat capacity, C_s is the concentration susceptibility, T_m is the mean fluid temperature and K_T is the thermal diffusion ratio. The last two terms on the right hand side of Eq. (2) stand for the first-order (Darcy) resistance and second-order porous inertia resistance, respectively. The microrotation represents the rotation in an average sense of the rigid particles centered in a small volume element about the centroid of the element.

The boundary conditions are

$$u = 0, \quad v = 0, \quad \omega = 0, \quad T = T_w, \quad C = C_w \quad \text{at } y = 0, \tag{6a}$$
$$u = u_\infty, \quad \omega = 0, \quad T = T_\infty, \quad C = C_\infty \quad \text{as } y \to \infty, \tag{6b}$$

where the subscripts w and ∞ indicate the conditions at the wall and at the outer edge of the boundary layer respectively and k is the thermal conductivity of the fluid. The boundary condition $\omega = 0$ in Eq. (6a), represents the case of concentrated particle flows in which the micro-elements close to the wall are not able to rotate.

In view of the continuity Eq. (1), we introduce the stream function ψ by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}. \tag{7}$$

Substituting Eq. (7) in Eqs. (2)–(5) and then using the following local similarity transformations

$$\begin{gathered} \eta = \frac{y}{x} Re_x^{1/2}, \quad \psi = \nu Re_x^{1/2} f(\eta), \quad \omega = \frac{\nu}{x^2} Re_x^{3/2} g(\eta), \\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \end{gathered} \tag{8}$$

where $Re_x = u_\infty x/\nu$ is the local Reynolds number, we get the following nonlinear system of differential equations.

$$\begin{aligned} &\frac{1}{\epsilon(1-N)} f''' + \frac{1}{2\epsilon^2} f f'' + \frac{N}{1-N} g' \\ &\quad + X\left[g_s\theta + g_c\phi + \frac{1}{DaRe}(1 - f') + \frac{Fs}{Da}\left(1 - f'^2\right)\right] = 0, \end{aligned} \tag{9}$$

$$\lambda g'' + \frac{1}{2\epsilon} f g' + \frac{1}{2\epsilon} f' g - \frac{N}{1-N} X \mathcal{J}\left(2g + \frac{1}{\epsilon} f''\right) = 0, \tag{10}$$

$$\frac{1}{Pr}\theta'' + \frac{1}{2}f\theta' + D_f\phi'' = 0, \tag{11}$$

$$\frac{1}{Sc}\phi'' + \frac{1}{2}f\phi' + S_r\theta'' = 0, \tag{12}$$

where the primes indicate partial differentiation with respect to η alone,

ν is the kinematic viscosity,
L is the characteristic length,
$X = x/L$ is the dimensionless coordinate along the plate,
$Gr = g^*\beta_T(T_w - T_\infty)L^3/\nu^2$ is the thermal Grashof number,
$Gc = g^*\beta_C(C_w - C_\infty)L^3/\nu^2$ is the solutal Grashof number,
$Re = u_\infty L/\nu$ is the Reynolds number,
$g_s = Gr/Re^2$ is the temperature buoyancy parameter,
$g_c = Gc/Re^2$ is the mass buoyancy parameter,
$Pr = \nu/\alpha$ is the Prandtl number,
$Sc = \nu/D$ is the Schmidt number,
$Da = K_p/L^2$ is the Darcy number,
$Fs = b/L$ is the Forchheimer number,
$\mathcal{J} = \nu L/(ju_\infty)$ is the micro-inertia density parameter,
$\lambda = \gamma/(j\rho\nu)$ is the spin-gradient viscosity parameter,
$N = \kappa/(\mu + \kappa)$ $(0 \le N < 1)$ is the Coupling number [19],
$D_f = DK_T(C_w - C_\infty)/(C_sC_p\nu(T_w - T_\infty))$ is the Dufour number,
$S_r = DK_T(T_w - T_\infty)/(T_m\nu(C_w - C_\infty))$ is the Soret number.

A close look at Eqs. (9) and (10) reveals that, in mixed convection due to micropolar fluid saturated non-Darcy porous medium, the velocity and angular momentum profiles are not similar because the x-coordinate cannot be eliminated from these equations. Although local non-similarity solutions have been found for some boundary layer flows dealing with viscous fluids, the technique is hard to extend to micropolar fluids. Thus, for ease of analysis, it was decided to proceed with finding local similarity solutions for the governing equation, Eqs. (9) and (12). Now, one can still study the effects of various parameters on different profiles at any given X-location.

Boundary conditions (6) in terms of f, g, θ and ϕ become

$$\eta = 0: \quad f(0) = 0, \quad f'(0) = 0, \quad g(0) = 0, \quad \theta(0) = 1, \quad \phi(0) = 1, \tag{13a}$$

$$\eta \to \infty: \quad f'(\infty) = 1, \quad g(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0. \tag{13b}$$

If $Da \to \infty$, $X = 1$, $\epsilon = 1$, $D_f = 0$ and $S_r = 0$, the problem reduces to mixed convection heat and mass transfer in a micropolar fluid without Soret and Dufour effects. In the limit, as $N \to 0$, the governing Eqs. (1)–(5) reduce to the corresponding equations for a mixed convection heat and mass transfer in a viscous fluids. Hence, the case of combined free-forced convective and mass transfer flow past a semi-infinite vertical plate of Kafoussias [20] can be obtained by taking $N = 0$, $Da \to \infty$, $\epsilon = 1$, $X = 1$, $D_f = 0$ and $S_r = 0$.

The wall shear stress, heat and mass transfers from the plate respectively are given by

$$\tau_w = \left[(\mu+\kappa)\frac{\partial u}{\partial y} + \kappa\omega\right]_{y=0}, \quad q_w = -k\left[\frac{\partial T}{\partial y}\right]_{y=0}, \quad q_m = -D\left[\frac{\partial C}{\partial y}\right]_{y=0} \tag{14}$$

The non-dimensional skin friction $C_f = 2\tau_w/\rho U_*^2$, the local Nusselt number $Nu_x = q_w x/(k(T_w - T_\infty))$ and local Sherwood number $Sh_x = q_m x/(D(C_w - C_\infty))$, where U_* is the characteristic velocity, are given by

$$C_f Re_x^{1/2} = \frac{2}{1-N} f''(0), \quad \frac{Nu_x}{Re_x^{1/2}} = -\theta'(0), \quad \frac{Sh_x}{Re_x^{1/2}} = -\phi'(0). \tag{15}$$

3 Results and discussions

The system of non-linear ordinary differential equations (9)–(12) together with the boundary conditions (13) are locally similar and solved numerically using Keller-box implicit method discussed in [18]. The method has the following four main steps:

1. Reduce the system of Eqs. (9) to (12) to a first order system;
2. Write the difference equations using central differences;
3. Linearize the resulting algebraic equations by Newton's method and write them in matrix-vector form;
4. Use the block-tridiagonal-elimination technique to solve the linear system.

This method has been proven to be adequate and give accurate results for boundary layer equations. A uniform grid was adopted, which is concentrated towards the wall. The calculations are repeated until some convergent criterion is satisfied and the calculations are stopped when $\delta f_0'' \le 10^{-8}$, $\delta g_0' \le 10^{-8}$, $\delta\theta_0' \le 10^{-8}$ and $\delta\phi_0' \le 10^{-8}$. In the present study, the boundary conditions for η at ∞ are replaced by a sufficiently large value of η where the velocity approaches one and microrotation, temperature and concentration approach zero. In order to see the effects of step size ($\Delta\eta$) we ran the code for our model with three different step sizes as $\Delta\eta = 0.001$, $\Delta\eta = 0.01$ and $\Delta\eta = 0.05$ and in each case we found very good agreement between them on different profiles. After some trials we imposed a maximal value of η at ∞ of 10 and a grid size of η as 0.01.

The dimensionless parameter $g_s = Gr/Re^2$ is used to represent the free, forced and combined (free-forced) convection regimes. The case $g_s \ll 1$ corresponds to pure forced convection, $g_s = 1$ corresponds to combined free-forced convection and $g_s \gg 1$ corresponds to pure free convection. As the mass Grashof number Gc is a measure of the buoyancy forces (due not to temperature but to concentration differences) to the viscous forces, the dimensionless parameter g_c has the same meaning as the parameter g_s. The dimensionless parameter g_s takes the values 0.1, 1 and 10 which correspond to three different flow regimes as already mentioned above. The corresponding parameter g_c takes the values 0.05, 0.10 and 0.20. The values of Soret number S_r and Dufour number D_f

are to be chosen in such a way that their product is constant according to their definition, provided that the mean temperature T_m is constant.

The coupling number N characterizes the coupling of linear and rotational motion arising from the micromotion of the fluid molecules. Hence N signifies the coupling between the Newtonian and rotational viscosities. With a large value of N the effect of microstructure becomes significant, whereas with a small value of N the individuality of the substructure is much less pronounced. As κ tends to zero, N also tends to zero, the micro-polarity is lost and the fluid behaves as non-polar fluid.

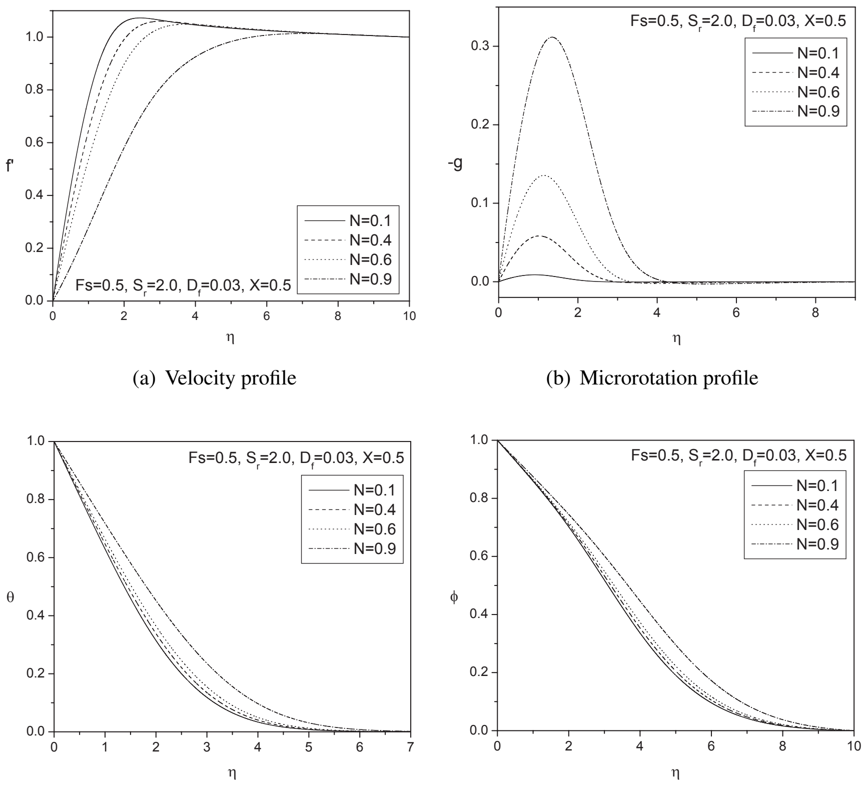
In the absence of coupling number N, Soret number S_r and Dufour number D_f with $Da \to \infty$, $X = 1$, $\epsilon = 1$, $\mathcal{J} = 0$, $\lambda = 0$, $Pr = 0.73$ and $Sc = 0.24$ for different values of buoyancy parameters g_s and g_c, the results have been compared with the case Kafoussias [20] and found that they are in good agreement, as shown in Table 1.

Table 1. Comparison of results for a vertical plate in viscous fluids without Soret and Dufour effects case [20].

		$f''(0)$		$-\theta'(0)$	
g_s	g_c	Kafoussias [20]	Present	Kafoussias [20]	Present
0.1	0.05	0.5538	0.5538	0.3296	0.3296
0.1	0.10	0.6317	0.6317	0.3404	0.3404
0.1	0.20	0.7776	0.7776	0.3589	0.3589
1.0	0.05	1.4452	1.4452	0.4129	0.4129
1.0	0.10	1.5007	1.5007	0.4179	0.4179
1.0	0.20	1.6096	1.6096	0.4274	0.4274
10.0	0.05	6.8389	6.8389	0.6449	0.6449
10.0	0.10	6.8715	6.8714	0.6461	0.6462
10.0	0.20	6.9366	6.9363	0.6487	0.6488

In the present study we have adopted the following default parameter values for the numerical computations: $Pr = 0.71$, $Sc = 0.22$, $Re = 200$, $\epsilon = 0.3$, $Da = 1.0$, $g_s = 1.0$, $g_c = 0.1$. The values $\mathcal{J} = 0.1$ and $\lambda = 1.0$ are chosen so as to satisfy the thermodynamic restrictions on the material parameters given by Eringen [9]. These values are used throughout the computations, unless otherwise indicated.

In Figs. 2(a)–2(d), the effects of the coupling number N on the dimensionless velocity, microrotation, temperature and concentration are presented for fixed values of Forchheimer, Soret and Dufour numbers and X-location. As N increases, it can be observed from Fig. 2(a) that the maximum velocity decreases in amplitude and the location of the maximum velocity moves farther away from the wall. Since $N \to 0$ corresponds to viscous fluid, the velocity in case of micropolar fluid is less compared to that of viscous fluid case. From Fig. 2(b), we observe that the microrotation is completely negative within the boundary layer. It is clear from Fig. 2(c) that the temperature increases with the increase of coupling number N. It can be seen from Fig. 2(d) that the concentration of the fluid increases with the increase of coupling number N. The temperature and concentration in case of micropolar fluids is more than that of the Newtonian fluid case.

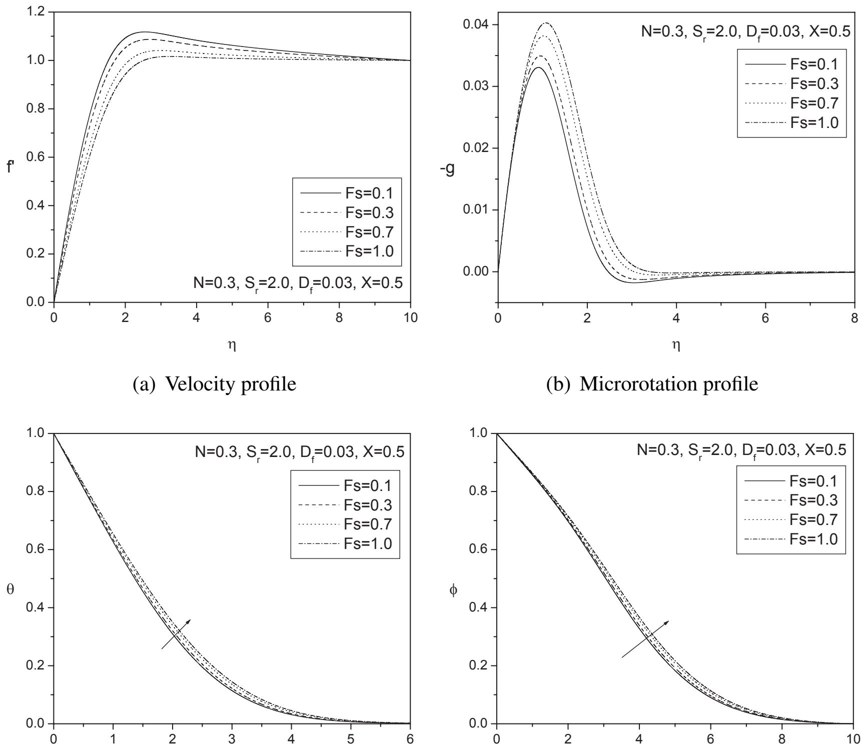


(a) Velocity profile (b) Microrotation profile

(c) Temperature profile (d) Concentration profile

Fig. 2. (a) Velocity, (b) microrotation, (c) temperature and (d) concentration profiles for various values of N.

The dimensionless velocity component for different values of Forchheimer number Fs with $N = 0.3$, $X = 0.5$, $S_r = 2.0$ and $D_f = 0.03$ is depicted in Fig. 3(a). It shows the effects of Forchheimer (inertial porous) number on the velocity. In the absence of Forchheimer number (i.e., when $Fs = 0$), the present investigation reduces to a mixed convection heat and mass transfer in a micropolar fluid saturated porous medium with Soret and Dufour effects. It is observed from Fig. 3(a) that velocity of the fluid decreases with increase in the value of the non-Darcy parameter Fs. The increase in non-Darcy parameter implies that the porous medium is offering more resistance to the fluid flow. This results in reduction of the velocity profile. From Fig. 3(b), it can be observed that the microrotation changes in sign from negative to positive within the boundary layer. The dimensionless temperature for different values of Forchheimer number Fs for $N = 0.3$, $X = 0.5$, $S_r = 2.0$ and $D_f = 0.03$, is displayed in Fig. 3(c). An increase in Forchheimer number Fs, increase temperature values, since as the fluid is decelerated, energy is dissipated as heat and serves to increase temperatures. As such the temperature is minimized for the lowest value of Fs and maximized for the highest value of Fs as shown in Fig. 3(c). Fig. 3(d) demonstrates the dimensionless concentration for different values of Forchheimer number with $N = 0.3$, $X = 0.5$, $S_r = 2.0$ and $D_f = 0.03$. It is clear that the concentration of the fluid increases with the increase of Forchheimer number. The increase in non-Darcy parameter reduces the intensity of the flow and increases the thermal and concentration boundary layer thickness.

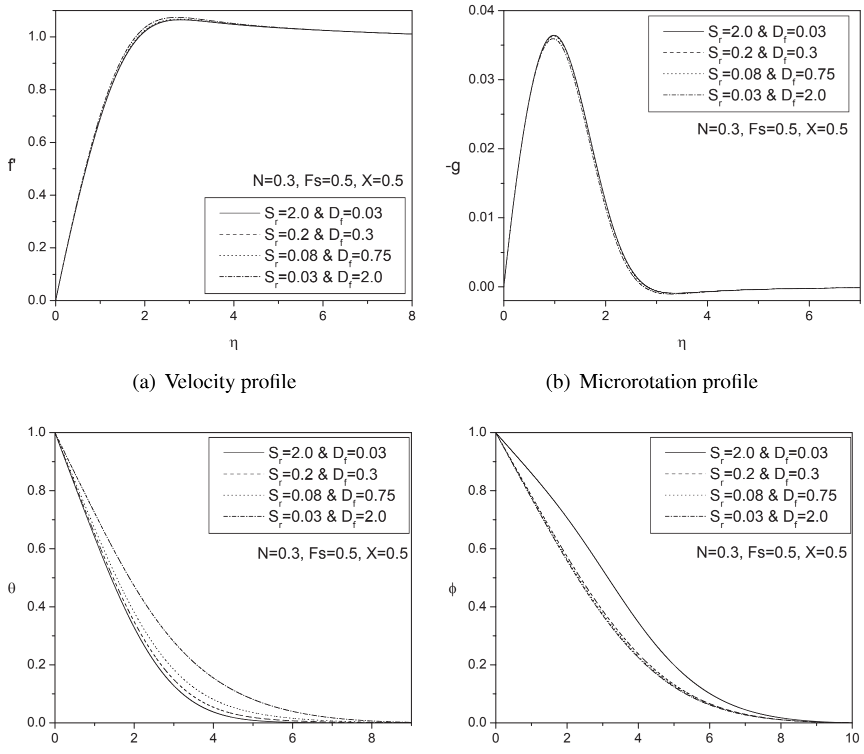


(a) Velocity profile (b) Microrotation profile

(c) Temperature profile (d) Concentration profile

Fig. 3. (a) Velocity, (b) microrotation, (c) temperature and (d) concentration profiles for various values of Fs.

Fig. 4(a) displays the non-dimensional velocity for different values of Soret number S_r and Dufour number D_f with fixed values of coupling number N, Forchheimer number Fs and X-location. It is observed that the velocity of the fluid increases with the increase of Dufour number (or decrease of Soret number). From Fig. 4(b), it can be noted that the microrotation changes sign from negative to positive within the boundary layer. The dimensionless temperature for different values of Soret number S_r and Dufour number D_f for $N = 0.3$, $Fs = 0.5$ and $X = 0.5$, is shown in Fig. 4(c). It is clear that the temperature of the fluid increases with the increase of Dufour number (or decrease of Soret number). Fig. 4(d) demonstrates the dimensionless concentration for different values of Soret number S_r and Dufour number D_f for $N = 0.3$, $Fs = 0.5$ and $X = 0.5$. It is seen that the concentration of the fluid decreases with increase of Dufour number (or decrease of Soret number).

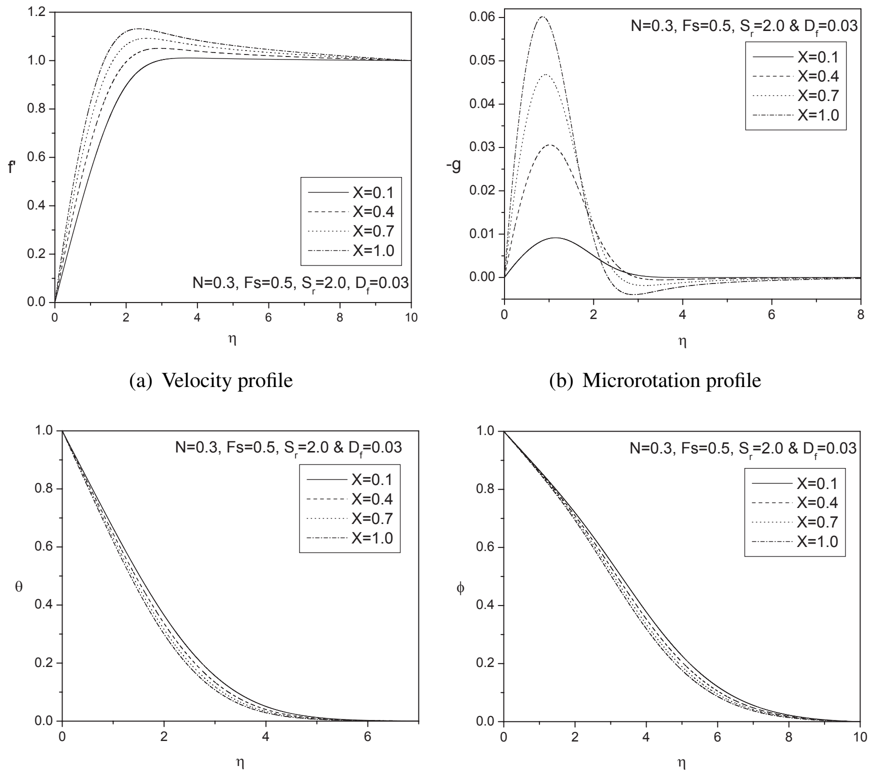


(a) Velocity profile (b) Microrotation profile

(c) Temperature profile (d) Concentration profile

Fig. 4. (a) Velocity, (b) microrotation, (c) temperature and (d) concentration profiles for various values of S_r and D_f.

In Figs. 5(a)–5(d), the effects of the X-location on the dimensionless velocity, microrotation, temperature and concentration are presented for fixed values of Coupling, Forchheimer, Soret and Dufour numbers. From the Fig. 5(a), it is noticed that the velocity increases with increase in the value of X in the momentum boundary layer. From Fig. 5(b), we observe that the microrotation is decreasing near the plate and increasing away from the plate within the boundary layer. It is clear from Fig. 5(c) that the thermal boundary layer thickness decreases with the increase of X. It can be seen from Fig. 5(d) that the solutal boundary layer thickness of the fluid decreases with the increase of X.



(a) Velocity profile

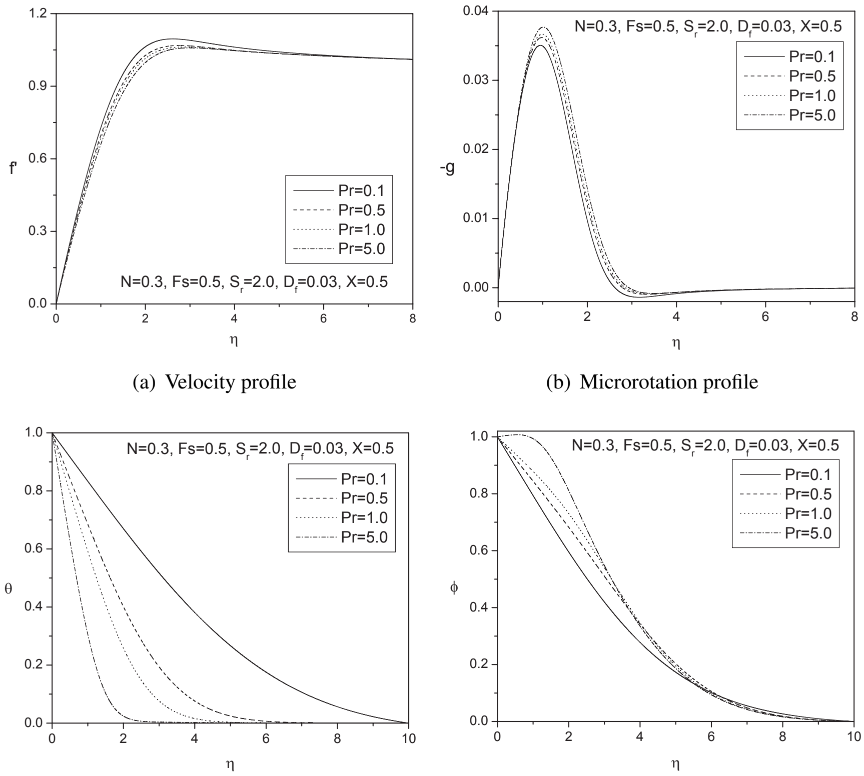
(b) Microrotation profile

(c) Temperature profile

(d) Concentration profile

Fig. 5. (a) Velocity, (b) microrotation, (c) temperature and (d) concentration profiles for various values of X-location.

The effects of the Pr on the dimensionless velocity, microrotation, temperature and concentration are depicted for fixed values of N, Fs, S_r and D_f and X-location in the Figs. 6(a)–6(d). Pr encapsulates the ratio of momentum diffusivity to thermal diffusivity. Larger Pr values imply a thinner thermal boundary layer thickness and more uniform temperature distributions across the boundary layer. Hence the thermal boundary layer will be much less in thickness than the hydrodynamic boundary layer. $Pr = 1$ implies that the thermal and velocity boundary layers are approximately equal [21]. Smaller Pr fluids have higher thermal conductivities so that heat can diffuse away from the vertical plate faster than for higher Pr fluids (thicker boundary layers). As Pr enhances, it can be seen from Fig. 6(a) that the velocity reduces since the fluid is increasingly viscous as Pr rises. Hence the micropolar fluid is decelerated with a rise in Pr. From Fig. 6(b), we observe that the microrotation is changes the sign from negative to positive within the boundary layer. Fig. 6(c) indicates that a rise in Pr substantially reduces the temperature in the micropolar fluid saturated porous regime. It can be found from Fig. 5(d) that the solutal boundary layer thickness of the fluid enhances with the enhance of Pr near the plate and opposite behavior observed far away from the plate.



(a) Velocity profile (b) Microrotation profile

(c) Temperature profile (d) Concentration profile

Fig. 6. (a) Velocity, (b) microrotation, (c) temperature and (d) concentration profiles for various values of Pr.

The variations of $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ which are proportional to the local skin-friction coefficient, rate of heat and mass transfers are shown in Table 2 for different values of the coupling number with fixed Forchheimer, Soret and Dufour numbers and X-location. From this table, it is observed that the the value of $f''(0)$ decreases with the increasing values of coupling number. Also, these values(local viscous drag) are higher for the Newtonian fluid ($N = 0$) than the micropolar fluid ($N \neq 0$). The heat and mass transfer rates decrease with the increasing values of coupling number. From this data, it is obvious that micropolar fluids presented lower heat and mass transfer values than those of Newtonian fluids. Since the skin-friction coefficient as well as heat and mass transfers are lower in the micropolar fluid comparing to the Newtonian fluid, which may be beneficial in flow, temperature and concentration control of polymer processing. Also, it can be observed from this table that, for fixed values of N, X-location, S_r and D_f, the velocity, heat and mass transfer coefficients are reducing with the increasing values of Forchheimer number Fs. Hence, the inertial effects in micropolar fluid saturated non-Darcy porous medium reduce the skin friction and heat and mass transfer coefficients. It is observed from this table that the skin friction and heat and mass transfer coefficients enhances as X increases with fixed values of N, Fs, S_r and D_f. The effect of increasing the value of Pr is to decrease the skin friction coefficient and mass transfer rate but increase heat transfer rate with fixed values of N, Fs, X, S_r and D_f. Finally, the effects of Dufour and Soret number on the local skin-friction coefficient and the rate of heat and mass transfer are shown in this table. The behavior of these parameters is self-evident from the Table 2 and hence are not discussed for brevity.

Table 2. Effects of skin friction, heat and mass transfer coefficients for varying values of coupling, Forchheimer, Prandtl, Soret and Dufour numbers, X-location.

N	Fs	Sr	Df	X	Pr	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.1	0.5	2.0	0.03	0.5	0.71	0.93617	0.37505	0.13923
0.2	0.5	2.0	0.03	0.5	0.71	0.87725	0.36979	0.13880
0.3	0.5	2.0	0.03	0.5	0.71	0.81437	0.36376	0.13825
0.4	0.5	2.0	0.03	0.5	0.71	0.74654	0.35675	0.13752
0.5	0.5	2.0	0.03	0.5	0.71	0.67227	0.34845	0.13654
0.6	0.5	2.0	0.03	0.5	0.71	0.58932	0.33835	0.13518
0.7	0.5	2.0	0.03	0.5	0.71	0.49397	0.32565	0.13323
0.8	0.5	2.0	0.03	0.5	0.71	0.37997	0.30874	0.13028
0.9	0.5	2.0	0.03	0.5	0.71	0.23663	0.28268	0.12499
0.3	0.1	2.0	0.03	0.5	0.71	0.68269	0.34864	0.13510
0.3	0.3	2.0	0.03	0.5	0.71	0.75113	0.35671	0.13676
0.3	0.7	2.0	0.03	0.5	0.71	0.87326	0.37000	0.13960
0.3	1.0	2.0	0.03	0.5	0.71	0.95477	0.37815	0.14141
0.3	0.5	2.0	0.03	0.1	0.71	0.57607	0.33802	0.13397
0.3	0.5	2.0	0.03	0.4	0.71	0.75915	0.33820	0.13730
0.3	0.5	2.0	0.03	0.7	0.71	0.91825	0.37366	0.13999
0.3	0.5	2.0	0.03	1.0	0.71	1.06085	0.38620	0.14225
0.3	0.5	2.0	0.03	0.5	0.1	0.85886	0.16720	0.20700
0.3	0.5	2.0	0.03	0.5	0.5	0.82232	0.31732	0.15527
0.3	0.5	2.0	0.03	0.5	1.0	0.80689	0.41451	0.11902
0.3	0.5	2.0	0.03	0.5	5.0	0.77567	0.75023	0.01801
0.3	0.5	2.0	0.03	0.5	0.71	0.81437	0.36376	0.13825
0.3	0.5	1.6	0.0375	0.5	0.71	0.81408	0.36339	0.15590
0.3	0.5	1.2	0.05	0.5	0.71	0.81383	0.36280	0.17355
0.3	0.5	1.0	0.06	0.5	0.71	0.81374	0.36234	0.18237
0.3	0.5	0.8	0.075	0.5	0.71	0.81370	0.36167	0.19120
0.3	0.5	0.5	0.12	0.5	0.71	0.81383	0.35971	0.20445
0.3	0.5	0.2	0.3	0.5	0.71	0.81518	0.35196	0.21779
0.3	0.5	0.1	0.6	0.5	0.71	0.81780	0.33906	0.22241

4 Conclusions

In this paper, a boundary layer analysis for mixed convection heat and mass transfer in a non-Darcy micropolar fluid over a vertical plate with uniform wall temperature and concentration conditions in the presence of Soret and Dufour effects is considered. Using the similarity variables, the governing equations are transformed into a set of non-similar parabolic equations where numerical solution has been presented for different values of parameters. The higher values of the coupling number N (i.e., the effect of microrotation becomes significant) result in lower velocity distribution but higher wall temperature; wall concentration distributions in the boundary layer compared to the Newtonian fluid case. The numerical results indicate that the skin friction coefficient as well as rate of heat and mass transfers in the micropolar fluid are lower compared to that of the Newtonian fluid. The higher values of the Forchheimer number Fs indicate lower velocity, skin friction coefficient as well as rate of heat and mass transfers but higher wall temperature and wall concentration distributions. The Velocity, skin friction coefficient as well as rate of heat and mass transfers increase where as the wall temperature and wall concentration distributions decrease with increase in the value of X in the boundary layer. Increasing the Prandtl number substantially decreases the velocity, skin friction coefficient, mass transfer rate and the temperature profile where as increase the rate of heat transfer. Microrotation is however increased at the wall with a rise in Prandtl number but reduced further from the wall as we approach the free stream. The present analysis has also shown that the flow field is appreciably influenced by the Dufour and Soret effects.

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