Solution of the Davey–Stewartson equation using homotopy analysis method

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Abstract. In this paper, the homotopy analysis method (HAM) proposed by Liao is adopted for solving Davey–Stewartson (DS) equations which arise as higher dimensional generalizations of the nonlinear Schrödinger (NLS) equation. The results obtained by HAM have been compared with the exact solutions and homotopy perturbation method (HPM) to show the accuracy of the method. Comparisons indicate that there is a very good agreement between the HAM solutions and the exact solutions in terms of accuracy.

Keywords: homotopy analysis method, Davey–Stewartson equations.

1 Introduction

Partial differential equations which arise in real-world physical problems are often too complicated to be solved exactly. In this paper, we consider the Davey–Stewartson (DS) equations [1–3]

$$
\begin{aligned}
&iq_t + \frac{1}{2}\sigma^2\left(q_{xx} + \sigma^2 q_{yy}\right) + \lambda|q|^2 q - \phi_x q = 0,\\
&\phi_{xx} - \sigma^2 \phi_{yy} - 2\lambda\left(|q|^2\right)x = 0.
\end{aligned}
\tag{1}
$$

The case $\sigma = 1$ is called the DS-I equation, while $\sigma = i$ is the DS-II equation. The parameter λ characterizes the focusing or defocusing case. The Davey–Stewartson I and II are two well-known examples of integrable equations in two space dimensions, which arise as higher dimensional generalizations of the nonlinear Schrödinger (NLS) equation [4]. They appear in many applications, for example in the description of gravity-capillarity surface wave packets in the limit of the shallow water. Davey and Stewartson first derived their model in the context of water waves, from purely physical considerations. In the context, $q(x, y, t)$ is the amplitude of a surface wave packet, while $\phi(x, y)$ reperesents the velocity potential of the mean flow interacting with the surface wave [4].

In 1992, Liao [5] employed the basic ideas of the homotopy in topology to propose a general analytic method for nonlinear problems, namely the homotopy analysis method

(HAM), [5–11]. This method has been successfully applied to solve many types of nonlinear problems [12–18]. The HAM offers certain advantages over routine numerical methods. The HAM is better since it does not involve discretization of the variables hence is free from rounding off errors and does not require large computer memory or time. In this paper, we will apply homotopy analysis method to the problem mentioned above.

2 Basic idea of HAM

Consider the following system of partial differential equations,

$$N_i\big[z_i(x,y,t)\big] = 0, \quad i = 1, 2, \ldots, n, \tag{2}$$

where N_i are nonlinear operators and $z_i(x,y,t)$ are unknowns functions. By means of generalizing the traditional homotopy method, Liao [9,10] constructed the so-called zero-order deformation equations

$$(1-q)L_i\big[\phi_i(x,y,t;q) - z_{i,0}(x,y,t)\big] = qh_iN_i\big[\phi_i(x,y,t;q)\big], \tag{3}$$
$$i = 1, 2, \ldots, n,$$

where $q \in [0,1]$ is an embedding parameter, h_i are nonzero auxiliary functions, L_i are auxiliary linear operators, $z_{i,0}(x,y,t)$ are initial guesses of $z_i(x,y,t)$ and $\phi_i(x,y,t;q)$ are unknown functions. It is important to note that, one has great freedom to choose auxiliary objects such as h_i and L_i in HAM. Obviously, when $q = 0$ and $q = 1$, both $\phi_i(x,y,t;0) = z_{i,0}(x,y,t)$ and $\phi_i(x,y,t;1) = z_i(x,y,t)$, hold. Thus as q increases from 0 to 1, the solutions $\phi_i(x,y,t;q)$ varies from the initial guesses $z_{i,0}(x,y,t)$ to the solutions $z_i(x,y,t)$. Expanding $\phi_i(x,y,t;q)$ in Taylor series with respect to q, one has

$$\phi_i(x,y,t;q) = z_{i,0}(x,y,t) + \sum_{m=1}^{+\infty} z_{i,m}(x,y,t)q^m, \tag{4}$$

where

$$z_{i,m} = \frac{1}{m!}\frac{\partial^m \phi_i(x,y,t;q)}{\partial q^m}\bigg|_{q=0}, \tag{5}$$

if the auxiliary linear operators, the initial guesses, the auxiliary parameters h_i, and the auxiliary functions are properly chosen, then the series equation (4) converges at $q = 1$ and

$$\phi_i(x,y,t;1) = z_{i,0}(x,y,t) + \sum_{m=1}^{+\infty} z_{i,m}(x,y,t), \tag{6}$$

which must be one of solutions of the original nonlinear equations, as proved by Liao [9,10]. As $h_i = -1$, Eq. (3) becomes

$$(1-q)L_i\big[\phi_i(x,y,t;q) - z_{i,0}(x,y,t)\big] + qN_i\big[\phi_i(x,y,t;q)\big] = 0, \tag{7}$$

which are mostly used in the homotopy-perturbation method.

According to (5), the governing equations can be deduced from the zero-order deformation equations (3). Define the vectors

$$\vec{z}_{i,n} = \big[z_{i,0}(x,y,t), z_{i,1}(x,y,t), \ldots, z_{i,n}(x,y,t)\big]. \tag{8}$$

Differentiating (3) m-times with respect to the embedding parameter q and then setting $q = 0$ and finally dividing them by $m!$, we have the so-called mth-order deformation equations

$$L_i\big[z_{i,m}(x,y,t) - \chi_m z_{i,m-1}(x,y,t)\big] = h_i R_{i,m}(\vec{z}_{i,m-1}), \tag{9}$$

where

$$R_{i,m}(\vec{z}_{i,m-1}) = \frac{1}{(m-1)!} \frac{\partial^{m-1} N_i[\phi_i(x,y,t;q)]}{\partial q^{m-1}}\bigg|_{q=0}, \tag{10}$$

The solution of the mth-order deformation Eq. (9) is readily found to be

$$z_{i,m}(x,y,t) = \chi_m z_{i,m-1} + h_i L^{-1}\big[R_{i,m}(z_{i,m-1})\big] \tag{11}$$

and

$$\chi_m = \begin{cases} 0, & m \le 1, \\ 1, & m > 1. \end{cases}$$

It should be emphasized that $z_{i,m}(x,y,t)$ ($m \ge 1$) is governed by linear equation (9) with the linear boundary condition that come from the original problem, which can be easily solved by symbolic computation softwares such as Mathematica and Maple. For the convergence of the above method we refer the reader to Liao's work [8]. If Eq. (2) admits unique solution, then this method will produce the unique solution. If Eq. (2) does not possess unique solution, the HAM will give a solution among many other (possible) solutions.

3 Analysis of the method by the HAM

Without loss of generality, first we separate the amplitude of a surface wave packet q into real part and imaginary part, i.e., $q = u + i\nu$. Then we rewrite the system (1) in the following form,

$$u_t = -\frac{1}{2}\sigma^2\big(\nu_{xx} + \sigma^2\nu_{yy}\big) - \lambda\big(u^2 + \nu^2\big)\nu + \phi_x\nu, \tag{12}$$

$$\nu_t = \frac{1}{2}\sigma^2\big(u_{xx} + \sigma^2 u_{yy}\big) + \lambda\big(u^2 + \nu^2\big)u - \phi_x u, \tag{13}$$

$$\phi_{xx} - \sigma^2\phi_{yy} - 2\lambda\big(u^2 + \nu^2\big)_x = 0. \tag{14}$$

To investigate the traveling wave solution of Eqs. (12)–(14), using homotopy analysis method, we choose the linear operators $L_1 = L_2 = L_3 = \partial^2/\partial y^2$ and nonlinear operators as

$$N_1 = \frac{\partial^2 \nu}{\partial y^2} + \frac{1}{\sigma^2}\left(\frac{\partial^2 \nu}{\partial x^2}\right) + \frac{2}{\sigma^4}\left(\frac{\partial u}{\partial t}\right) + \frac{2\lambda}{\sigma^4}\left(u^2\nu + \nu^3\right) - \frac{2}{\sigma^4}\left(\frac{\partial \phi}{\partial x}\nu\right),$$

$$N_2 = \frac{\partial^2 u}{\partial y^2} + \frac{1}{\sigma^2}\left(\frac{\partial^2 u}{\partial x^2}\right) - \frac{2}{\sigma^4}\left(\frac{\partial \nu}{\partial t}\right) + \frac{2\lambda}{\sigma^4}\left(u^3 + \nu^2 u\right) - \frac{2}{\sigma^4}\left(\frac{\partial \phi}{\partial x}u\right),$$

$$N_3 = \frac{\partial^2 \phi}{\partial y^2} - \frac{1}{\partial^2}\left(\frac{\partial^2 \phi}{\partial x^2}\right) + \frac{2\lambda}{\sigma^2}\left(\frac{\partial}{\partial x}\left(u^2 + \nu^2\right)\right),$$

for Eqs. (12)–(14) respectively. The initial guesses are considered as follow,

$$\nu_0(x, y, t) = \nu(x, 0, t),$$
$$u_0(x, y, t) = u(x, 0, t), \tag{15}$$
$$\phi_0(x, y, t) = \phi(x, 0, t).$$

According to HAM ν_1, u_1, ϕ_1 are obtained as follow:

$$\nu_1 = h \int_0^y \int_0^y R_1(\nu_0)\, dy\, dy,$$

$$u_1 = h \int_0^y \int_0^y R_1(u_0)\, dy\, dy, \tag{16}$$

$$\phi_1 = h \int_0^y \int_0^y R_1(\phi_0)\, dy\, dy.$$

In view of (11) other components of ν, u, ϕ are obtained using the following recursive relations:

$$\nu_i = \nu_{i-1} + h \int_0^y \int_0^y R_i(\nu_{i-1})\, dy\, dy,$$

$$u_i = u_{i-1} + h \int_0^y \int_0^y R_i(u_{i-1})\, dy\, dy, \tag{17}$$

$$\phi_i = \phi_{i-1} + h \int_0^y \int_0^y R_i(\phi_{i-1})\, dy\, dy, \quad i = 2, 3, \ldots.$$

In view of Eq. (10), we obtained

$$R_1(\nu_0) = \left(\frac{\partial^2 \nu_0}{\partial y^2}\right) + \frac{1}{\sigma^2}\left(\frac{\partial^2 \nu_0}{\partial x^2}\right) + \frac{2}{\sigma^4}\left(\frac{\partial u_0}{\partial t}\right) + \frac{2\lambda}{\sigma^4}\left(u_0^2\nu_0 + \nu_0^3\right) - \frac{2}{\sigma^4}\left(\frac{\partial \phi_0}{\partial x}\nu_0\right),$$

$$R_1(u_0) = \left(\frac{\partial^2 u_0}{\partial y^2}\right) + \frac{1}{\sigma^2}\left(\frac{\partial^2 u_0}{\partial x^2}\right) - \frac{2}{\sigma^4}\left(\frac{\partial \nu_0}{\partial t}\right) + \frac{2\lambda}{\sigma^4}\left(u_0^3 + \nu_0^2 u_0\right) - \frac{2}{\sigma^4}\left(\frac{\partial \phi_0}{\partial x}u_0\right),$$

$$R_1(\phi_0) = \left(\frac{\partial^2 \phi_0}{\partial y^2}\right) - \frac{1}{\sigma^2}\left(\frac{\partial^2 \phi_0}{\partial x^2}\right) + \frac{2\lambda}{\sigma^2}\left(\frac{\partial}{\partial x}\left(u_0^2 + \nu_0^2\right)\right),$$

$$R_2(\nu_1) = \left(\frac{\partial^2 \nu_1}{\partial y^2}\right) + \frac{1}{\sigma^2}\left(\frac{\partial^2 \nu_1}{\partial x^2}\right) + \frac{2}{\sigma^4}\left(\frac{\partial u_1}{\partial t}\right) - \frac{2}{\sigma^4}\left(\frac{\partial \phi_1}{\partial x}\nu_0 + \nu_1\frac{\partial \phi_0}{\partial x}\right)$$
$$+ \frac{2\lambda}{\sigma^4}\left(2u_0 u_1 \nu_0 + \nu_1 u_0^2 + 3\nu_0^2 \nu_1\right),$$

$$R_2(u_1) = \left(\frac{\partial^2 u_1}{\partial y^2}\right) + \frac{1}{\sigma^2}\left(\frac{\partial^2 u_1}{\partial x^2}\right) - \frac{2}{\sigma^4}\left(\frac{\partial \nu_1}{\partial t}\right) - \frac{2}{\sigma^4}\left(\frac{\partial \phi_1}{\partial x}u_0 + u_1\frac{\partial \phi_0}{\partial x}\right)$$
$$+ \frac{2\lambda}{\sigma^4}\left(3u_0^2 u_1 + 2\nu_0 \nu_1 u_0 + u_1 \nu_0^2\right),$$

$$R_2(\phi_1) = \left(\frac{\partial^2 \phi_1}{\partial y^2}\right) - \frac{1}{\sigma^2}\left(\frac{\partial^2 \phi_1}{\partial x^2}\right) + \frac{2\lambda}{\sigma^2}\left(\frac{\partial}{\partial x}\left(2u_0 u_1 + 2\nu_0 \nu_1\right)\right),$$

$$R_3(\nu_2) = \frac{\partial^2 \nu_2}{\partial y^2}) + \frac{1}{\sigma^2}\left(\frac{\partial^2 \nu_2}{\partial x^2}\right) + \frac{2}{\sigma^4}\left(\frac{\partial u_2}{\partial t}\right) - \frac{2}{\sigma^4}\left(\frac{\partial \phi_2}{\partial x}\nu_0 + \nu_1\frac{\partial \phi_1}{\partial x} + \nu_2\frac{\partial \phi_0}{\partial x}\right)$$
$$+ \frac{2\lambda}{\sigma^4}\left(u_1^2 \nu_0 + 2u_2 u_0 \nu_0 + 2\nu_1 u_1 u_0 + \nu_2 u_0^2 + 3\nu_0 \nu_1^2 + 3\nu_2 \nu_0^2\right),$$

$$R_3(u_2) = \left(\frac{\partial^2 u_2}{\partial y^2}\right) + \frac{1}{\sigma^2}\left(\frac{\partial^2 u_2}{\partial x^2}\right) - \frac{2}{\sigma^4}\left(\frac{\partial \nu_2}{\partial t}\right) - \frac{2}{\sigma^4}\left(\frac{\partial \phi_2}{\partial x}u_0 + u_1\frac{\partial \phi_1}{\partial x} + u_2\frac{\partial \phi_0}{\partial x}\right)$$
$$+ \frac{2\lambda}{\sigma^4}\left(3u_0 u_1^2 + 3u_0^2 u_2 + \nu_1^2 u_0 + 2\nu_2 \nu_0 u_0 + 2u_1 \nu_0 \nu_1 + u_2 \nu_0^2\right),$$

$$R_3(\phi_2) = \left(\frac{\partial^2 \phi_2}{\partial y^2}\right) - \frac{1}{\sigma^2}\left(\frac{\partial^2 \phi_2}{\partial x^2}\right) + \frac{2\lambda}{\sigma^2}\left(\frac{\partial}{\partial x}\left(u_1^2 + 2u_2 u_0 + \nu_1^2 + 2\nu_2 \nu_0\right)\right),$$

and so on. We will obtain

$$\nu(x,y,t) = \lim_{n\to\infty}\sum_{i=0}^{n}\nu_i(x,y,t),$$

$$u(x,y,t) = \lim_{n\to\infty}\sum_{i=0}^{n}u_i(x,y,t), \tag{18}$$

$$\phi(x,y,t) = \lim_{n\to\infty}\sum_{i=0}^{n}\phi_i(x,y,t).$$

4 Application

For solving Eqs. (12)–(14) using HAM, we consider the following initial conditions:

$$\nu(x,0,t) = r \operatorname{sech}\big[s(x-ct)\big]\sin[k_1 x + k_3 t],$$
$$u(x,0,t) = r \operatorname{sech}\big[s(x-ct)\big]\cos[k_1 x + k_3 t], \tag{19}$$
$$\phi(x,0,t) = f \tanh\big[s(x-ct)\big],$$

where $c = k_2 + \sigma^2 k_1$, $r = \sqrt{-(2k_3 + k_1^2\sigma^2 + k_2^2)/\lambda}$, $s = \sqrt{(2k_3 + k_1^2\sigma^2 + k_2^2)/\sigma^2}$, $f = 2\sigma\sqrt{-\lambda}/(1-\sigma^2)$ and k_i $(i = 1,2,3)$ are arbitrary constants. Now we substitute the initial conditions (19) into the system (16), we obtain

$$
\begin{aligned}
\nu_1 = h\Bigg[& -\frac{frsy^2 \operatorname{sech}[s(-ct+x)]^3 \sin[xk_1 + tk_3]}{\sigma^4} \\
& -\frac{rs^2 y^2 \operatorname{sech}[s(-ct+x)]^3 \sin[xk_1 + tk_3]}{2\sigma^2} \\
& +\frac{r^3 y^2 \lambda \cos[xk_1 + tk_3]^2 \operatorname{sech}[s(-ct+x)]^3 \sin[xk_1 + tk_3]}{\sigma^4} \\
& +\frac{r^3 y^2 \lambda \operatorname{sech}[s(-ct+x)]^3 \sin[xk_1 + tk_3]^3}{\sigma^4} \\
& -\frac{ry^2 \operatorname{sech}[s(-ct+x)] \sin[xk_1 + tk_3]k_1^2}{2\sigma^2} \\
& -\frac{ry^2 \operatorname{sech}[s(-ct+x)] \sin[xk_1 + tk_3]k_3}{\sigma^4} \\
& +\frac{crsy^2 \cos[xk_1 + tk_3] \operatorname{sech}[s(-ct+x)] \tanh[s(-ct+x)]}{\sigma^4} \\
& -\frac{rsy^2 \cos[xk_1 + tk_3] \operatorname{sech}[s(-ct+x)]k_1 \tanh[s(-ct+x)]}{\sigma^2} \\
& +\frac{rs^2 y^2 \operatorname{sech}[s(-ct+x)] \sin[xk_1 + tk_3] \tanh[s(-ct+x)]^2}{2\sigma^2}\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
u_1 = h\Bigg[& -\frac{frsy^2 \operatorname{sech}[s(-ct+x)]^3 \cos[xk_1 + tk_3]}{\sigma^4} \\
& -\frac{rs^2 y^2 \operatorname{sech}[s(-ct+x)]^3 \cos[xk_1 + tk_3]}{2\sigma^2} \\
& +\frac{r^3 y^2 \lambda \cos[xk_1 + tk_3]^3 \operatorname{sech}[s(-ct+x)]^3}{\sigma^4} \\
& +\frac{r^3 y^2 \lambda \operatorname{sech}[s(-ct+x)]^3 \sin[xk_1 + tk_3]^2 \cos[xk_1 + tk_3]}{\sigma^4} \\
& -\frac{ry^2 \operatorname{sech}[s(-ct+x)] \cos[xk_1 + tk_3]k_1^2}{2\sigma^2}
\end{aligned}
$$

$$- \frac{ry^2 \operatorname{sech}[s(-ct+x)] \cos[xk_1 + tk_3]k_3}{\sigma^4}$$

$$- \frac{crsy^2 \sin[xk_1 + tk_3] \operatorname{sech}[s(-ct+x)] \tanh[s(-ct+x)]}{\sigma^4}$$

$$+ \frac{rsy^2 \sin[xk_1 + tk_3] \operatorname{sech}[s(-ct+x)]k_1 \tanh[s(-ct+x)]}{\sigma^2}$$

$$+ \left. \frac{rs^2y^2 \operatorname{sech}[s(-ct+x)] \cos[xk_1 + tk_3] \tanh[s(-ct+x)]^2}{2\sigma^2} \right],$$

$$\phi_1 = \frac{hsy^2(fs - 2r^2\lambda) \operatorname{sech}[s(-ct+x)]^2 \tanh[s(-ct+x)]}{\sigma^2}.$$

In the same manner, using recurnce relations in (17) the other components $\nu_2(x, y, t)$, $\nu_3(x, y, t), \ldots$, $u_2(x, y, t), u_3(x, y, t), \ldots$ and $\phi_2(x, y, t), \phi_3(x, y, t), \ldots$ can be obtained. Substituting these components into Eqs. (18) to be obtain $\nu(x, y, t)$, $u(x, y, t)$, and $\phi(x, y, t)$. Using a Taylor series, then the closed form solutions yields as follows:

$$u(x, y, t) = r \operatorname{sech}\left[s(x + y - ct)\right] \cos\left[(k_1 x + k_2 y + k_3 t)\right],$$
$$\nu(x, y, t) = r \operatorname{sech}\left[s(x + y - ct)\right] \sin\left[(k_1 x + k_2 y + k_3 t)\right], \qquad (20)$$
$$\phi(x, y, t) = f \tanh\left[s(x + y - ct)\right],$$

where $c = k_2 + \sigma^2 k_1$, $r = \sqrt{-(2k_3 + k_1^2\sigma^2 + k_2^2)/\lambda}$, $s = \sqrt{(2k_3 + k_1^2\sigma^2 + k_2^2)/\sigma^2}$, $f = 2\sigma\sqrt{-\lambda}/(1 - \sigma^2)$, k_1, k_2 and k_3 are arbitrary constants.

5 Comparing the HAM results with the exact solutions

To demonstrate the convergence of the HAM, the results of the numerical example are presented and only few terms are required to obtain accurate solution. Tables 1–3 show the absolute errors between the analytical solutions and the 3th-order HAM solutions of DS for $y = 0.2$, $k_1 = 0.1$, $k_2 = 0.03$, $k_3 = -0.3$, $\sigma = I$, $\lambda = 1$, and $h = -.75$. Both the exact solutions and the approximate solution of $u(x, y, t)$, $\nu(x, y, t)$ and $\phi(x, y, t)$ (for the same parameters as mentioned before) are plotted in Figs. 1–3.

Table 1. Absolute errors of $u(x, y, t)$.

$x \setminus t$	0.1	0.2	0.3	0.4	0.5
20	1.27831×10^{-8}	1.16602×10^{-8}	1.05388×10^{-8}	9.41974×10^{-9}	8.30419×10^{-9}
17	1.90799×10^{-8}	7.39331×10^{-9}	4.17318×10^{-9}	1.56101×10^{-8}	2.69083×10^{-8}
14	1.00238×10^{-6}	1.11316×10^{-6}	1.22171×10^{-6}	1.32795×10^{-6}	1.4318×10^{-6}
11	2.19709×10^{-5}	2.29084×10^{-5}	2.38145×10^{-5}	2.46887×10^{-5}	2.55306×10^{-5}
8	3.28063×10^{-4}	3.34711×10^{-4}	3.40978×10^{-4}	3.46861×10^{-4}	3.5236×10^{-4}

Table 2. Absolute errors of $\nu(x,y,t)$.

$x \setminus t$	0.1	0.2	0.3	0.4	0.5
20	3.51132×10^{-8}	3.52876×10^{-8}	3.54247×10^{-8}	3.55329×10^{-8}	3.56044×10^{-8}
17	3.87977×10^{-7}	3.86258×10^{-7}	3.84202×10^{-7}	3.81812×10^{-7}	3.79095×10^{-7}
14	3.91158×10^{-6}	3.85862×10^{-6}	3.80266×10^{-6}	3.74379×10^{-6}	3.68211×10^{-6}
11	3.57663×10^{-5}	$3.49\quad\times10^{-5}$	3.40109×10^{-5}	3.30999×10^{-5}	3.21682×10^{-5}
8	2.87696×10^{-4}	2.76213×10^{-4}	2.64599×10^{-4}	2.52867×10^{-4}	2.41028×10^{-4}

Table 3. Absolute errors of $\phi(x,y,t)$.

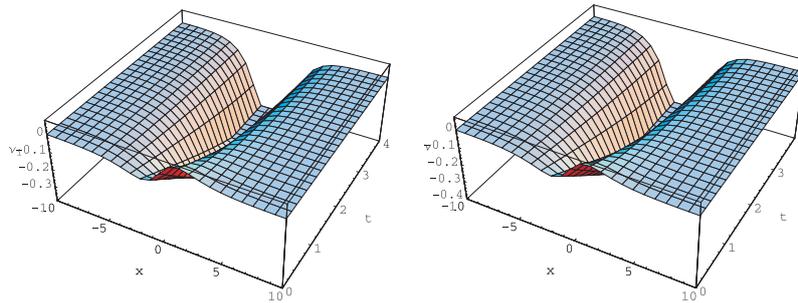| $x \setminus t$	0.1	0.2	0.3	0.4	0.5
20	1.11022×10^{-16}	1.11022×10^{-16}	1.11022×10^{-16}	1.11022×10^{-16}	2.22045×10^{-16}
17	1.4877×10^{-14}	1.46549×10^{-14}	1.43219×10^{-14}	1.43219×10^{-14}	1.39888×10^{-14}
14	1.58651×10^{-12}	1.5693×10^{-12}	1.5522×10^{-12}	1.53555×10^{-12}	1.51901×10^{-12}
11	1.70145×10^{-10}	1.69595×10^{-10}	1.67752×10^{-10}	1.65929×10^{-10}	1.64126×10^{-10}
8	1.8514×10^{-8}	1.8313×10^{-8}	1.81141×10^{-8}	1.79174×10^{-8}	1.72228×10^{-8}

Fig. 1. Comparison between the exact solution and the 3th-order HAM solution of $u(x,y,t)$.

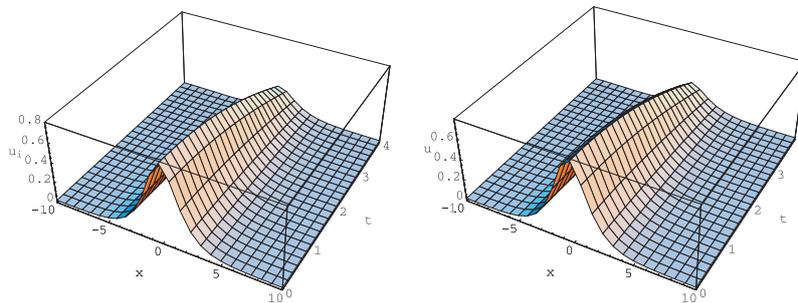

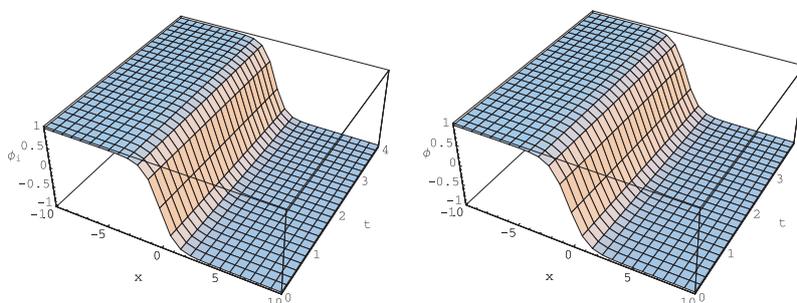Fig. 2. Comparison between the exact solution and the 3th-order HAM solution of $\nu(x,y,t)$.

Fig. 3. Comparison between the exact solution and the 3th-order HAM solution of
$\phi(x, y, t)$.

Remark 1. The numerical result given in the Tables 1–3 have better approximate with those given in [3] using HPM.

Remark 2. When we select $h = -1$ we get same result as obtained by Zedan and Tantawy [3].

6   Conclusions

In this paper, the homotopy analysis method has been successfully applied to finding the solution of Davey–Stewartson equations. The solution obtained by the homotopy analysis method is an infinite power series for appropriate initial condition, which can, in turn, be expressed in a closed form, the exact solution. We found that HAM logically contains HPM. Besides, if the same initial value and the same auxiliary linear operator are chosen, the approximations given by HPM are exactly a special case of those given by HAM when $h = -1$ and $H = 1$. In comparison with HPM [3] and VIM [2] methods we will find better approximations. The results show that the homotopy analysis method is a powerful Mathematical tool for solving Davey–Stewartson equations. It is also a promising method to solve other nonlinear equations. Mathematica has been used for computations in this paper.

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