Effects of Viscous and Joules Dissipation on MHD Flow, Heat and Mass Transfer past a Stretching Porous Surface Embedded in a Porous Medium

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Abstract. This paper investigates the influence of both viscous and joules dissipation on the problem of magnetohydrodynamic flow past a stretching porous surface embedded in a porous medium. Analytic solutions of the resulting nonlinear non-homogeneous boundary value problem in the case when the plate stretches with a velocity varying linearly with distance, expressed in terms of confluent hypergeometric functions, are presented for the case of prescribed surface temperature. Numerical calculations have been carried out for various values of suction parameter, magnetic field, Prandtl number, Eckert number and Schmidt number. The results show that increases in magnetic parameter decrease both the dimensionless transverse velocity, longitudinal velocity and also the skin friction coefficient. Also, formation of thin boundary layer is observed for higher value of magnetic parameter.

Keywords: MHD, porous medium, joules dissipation, viscous dissipation.

1 Introduction

The study of two-dimensional boundary layer flow, heat and mass transfer over a porous stretching surface is very important as it finds many practical applications in different areas. To be more specific, it may be pointed out that many metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid and that in the process of drawing these strips are sometimes stretched. Viscous dissipation changes the temperature distributions by playing a role like an energy source, which leads to affected heat transfer rates. The merit of the effect of viscous dissipation depends on whether the plate is being cooled or heated. Apart from the viscous dissipation in MHD flows, the Joules dissipation also acts as a volumetric heat source. Heat transfer analysis over porous surface is of much practical interest due to its abundant applications. To be more specific, heat-treated materials traveling between a feed roll and wind-up roll or materials manufactured by extrusion, glass-fiber and paper production, cooling of metallic sheets or electronic chips, crystal growing just to name a few. In these cases, the final product of desired characteristics depends on the rate of cooling in the process and the process of stretching. In view of all these aspects, the present work deals with the effect of viscous and Joules dissipation on MHD flow, heat and mass transfer over a porous surface embedded in a porous medium.

Recently, attention has been made on the effect of transversely applied magnetic field on the flow of electrically conducting fluids with various properties associated with the interplay of magnetic fields and thermal perturbation in porous medium past vertical plate find usual applications in astrophysics, geophysical fluid dynamics and engineering. Researches in these fields have been conducted by many investigators. For example, analytical results were carryout by Vajravelu and Hadjinicolaou [1] who took into account the effects of viscous dissipation and internal heat generation. An analysis of thermal boundary layer in an electrically conducting fluid over a linearly stretching sheet in the presence of a constant transverse magnetic field with suction or blowing at the sheet was carried out by Chaim [2]. The viscous and joules dissipation and internal heat generation was taken into account in the energy equation.

Very recently, the viscous and joules dissipation and internal heat generation was taken into account in the energy equation. Sajid et al. [3] investigated the non-similar analytic solution for MHD flow and heat transfer in a third-order fluid over a stretching sheet. He found that the skin friction coefficient decreases as the magnetic parameter or the third grade parameter increases. A mathematical analysis has been carried out on momentum and heat transfer characteristics in an incompressible, electrically conducting viscoelastic boundary layer fluid flow over a linear stretching sheet by Abel et al. [4]. A numerical reinvestigation of MHD boundary layer flow over a heated stretching sheet with variable viscosity has been analyzed by Pantokratoras [5].

The problem of viscous dissipation, Joule heating and heat source/sink on non-Darcy MHD natural convection flow over an isoflux permeable sphere in a porous medium is numerically analyzed by Yih [6]. The work of Sonth et al. [7] deals with the effect of the viscous dissipation term along with temperature dependent heat source/sink on momentum, heat and mass transfer in a visco-elastic fluid flow over an accelerating surface. Chen [8] examined the effect of combined heat and mass transfer on MHD free convection from a vertical surface with ohmic heating and viscous dissipation.

Very Recently, the effect of viscous dissipation and Joule heating on MHD free convection flow past a semi-infinite vertical flat plate in the presence of the combined effect of Hall and non-slip currents for the case of power-law variation of the wall temperature is analyzed by Abo-Eldahab and El Aziz [9]. In 2005, Tak and Lodha [10] analyzed the flow and heat transfer due to a stretching porous surface in presence of transverse magnetic field including heat due to viscous dissipation. The effects of viscous dissipation on natural convection flow over a sphere in the presence of magnetic field and heat generation for an electrically conducting fluid have been investigated theoretically by Alam et al. [11]. Barletta and Celli [12] investigated the mixed convection MHD flow in a vertical channel with Joules and viscous dissipation effects. The study of nonlinear hydromagnetic flow and heat transfer due to a stretching porous surface with prescribed heat flux and viscous dissipation effects was analyzed by Ganga et al. [13]. Hence the present study investigates the effect of viscous and Joules dissipation on MHD flow in a porous medium with heat and mass transfer

2 Mathematical analysis

Two-dimensional, nonlinear, steady, MHD laminar boundary layer flow with heat and mass transfer of a viscous, incompressible and electrically conducting fluid over a porous surface embedded in a porous medium in the presence of a transverse magnetic field including viscous and Joules dissipation is considered for investigation. An uniform transverse magnetic field of strength B_0 is applied parallel to y-axis. Consider a polymer sheet emerging out of a slit at $x = 0$, $y = 0$ and subsequently being stretched, as in a polymer extrusion process. Let us assume that the speed at a point in the plate is proportional to the power of its distance from the slit and the boundary layer approximations are applicable. In writing the following equations, it is assumed that the induced magnetic field, the external electric field and the electric field due to the polarization of charges are negligible. Under these conditions, the governing boundary layer equations of momentum, energy and diffusion with viscous and Joules dissipation are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho} - \frac{\nu}{K_p}u, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{K}{\rho\, C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\sigma B_0^2}{\rho C_p}\right)u^2, \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2}. \tag{4}$$

The boundary conditions are

$$\begin{aligned} &u = ax^m, \quad v = v_w(x), \quad T = T_w(x) = T_\infty + T_0 x^n, \\ &C = C_w(x) = C_\infty + C_0 x^n \qquad \text{at } \ y = 0, \\ &u = 0, \quad T = T_\infty, \quad C = C_\infty \quad \text{at } \ y \to \infty. \end{aligned} \tag{5}$$

Here u, v are components of velocity components in the x and y directions, ν is kinematic coefficient of viscosity, K_p is permeability of the medium, σ is electrical conductivity of the fluid, B_0 is applied magnetic field, ρ is density of the fluid, T is temperature of the fluid, T_w is wall temperature, T_∞ is temperature far away from the surface, K is thermal conductvity, C_p is specific heat at constant pressure, C is species concentration of the fluid, C_w is species concentration near the wall C_∞ is species concentration of the fluid away from the wall, D is diffusivity coefficient, $a,\ T_0$ and C_0 are dimensional constants, m is index of power-law velocity and n is index of power-law variation of wall temperature which is constant.

As in [10], we introduce the following similarity transformations

$$\Psi(x,y) = \left[\frac{2\nu x U(x)}{1+m}\right]^{1/2} F(\eta),$$
$$\eta = \left[\frac{(1+m)U(x)}{2\nu x}\right]^{1/2} y, \tag{6}$$
$$v_w(x) = -\lambda\sqrt{\frac{\nu a(m+1)}{2}}\, x^{(m-1)/2},$$
$$n = 2m,$$

where $\lambda > 0$ for suction at the stretching plate and Ψ is the stream function.

The velocity components are given by

$$u = \frac{\partial \Psi}{\partial y}, \quad v = -\frac{\partial \Psi}{\partial x}. \tag{7}$$

It can be easily verified that the continuity equation (1) is identically satisfied and introduce the non-dimensional form of temperature and the concentration as

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad h = \frac{C - C_\infty}{C_w - C_\infty}. \tag{8}$$

Now the equations (2) to (4) become

$$F''' + FF'' - \beta F'^2 - \left(R_1^{-1} + M^2\right)F' = 0, \tag{9}$$
$$\theta'' + PrF\theta' - 2\beta PrF'\theta = -Ec\,Pr\left[F''^2 + M^2 F'^2\right], \tag{10}$$
$$h'' + ScFh' - Sc\,m\,hF' = 0 \tag{11}$$

with boundary conditions

$$F(0) = \lambda, \quad F'(0) = 1, \quad F'(\infty) = 0,$$
$$\theta(0) = 1, \quad \theta(\infty) = 0, \tag{12}$$
$$h(0) = 1, \quad h(\infty) = 0,$$

where

$$\beta = \frac{2m}{m+1} \qquad \text{stretching parameter,}$$

$$M^2 = v\frac{2\sigma B_0^2}{\rho a(1+m)} \qquad \text{magnetic parameter,}$$

$$R_1 = \frac{K_p a}{\nu} \qquad \text{permeability parameter,}$$

$$Pr = \frac{\mu C_p}{K} \qquad \text{Prandtl number,}$$

$$E_c = \frac{a^2}{C_p T_0} \qquad \text{Eckert number,}$$

$$Sc = \frac{\nu}{D} \qquad \text{Schmidt number.}$$

Equations (9), (10) and (11) with boundary conditions (12) constitute a non-linear BVP, the analytical solution of which is not feasible for general value of parameter β. However, an analytical solution of these equations can be obtained when $\beta = 1$ (i.e. $m = 1$), as follows.

It may be noted that, when $\beta = 1$ or $m = 1$, the velocity of the stretching plate is ax, i.e. the plate stretches with a velocity varying linearly with distance. In this case, the equations (9), (10) and (11) are reduced to

$$F''' + FF'' - F'^2 - \left(M^2 + R_1^{-1}\right)F' = 0, \tag{13}$$

$$\theta'' + Pr\,F\,\theta' - 2Pr\,F'\theta = -Ec\,Pr\left[F''^2 + M^2F'^2\right], \tag{14}$$

$$h'' + Sc F h' - 2Sc\,h F' = 0 \tag{15}$$

with the boundary conditions given in (12).

Equation (13) with boundary conditions (12), is independent of (14) and admits a solution of the form (following Chakrabarti and Gupta [14])

$$F(\eta) = A + Be^{-\alpha\eta},$$

where

$$A = \frac{\alpha^2 - (R_1^{-1} + M^2)}{\alpha}, \quad B = \frac{-1}{\alpha}, \quad \alpha = \frac{\lambda + \sqrt{\lambda^2 + 4(1 + M^2 + R_1^{-1})}}{2}.$$

Hence the exact solution is

$$F(\eta) = \frac{1}{\alpha}\left[\alpha^2 - \left(R_1^{-1} + M^2\right) - e^{-\alpha\eta}\right]. \tag{16}$$

In order to solve energy equation (14), a new independent variable ξ is introduced

$$\xi = \frac{-Pr}{\alpha^2}\,e^{-\alpha\eta}. \tag{17}$$

Using (16) and (17), equation (14) yields

$$\xi\frac{\mathrm{d}^2\theta}{\mathrm{d}\xi^2}+[(1-K_1)-\xi]\frac{\mathrm{d}\theta}{\mathrm{d}\xi}+2\theta=-\frac{E_c\alpha^2(\alpha^2+M^2)}{Pr}\xi \tag{18}$$

with corresponding boundary conditions

$$\theta(\xi=0)=0,\quad \theta\biggl(\xi=\frac{-Pr}{\alpha^2}\biggr)=1,$$

where $K_1=\frac{Pr}{\alpha^2}[\alpha^2-(M^2+R_1^{-1})]$

Equation (18) is confluent hypergeometric equation with non-homogeneous part, the solution of which may be expressed as follows

$$\theta(\xi)=-\frac{Ec\,\alpha^2(\alpha^2+M^2)}{2Pr(2-K_1)}\xi^2+\frac{\bigl[1+\frac{Ec\,Pr(\alpha^2+M^2)}{2\alpha^2(2-K_1)}\bigr]\xi^{K_1}\,{}_1F_1(-2+K_1;1+K_1;\xi)}{\bigl(\frac{-Pr}{\alpha^2}\bigr)^{K_1}\,{}_1F_1\bigl(-2+K_1;1+K_1;\frac{-Pr}{\alpha^2}\bigr)}.$$

Function θ in terms of variable η can now be expressed as

$$\begin{aligned}\theta(\eta)=&-\frac{Ec\,Pr(\alpha^2+M^2)}{2\,\alpha^2(2-K_1)}\,e^{-2\alpha\eta}\\&+\frac{\bigl[1+\frac{Ec\,Pr(\alpha^2+M^2)}{2\,\alpha^2(2-K_1)}\bigr]\,e^{-\alpha K_1\eta}\,{}_1F_1\bigl(-2+K_1;1+K_1;\frac{-Pr}{\alpha^2}e^{-\alpha\eta}\bigr)}{{}_1F_1\bigl(-2+K_1;1+K_1;\frac{-Pr}{\alpha^2}\bigr)}.\end{aligned} \tag{19}$$

The dimensionless surface heat transfer rate may be derived as

$$\begin{aligned}\theta'(0)=&\frac{Ec\,Pr(\alpha^2+M^2)}{\alpha(2-K_1)}-\alpha K_1\biggl[1+\frac{Ec\,Pr(\alpha^2+M^2)}{2\alpha^2(2-K_1)}\biggr]\\&+\frac{Pr}{\alpha}\biggl(\frac{K_1-2}{1+K_1}\biggr)\frac{\bigl[1+\frac{Ec\,Pr(\alpha^2+M^2)}{2\alpha^2(2-K_1)}\bigr]\,{}_1F_1\bigl(-1+K_1;2+K_1;\frac{-Pr}{\alpha^2}\bigr)}{{}_1F_1\bigl(-2+K_1;1+K_1;\frac{-Pr}{\alpha^2}\bigr)}.\end{aligned} \tag{20}$$

To obtain the solution of equation (15), a new variable ζ is introduced which is defined as

$$\zeta^{-1}=-\frac{\alpha^2e^{\alpha\eta}}{Sc}.$$

Now equation (15) can be written as

$$\zeta\frac{\mathrm{d}^2h}{\mathrm{d}\zeta^2}+\frac{\mathrm{d}h}{\mathrm{d}\zeta}[(1-K_2)-\zeta]+2h=0, \tag{21}$$

where $K_2=\frac{Sc}{\alpha^2}[\alpha^2-(R_1^{-1}+M^2)]$ with the corresponding boundary conditions

$$h\biggl(\zeta=\frac{-Sc}{\alpha^2}\biggr)=1,\quad h(\zeta=0)=0. \tag{22}$$

The solution of equation (21) subject to the boundary condition (22) is obtained in terms of confluent hypergeometric function

$$h(\zeta) = \frac{\zeta^{K_2}\ {}_1F_1(K_2-2; 1+K_2; \zeta)}{\left(\frac{-Sc}{\alpha^2}\right)^{K_2}\ {}_1F_1\left(K_2-2; 1+K_2; \frac{-Sc}{\alpha^2}\right)}.$$

Function h in terms of variable η can now be expressed as

$$h(\eta) = \frac{e^{-\alpha K_2 \eta}\ {}_1F_1\left(K_2-2; 1+K_2; \frac{-Sc}{\alpha^2} e^{-\alpha\eta}\right)}{{}_1F_1\left(K_2-2; 1+K_2; \frac{-Sc}{\alpha^2}\right)}. \tag{23}$$

The dimensionless surface mass transfer rate may be derived as

$$h'(0) = -\alpha K_2 + \frac{Sc}{\alpha}\left(\frac{K_2-2}{1+K_2}\right)\frac{{}_1F_1\left(K_2-1; 2+K_2; \frac{-Sc}{\alpha^2}\right)}{{}_1F_1\left(K_2-2; 1+K_2; \frac{-Sc}{\alpha^2}\right)}. \tag{24}$$

The non-dimensional form of skin-friction at the wall can be calculated as

$$\tau^* = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = F''(0) = -\alpha. \tag{25}$$

3 Results and discussion

In order to have a physical point of view of the problem, numerical calculations were carried out for different values of suction parameter λ, magnetic parameter (M^2), Prandtl number (Pr), Eckert number (Ec) and Schmidth number (Sc).

The dimensionless transverse and longitudinal velocity profiles for different values of magnetic parameter with constant suction parameter and permeability parameter are presented in Figs. 1 and 2. It is observed that the velocity rose steadily and then converge closely for transverse velocity but different trend is noticed in the longitudinal velocity and also observed that both the transverse and longitudinal velocity of the fluid are decreased for increase in the value of M^2, since the magnetic field exerts a restraining force on the fluid which tends to impede its motion.

The influence of suction parameter over the non-dimensional transverse and longitudinal velocity profiles are shown in Figs. 3 and 4. It is seen that the effect of suction parameter enhances the transverse velocity but it decelerates the longitudinal velocity. However the trend of suction effect over longitudinal velocity is different from the transverse velocity.

In the subsequent analysis, the temperature profiles are discussed due to its primary importance in astrophysical environments. Figs. 5–8 display the temperature profiles for different values suction parameter, magnetic parameter, Prandtl number and Eckert number. Fig. 5 shows that higher value of magnetic parameter caused a rise in temperature. But increase in the suction parameter recorded a decrease in temperature (which is shown in Fig. 6). An increase in Prandtl number Pr is associated with a decrease in the

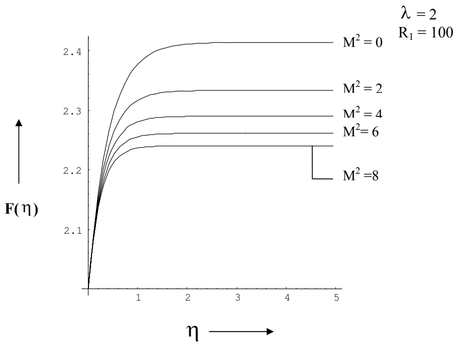

Fig. 1. Non-dimensional transverse velocity profiles for different M^2.

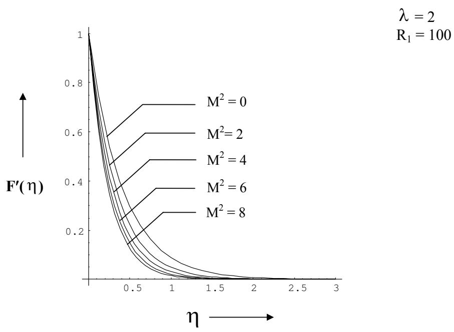

Fig. 2. Non-dimensional longitudinal velocity profiles for various M^2.

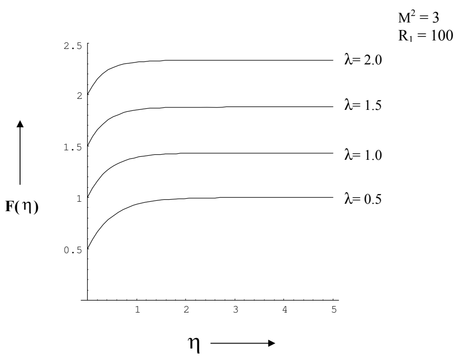

Fig. 3. Effect of λ on dimensionless transverse velocity.

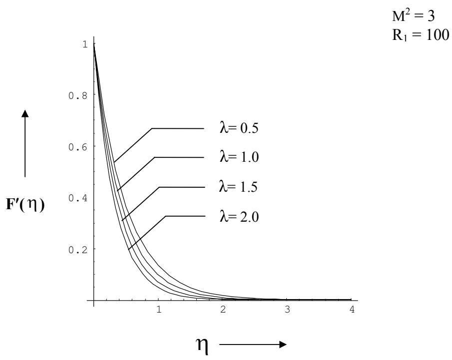

Fig. 4. Dimensionless longitudinal velocity profiles for different λ.

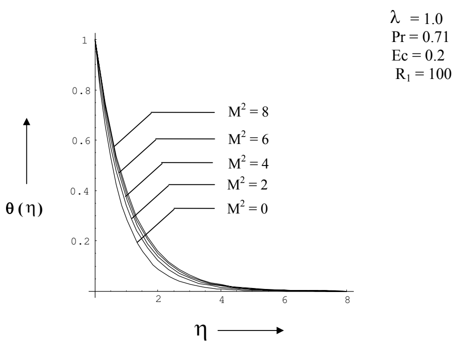

Fig. 5. Temperature distribution for various values of M^2.

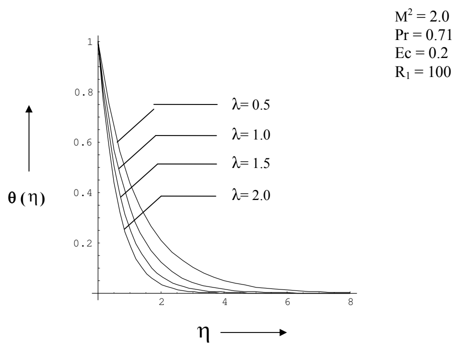

Fig. 6. Temperature distribution for different λ.

temperature distribution which is displayed in Fig. 7. We observed from Fig. 8 that an increase in Eckert number Ec enhances the temperature because the heat energy is stored in the liquid due to the frictional heating. Comparing the Figs. 5–8 it is observed that the formation of the thin boundary layer is observed far away from the wall for higher values of magnetic parameter.

In Figs. 9–11, the behavior of the dimensionless concentration is presented for various material parameters; M, λ and Sc. It is noticed that the concentration of the fluid increase with increase of magnetic parameter as shown in Fig. 9. The concentration of the fluid decreases with increase of suction parameter (which is evident from Fig.10). Fig. 11 display the effect of Schmidt number Sc, on the concentration profile. Increase in Schmidt number decreases the concentration. Comparing the Figs. 8 and 11, it is observed that while the effect of Eckert number is to enhance the temperature, the effect of Schmidth number is to decrease the concentration.

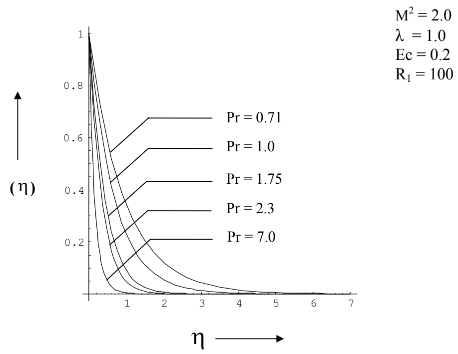

Fig. 7. Effect of Pr over temperature distribution.

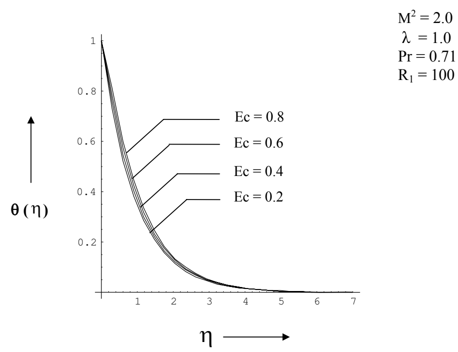

Fig. 8. Temperature distribution for different Ec.

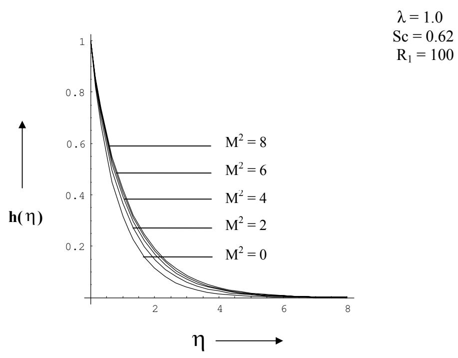

Fig. 9. Dimensionless concentration distribution for different M^2.

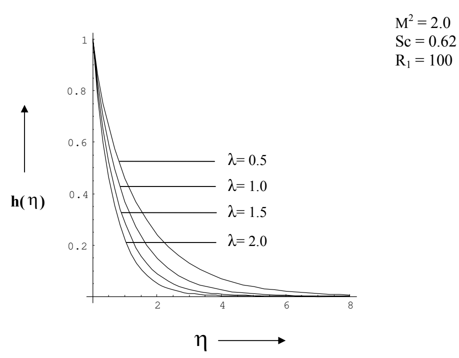

Fig. 10. Dimensionless concentration distribution for various λ.

Variation in skin friction coefficient against magnetic parameter and suction parameter are displayed in Figs. 12 and 13. It is noted that the skin friction decreases as magnetic parameter increases. The effect of suction is to decrease the skin friction coefficient (which is evident from Fig. 13). The rate of heat transfer $\theta'(0)$ as a function of the Eckert number for different magnetic parameter are shown in Fig. 14. It is observed that the rate of heat transfer enhances for increase in both magnetic parameter and Eckert number. Fig. 15 illustrates the variation of rate of heat transfer $\theta'(0)$ against the Ec for different suction parameter. The rate of heat transfer $\theta'(0)$ of the fluid reduces with increase of suction parameter. In Fig. 16, the effect of Eckert number Ec over the rate of heat transfer $\theta'(0)$ for different values of Prandtl number Pr is demonstrated. It is apparent that increase in Pr decreases the rate of heat transfer $\theta'(0)$.

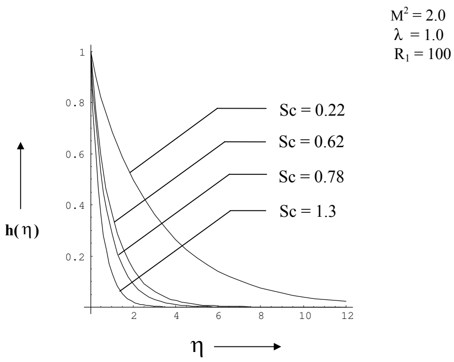

Fig. 11. Effect of Sc over dimensionless concentration distribution.

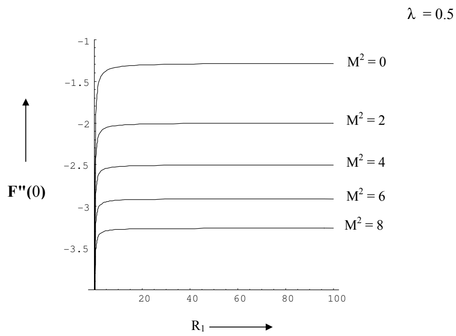

Fig. 12. Skin friction coefficient for various M^2.

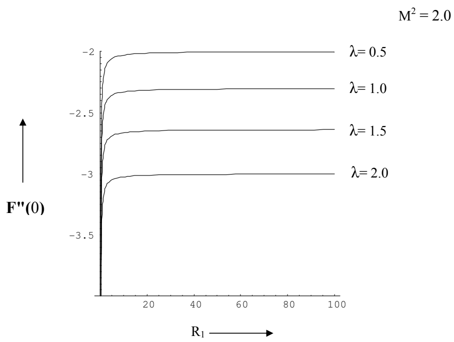

Fig. 13. Skin friction coefficient for various values of λ.

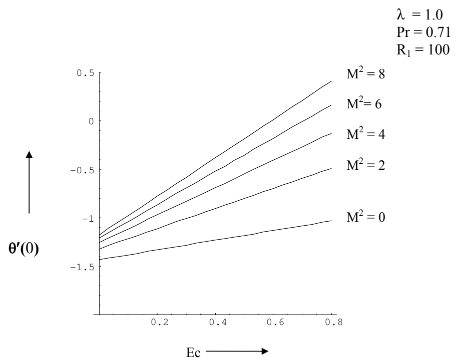

Fig. 14. Influence of M^2 over rate of heat transfer.

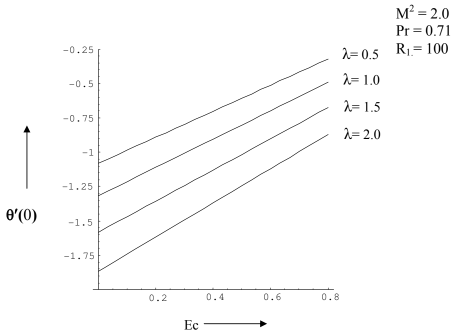

Fig. 15. Influence of λ over rate of heat transfer.

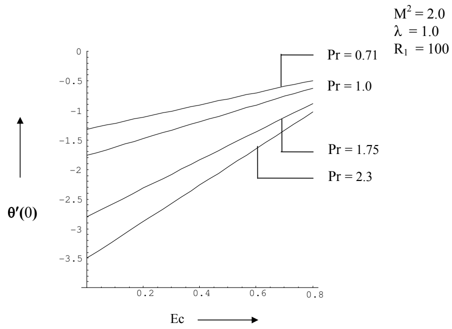

Fig. 16. Surface heat transfer rate against Eckert number for different Pr.

4 Conclusion

The effects of viscous and joules dissipation on MHD flow with heat and mass transfer past a stretching porous surface embedded in a porous medium is analyzed in the present study. In the absence of porous medium, Joules dissipation and mass transfer these results agree quantitatively with the earlier result of Tak and Lodha [10]. The important conclusions of the study are summarized below:

- The effect of magnetic parameter is to decrease both the dimensionless transverse velocity, longitudinal velocity and also the skin friction coefficient.
- In the presence of viscous and Joules dissipation, the effect of magnetic parameter is to increase the temperature, concentration and the heat transfer rate.
- While the effect of suction parameter is to decrease the non-dimensional longitudinal velocity, temperature, concentration, skin friction and rate of heat transfer, its effect is to accelerate the dimensionless transverse velocity.
- Prandtl number reduces both the temperature distribution and heat transfer rate for its increasing values.
- Formation of thin thermal boundary layer is observed faraway from the plate for higher value of suction parameter.
- The flow of heat becomes faster when the Eckert number increases. An increase in Schmidt number results in lowering the concentration distribution steadily.

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