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Multiobjective nonfragile fuzzy control for nonlinear stochastic financial systems with mixed time delays*

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Abstract. In this study, a multiobjective nonfragile control is proposed for a class of stochastic Takagi and Sugeno (T–S) fuzzy systems with mixed time delays to guarantee the optimal H_2 and H_∞ performance simultaneously. Firstly, based on the T–S fuzzy model, two form of nonfragile state feedback controllers are designed to stabilize the T–S fuzzy system, that is to say, nonfragile state feedback controllers minimize the H_2 and H_∞ performance simultaneously. Then, by applying T–S fuzzy approach, the multiobjective H_2/H_∞ nonfragile fuzzy control problem is transformed into linear matrix inequality (LMI)-constrained multiobjective problem (MOP). In addition, we efficiently solve Pareto optimal solutions for the MOP by employing LMI-based multiobjective evolution algorithm (MOEA). Finally, the validity of this approach is illustrated by a realistic design example.

Keywords: multiobjective nonfragile control, mixed time delays, Pareto optimal solutions, linear matrix inequality.

1 Introduction

Stochastic systems have attracted much attention due to their practical application, such as mathematical finance, gene networks, signal processing, etc. [2, 3, 21, 32, 39, 40]. The

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time delays are frequently encountered in many practical engineering systems, such as communication, chemical systems, financial and economic systems, and so on. It has been shown that the existence of time delays in dynamical systems often leads to instability and performance degradation. Therefore, stability analysis and controller synthesis for control systems with time delays has been extensively investigated in the past years, and a great deal of results related to time-delay systems have been reported in the literature [4, 5, 12–14, 27]. Naturally, the study on stochastic systems with time delays has received considerable attention [10, 17–19, 34–36, 42].

During the past two decades, the well-known T–S fuzzy model [6, 30] was considered as a popular and powerful tool for approximating complex nonlinear dynamical systems. Using the fuzzy model, the nonlinear system can be described as a weighted sum of simple linear subsystems, and then the nonlinear system can be stabilized based on fuzzy control. Therefore, the processing method in the traditional linear system theory can be applied to T–S fuzzy systems. In the recent years, the research on T–S fuzzy system has been deeply researched, and a lot of significant results have been obtained, see [8, 9, 15, 16, 22–25, 37, 41] and the references therein. In particular, multiobjective optimization problem of T–S fuzzy systems was investigated in [1, 28] recently. It is worth mentioning that in many industrial applications, inaccuracy will inevitably occur in the implementation of the controller due to numerical rounding errors and actuator degradation, which leads to the study of nonfragile controllers [33]. The desired controller can provide sufficient tuning margin and tolerate uncertainties in its coefficients. Nonfragile fuzzy control was proposed to design a feedback control that will be insensitive to some error or variation in gains of feedback control [20, 31, 43].

Meanwhile, several papers discussed the multiobjective optimization problem for the nonlinear stochastic financial systems in recent literature [29, 38]. Nevertheless, few work devoted to the nonlinear stochastic financial systems with discrete delay and distributed time delay. In fact, the nonlinear stochastic financial systems with mixed time delays can more effectively describe the real economic system. Moreover, the existing literature about the nonlinear stochastic financial systems only considered simple state feedback controllers without taking into account changes in controller parameters. As we known, the design of multiobjective nonfragile controllers for nonlinear stochastic mixed time-delays systems has not been fully studied, which motivates our current research.

Motivated by aforementioned observation, this paper focuses on the nonfragile control for stochastic T–S fuzzy systems with mixed time delays to guarantee the optimal H_2 and H_∞ performance simultaneously. Firstly, nonlinear stochastic T–S fuzzy model with mixed delays, H_2/H_∞ performance, nonfragile state feedback controller with either additive or multiplicative norm-bounded uncertainties are introduced. Secondly, the multiobjective optimization problem of nonlinear stochastic T–S fuzzy system is transformed into a LMI-constrained multiobjective optimization problem by two kinds of nonfragile fuzzy controllers. The stability of nonlinear stochastic mixed-times-delays systems is also analyzed. Third, Multiobjective nonfragile fuzzy control for nonlinear stochastic T–S fuzzy systems with mixed time delays is solved via LMI-constrained MOEA algorithm. Finally, a financial system example is given to show the effectiveness and applicability of the proposed methods.

Notations. The notations used in this paper are standard. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, A^{T} represents the transpose of the matrix A; The notations $A > 0$ ($A \geqslant 0$) is used to denote a symmetric positive-definite (positive-semidefinite) matrix. $*$ denotes the symmetric block in symmetric matrix. $\|X\|$ represent the Euclidean norm of the matrix X. $\mathcal{L}^2_{\mathcal{F}}(\mathbb{R}^+, \mathbb{R}^n)$ is the space of nonanticipative stochastic processes. $y(t) \in \mathbb{R}^l$ with respect to an increasing σ-algebras $\mathcal{F}_t$ ($t \geqslant 0$) satisfies $\|y(t)\|_{\mathcal{L}^2(\mathbb{R}^+;\mathbb{R}^n;Q)} \triangleq \mathbf{E}\{\int_0^\infty y^{\mathrm{T}}(t)Qy(t)\,\mathrm{d}t\}^{1/2}$. $\mathbf{E}$ is the expectation operator. $\mathrm{diag}\{\cdot\}$ stands for a block-diagonal matrix. $\bar{\lambda}(P)$ is the maximum eigenvalue of real-valued matrix P.

2 Problem formulation

Consider the following stochastic financial system with mixed time delays:

$$\mathrm{d}\mathbf{x}(t) = \big(\bar{f} + Bu(t) + v(t)\big)\,\mathrm{d}t + \bar{g}\,\mathrm{d}\omega(t), \tag{1}$$

with

$$\begin{aligned}
\mathbf{x}(t) &= \big[x(t), y(t), z(t), \phi(t)\big]^{\mathrm{T}},\\
u(t) &= \big[u_1(t), u_2(t), u_3(t), 0\big]^{\mathrm{T}},\\
v(t) &= \big[v_1(t), v_2(t), v_3(t), 0\big]^{\mathrm{T}},\\
\bar{f}_1 &= z(t) + \big(y(t) - a\big)x(t) + h_1\phi(t),\\
\bar{f}_2 &= 1 - by(t) - x^2(t) + h_2\big(y(t) - y(t-\tau_1)\big),\\
\bar{f}_3 &= -x(t) - cz(t), \qquad \bar{f}_4 = \mu\big(x(t) - \phi(t)\big),\\
\bar{f} &= [\bar{f}_1,\ \bar{f}_2,\ \bar{f}_3,\ \bar{f}_4]^{\mathrm{T}}, \qquad \bar{g} = [\bar{g}_1,\ \bar{g}_2, \bar{g}_3,\ 0]^{\mathrm{T}},
\end{aligned}$$

where $x(t)$, $y(t)$ and $z(t)$ represent the interest rate, the investment demand and the price index, respectively. $a \geqslant 0$ represents the saving amount, $b \geqslant 0$ is the cost per investment, and $c \geqslant 0$ is the elasticity of demand of commercial markets. $\bar{f} : \mathbb{R}^3 \to \mathbb{R}^3$ and $\bar{g} : \mathbb{R}^3 \to \mathbb{R}^3$ are nonlinear Borel measurable continuous functions, which are satisfied with Lipschitz continuity. B is a 3×3 real-valued constant matrix. The $\mathbf{x}(t) \in \mathbb{R}^3$ is the state vector; the initial state vector $\mathbf{x}(0) = \mathbf{x}_0$. $\tau_1 > 0$ is a constant time delay; distributed time delay (continuous delay) $\phi(t) = \int_{-\infty}^t \mu \exp(-\mu(t-\tau))x(\tau)\,\mathrm{d}\tau$, $\mu \geqslant 0$. Both h_1 and h_2 indicate the feedback intensity. The input vector $u(t) \in \mathcal{L}^2_{\mathcal{F}}(\mathbb{R}^+; \mathbb{R}^3)$ is the admissible regulation effort with respect to $\{\mathcal{F}_t\}_{t\geqslant 0}$. $v(t) \in \mathcal{L}^2_{\mathcal{F}}(\mathbb{R}^+; \mathbb{R}^3)$ is regarded as an unknown finite energy stochastic external disturbance and denotes the external disturbance caused by the international situation like war or natural disaster. The term $\bar{g}\,\mathrm{d}\omega(t)$ can be regarded as a continuous state-dependent internal random fluctuation.

Assume that $\mathbf{x}_d$ is an equilibrium point of nonlinear stochastic system in (1). One can derive from (1) that the transformation $x(t) = \mathbf{x}(t) - \mathbf{x}_d$ transforms nonlinear stochastic system (1) into the following nonlinear stochastic system:

$$\mathrm{d}x(t) = \big(f + Bu(t) + v(t)\big)\,\mathrm{d}t + g\,\mathrm{d}\omega(t), \tag{2}$$

where $f = \bar{f}(x(t) + \mathbf{x}_d,\, x(t - \tau_1) + \mathbf{x}_d)$ and $g = \bar{g}(x(t) + \mathbf{x}_d,\, x(t - \tau_1) + \mathbf{x}_d)$. Thus, the control design problem of nonlinear stochastic system in (1) to the desired $\mathbf{x}_d$ is transformed to the stabilization problem of the nonlinear stochastic system in (2).

Next, we approach the stochastic financial system with discrete delay and distributed time delay (2) by fuzzy interpolation method:

Plant rule:
for $i = 1, 2 \ldots, l$,
if z_1 is G_{i1} and … and z_g is G_{ig}, then

$$\mathrm{d}x(t) = \left[A_i x(t) + A_{1i} x(t - \tau_1) + A_{2i} \int_{t-\tau_2}^{t} x(s)\,\mathrm{d}s + B_i u(t) + C_i v(t)\right] \mathrm{d}t + \left[D_i x(t) + D_{1i} x(t - \tau_1) + D_{2i} \int_{t-\tau_2}^{t} x(s)\,\mathrm{d}s\right] \mathrm{d}\omega(t), \tag{3}$$

where l is the number of fuzzy rules, $z_1 \ldots z_g$ are premise variables, G_{ij} is the fuzzy set. The matrices $A_i, A_{1i}, A_{2i}, B_i, C_i, D_i, D_{1i}, D_{2i} \in \mathbb{R}^{n\times n}$ are constant matrices, and τ_1 and τ_2 are the discrete and distributed time delay, respectively.

By using singleton fuzzifier, product inference and center average defuzzifier, the dynamic model (3) can be expressed by the following global model:

$$\mathrm{d}x(t) = \sum_{i=1}^{l} h_i(z) \left[A_i x(t) + A_{1i} x(t - \tau_1) + A_{2i} \int_{t-\tau_2}^{t} x(s)\,\mathrm{d}s + B_i u(t) + C_i v(t)\right] \mathrm{d}t + \left[D_i x(t) + D_{1i} x(t - \tau_1) + D_{2i} \int_{t-\tau_2}^{t} x(s)\,\mathrm{d}s\right] \mathrm{d}\omega(t), \tag{4}$$

where $z = [z_1^{\mathrm{T}}, z_2^{\mathrm{T}}, \ldots, z_g^{\mathrm{T}}]^{\mathrm{T}}$, $\mu_i(z) = \prod_{j=1}^{g} G_{ij}(z_j)$, $h_i(z) = \mu_i(z)/\sum_{i=1}^{l} \mu_i(z)$, and $G_{ij}(z_j)$ is the membership grade of z_j in G_{ij}. Suppose $\mu_i(z) \geqslant 0$, $i = 1, 2, \ldots, l$, $\sum_{i=1}^{l} \mu_i(z) > 0$. Therefore, $h_i(z) \geqslant 0$ for $i = 1, 2, \ldots, l$, and $\sum_{i=1}^{l} h_i(z) = 1$.

Remark 1. Recently, T–S fuzzy systems have been used to efficient approximated nonlinear dynamic systems [1,9,15,28,30]. We also give an example to show that stochastic T–S fuzzy system (4) can effectively approximate nonlinear stochastic financial system (2) in the final simulation.

Similarly, a nonfragile state feedback controller is designed for a given T–S fuzzy system (4) as follows:

Control rule j:
for $i = 1, 2 \ldots, l$,
if z_1 is G_{j1} and … and z_g is G_{jg}, then
$u(t) = (K_j + \varDelta K_j) x(t)$,

The matrix K_j is to be designed such that the closed-loop system is stable. ΔK_j is the gain variation of K_j and ΔK_j is assumed to be of the two form:

Case 1. ΔK_j has an additive uncertainty:

$$\Delta K_j = M_{1j} F_{1j}(t) N_{1j}; \tag{5}$$

Case 2. ΔK_j has a multiplicative uncertainty:

$$\Delta K_j = M_{2j} F_{2j}(t) N_{2j} K_j, \tag{6}$$

where M_{1j}, N_{1j}, M_{2j} and N_{2j} are known real constant matrices, F_{1j} and F_{2j} are unknown matrix functions satisfying $F_{1j}^{\mathrm{T}} F_{1j} \leqslant I$, $F_{2j}^{\mathrm{T}} F_{2j} \leqslant I$.

Hence, the overall fuzzy controller is given by

$$u(t) = \sum_{j=1}^{l} h_j(z)(K_j + \Delta K_j)x(t), \tag{7}$$

where $h_j(z)$ is designed as (4).

Under control (7), the overall closed-loop system is obtained as follows:

$$\begin{aligned} \mathrm{d}x(t) = \sum_{i=1}^{l} \sum_{j=1}^{l} h_i(z) h_j(z) \Bigg[& A_i x(t) + A_{1i} x(t - \tau_1) \\ & + A_{2i} \int_{t-\tau_2}^{t} x(s)\,\mathrm{d}s + B_i(K_j + \Delta K_j)x(t) + C_i v(t) \Bigg] \mathrm{d}t \\ & + \Bigg[D_i x(t) + D_{1i} x(t - \tau_1) + D_{2i} \int_{t-\tau_2}^{t} x(s)\,\mathrm{d}s \Bigg] \mathrm{d}\omega(t), \end{aligned} \tag{8}$$

Based on the fuzzy augmented system (8), the H_2 control performance $J_2(u(t))$ index without considering the effect of external disturbance $v(t)$ can be represented by

$$J_2\big(u(t)\big) = \big\|x(t)\big\|^2_{\mathcal{L}^2_{\mathcal{F}}(\mathbb{R}^+;\mathbb{R}^n,Q_1)} + \big\|u(t)\big\|^2_{\mathcal{L}^2_{\mathcal{F}}(\mathbb{R}^+;\mathbb{R}^n,R_1)}, \tag{9}$$

where $Q_1 > 0$ and $R_1 > 0$ are weighting matrices to tradeoff between regulation error $x(t)$ and control input $u(t)$. Further, in order to efficiently mitigate the effects of external disturbance $v(t)$, the H_∞ control performance index $J_\infty(u(t))$ of the fuzzy system in (8) is defined as follows:

$$J_\infty\big(u(t)\big) = \sup_{\substack{v(t) \in \mathcal{L}^2_{\mathcal{F}}(\mathbb{R}^+,\mathbb{R}^n) \\ v \neq 0,\, x_0 \neq 0}} \frac{\|x(t)\|^2_{\mathcal{L}^2_{\mathcal{F}}(\mathbb{R}^+;\mathbb{R}^n,Q_2)} + \|u(t)\|^2_{\mathcal{L}^2_{\mathcal{F}}(\mathbb{R}^+;\mathbb{R}^n,R_2)} - \mathbf{E}\{V(0)\}}{\|v(t)\|^2_{\mathcal{L}^2_{\mathcal{F}}(\mathbb{R}^+;\mathbb{R}^n)}}, \tag{10}$$

where $Q_2 > 0$, $R_2 > 0$. The robust H_∞ performance index $J_\infty(u(t))$ in (8) is denoted as the effect of external disturbance.

Remark 2. Because the H_2 performance and H_∞ performance of the system are often two mutually-constrained performance indicators, it is not possible to reach the maximum or minimum at the same time, so it can be regarded as a multiobjective optimization problem.

The multiobjective H_2/H_∞ control design problem is to specify the control input $u(t)$ in (7) such that the H_2 performance (9) and robust H_∞ performance (10) are all minimized simultaneously. Throughout this paper, the following definitions and lemmas are used to derive the main results.

Definition 1. (See [11].) The solution $x(t) \equiv 0$ of system (4) is said to be exponentially 2-stable (stability in mean square) if, for some positive constants A and k,

$$\mathbf{E}\{\|x(t)\|^2\} \leqslant A\|x_0\|^2 \exp(-kt).$$

Definition 2. The multiobjective H_2/H_∞ control design problem of a nonlinear stochastic T–S fuzzy system with mixed delays (4) is to design an admissible control design $u(t)$ in (7), which could make the H_2 and H_∞ performance indices minimum in the Pareto optimal sense, simultaneously, i.e.,

$$\min_{u(t)\in\mathcal{U}} \big(J_2\big(u(t)\big), J_\infty\big(u(t)\big)\big) \quad \text{s.t. (4)}, \tag{11}$$

where $\mathcal{U}$ is the set of all the admissible control; the objective functional $J_2(u(t))$ and $J_\infty(u(t))$ are defined in (9) and (10), respectively; the vector of the objective functionals $(J_2(u(t)), J_\infty(u(t)))$ is called objective vector of $u(t)$.

Lemma 1. (*See* [28].) *Suppose α and β are the upper bounds of the H_2 and H_∞ performance indices, respectively, i.e., $J_2(u(t)) \leqslant \alpha$, and $J_\infty(u(t)) \leqslant \beta$. The MOP in* (11) *is equivalent to the MOP given in the following*:

$$\min_{u(t)\in\mathcal{U}} (\alpha, \beta) \quad \textit{s.t. } J_2\big(u(t)\big) \leqslant \alpha \textit{ and } J_\infty\big(u(t)\big) \leqslant \beta.$$

Lemma 2. (*See* [38].) *For any two real matrices X, Y with appropriate dimensions and for any constant $\eta > 0$, we have*:

$$X^{\mathrm{T}}Y + Y^{\mathrm{T}}X \leqslant \eta^2 X^{\mathrm{T}}Y + \frac{1}{\eta^2} Y^{\mathrm{T}}Y.$$

Lemma 3. (*See* [19].) *Let K, M, N and $R > 0$ be real matrices of appropriate dimensions, and $F(t)$ be function matrices satisfying $F^{\mathrm{T}}(t)F(t) \leqslant I$. Then the following statements hold*:

(i) *For any scalar $\varepsilon > 0$ and vectors $x, y \in \mathbb{R}^n$,*

$$2x^{\mathrm{T}}MF(t)Ny \leqslant \varepsilon x^{\mathrm{T}}MM^{\mathrm{T}}x + \varepsilon^{-1}y^{\mathrm{T}}N^{\mathrm{T}}Ny.$$

(ii) *For any scalar $\varepsilon > 0$ such that $R^{-1} - \varepsilon MM^{\mathrm{T}} > 0$,*

$$\big(K + MF(t)N\big)^{\mathrm{T}}R\big(K + MF(t)N\big) \leqslant K^{\mathrm{T}}\big(R^{-1} - \varepsilon MM^{\mathrm{T}}\big)^{-1}K + \varepsilon^{-1}N^{\mathrm{T}}N.$$

Lemma 4. (*See* [26].) *For any matrix D_i with appropriate dimension and the scheduling functions $h_i(z)$ with $0 \leqslant h_i(z) \leqslant 1$, for $i \in N_+$, $1 \leqslant i \leqslant l$, $P > 0$, and $\sum_{i=1}^{l} h_i(z) = 1$, we have*

$$\left(\sum_{j=1}^{l} h_j(z)D_j\right)^{\mathrm{T}} P \sum_{i=1}^{l} h_i(z)D_i \leqslant \sum_{i=1}^{l} h_i(z)D_i^{\mathrm{T}} P D_i.$$

3 Main results

3.1 Multiobjective H_2/H_∞ control design

In this section, there are two sufficient conditions for the nonfragile control input $u(t)$ in (5) and (6) to solve the multiobjective H_2/H_∞ control problem for nonlinear stochastic T–S fuzzy model with mixed time delays in (4), respectively.

Case 1. For the additive gain variation model $\Delta K_j = M_{1j}F_{1j}(t)N_{1j}$, we have the following design condition.

Theorem 1. *For given scalars τ_1 and τ_2, if there exist positive scalars ε_1, ε_2 and positive definite symmetric matrices P, Z_1, Z_2 and Z_3 with appropriate dimensions, such that the following LMIs-constrained MOP can be solved:*

$$\min_{P,Z_1,Z_2,Z_3,K_1,\dots,K_l} (\alpha,\beta) \quad \text{s.t. the following LMIs for all } i,j = 1,2,\dots,l, \tag{12}$$

$$P + \tau_1 Z_1 + \tau_1^2 Z_2 + \frac{3}{2}\tau_2^3 Z_3 \leqslant \alpha\gamma^{-1} I, \tag{13}$$

$$\begin{bmatrix} \Pi_{11}^{(1)} & \Pi_{12}^{(1)} \\ * & \Pi_{22}^{(1)} \end{bmatrix} \leqslant 0, \tag{14}$$

$$\begin{bmatrix} \Pi_{11}^{(2)} & \Pi_{12}^{(2)} \\ * & \Pi_{22}^{(2)} \end{bmatrix} \leqslant 0, \tag{15}$$

where

$$W = P^{-1}, \qquad Z_1 = V^{-1}, \qquad Z_2 = U^{-1}, \qquad Z_3 = T^{-1}, \qquad Y_j = K_j W,$$

$$\Psi_1 = A_i W + W A_i^{\mathrm{T}} + B_i Y_j + (B_i Y_j)^{\mathrm{T}} + \varepsilon_2 B_i M_{1j} M_{1j}^{\mathrm{T}} B_i,$$

$$\Psi_2 = A_i W + W A_i^{\mathrm{T}} + B_i Y_j + (B_i Y_j)^{\mathrm{T}} + \varepsilon_2 B_i M_{1j} M_{1j}^{\mathrm{T}} B_i + \frac{1}{\beta} C_i C_i^{\mathrm{T}},$$

$$\Pi_{11}^{(1)} = \begin{bmatrix} \Psi_1 & A_{1i}V & A_{2i}T & 0 \\ * & -V & 0 & 0 \\ * & * & -T & 0 \\ * & * & * & -U \end{bmatrix}, \qquad \Pi_{11}^{(2)} = \begin{bmatrix} \Psi_2 & A_{1i}V & A_{2i}T & 0 \\ * & -V & 0 & 0 \\ * & * & -T & 0 \\ * & * & * & -U \end{bmatrix},$$

$$\Pi_{12}^{(1)} = \Pi_{12}^{(2)} = \begin{bmatrix} W D_i^{\mathrm{T}} & W & Y_j^{\mathrm{T}} & W N_{1j}^{\mathrm{T}} \\ V D_{1i}^{\mathrm{T}} & 0 & 0 & 0 \\ T D_{2i}^{\mathrm{T}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\varPi_{22}^{(1)} = \operatorname{diag}\biggl\{-W,\ -Q_1^{-1} - V - \frac{1}{\tau_1^2}U - \frac{1}{\tau_2^2}T,\ -R_1^{-1} + \varepsilon_1 M_{1j} M_{1j}^{\mathrm{T}},\ -(\varepsilon_1 + \varepsilon_2)I\biggr\},$$
$$\varPi_{22}^{(2)} = \operatorname{diag}\biggl\{-W,\ -Q_2^{-1} - V - \frac{1}{\tau_1^2}U - \frac{1}{\tau_2^2}T,\ -R_2^{-1} + \varepsilon_1 M_{1j} M_{1j}^{\mathrm{T}},\ -(\varepsilon_1 + \varepsilon_2)I\biggr\},$$

then the multiobjective H_2/H_∞ control $u(t) = \sum_{j=1}^{l} h_j(z)(K_j + M_{1j}F_{1j}(t)N_{1j})x(t)$ with $K_j = Y_j W^{-1}$ for the stochastic T–S fuzzy model with mixed delays in (8) *can be solved.*

Proof. Define the following Lyapunov function candidate for nonlinear stochastic T–S fuzzy model with mixed delays (8):

$$V(t) = \sum_{k=1}^{5} V_k(t), \tag{16}$$

where

$$V_1(t) = x^{\mathrm{T}}(t)Px(t), \qquad V_2(t) = \int_{t-\tau_1}^{t} x^{\mathrm{T}}(s)Z_1 x(s),$$
$$V_3(t) = \int_{-\tau_1}^{0}\int_{t+\theta}^{t} x^{\mathrm{T}}(s)Z_2 x(s)\,\mathrm{d}s\,\mathrm{d}\theta,$$
$$V_4(t) = \int_{t-\tau_2}^{t}\Biggl[\int_{s}^{t} x^{\mathrm{T}}(\theta)\,\mathrm{d}\theta\Biggr] Z_3 \Biggl[\int_{s}^{t} x(\theta)\,\mathrm{d}\theta\Biggr]\mathrm{d}s,$$
$$V_5(t) = \int_{0}^{\tau_2}\int_{t-s}^{t} (\theta - t + s)x^{\mathrm{T}}(\theta)Z_3 x(\theta)\,\mathrm{d}\theta\,\mathrm{d}s.$$

Using Lemmas 2–4 and Itô formula, we have

$$\begin{aligned}
LV_1(t) \leqslant{}& 2\sum_{i=1}^{l}\sum_{j=1}^{l} h_i(z)h_j(z)x^{\mathrm{T}}P\Biggl[A_i x(t) + A_{1i}x(t-\tau_1) + A_{2i}\int_{t-\tau_2}^{t} x(s)\,\mathrm{d}s \\
&+ B_i(K_j + M_{1j}F_{1j}(t)N_{1j})x(t) + C_i v(t)\Biggr] + \eta^{\mathrm{T}}D^{\mathrm{T}}PD\eta \\
\leqslant{}& 2\sum_{i=1}^{l}\sum_{j=1}^{l} x^{\mathrm{T}}P\Biggl[A_i x(t) + A_{1i}x(t-\tau_1) \\
&+ A_{2i}\int_{t-\tau_2}^{t} x(s)\,\mathrm{d}s + B_i K_j x(t) + C_i v(t)\Biggr] \\
&+ x^{\mathrm{T}}(t)\bigl[\varepsilon_2 x^{\mathrm{T}}(t)PB_i M_{1j}(PB_i M_{1j})^{\mathrm{T}} + \varepsilon_2^{-1}N_{1j}^{\mathrm{T}}N_{1j}\bigr]x(t) + \eta^{\mathrm{T}}D^{\mathrm{T}}PD\eta,
\end{aligned}$$

$$
\begin{aligned}
LV_2(t) &= x^{\mathrm{T}}(t)Z_1x(t) - x^{\mathrm{T}}(t-\tau_1)Z_1x(t-\tau_1),\\
LV_3(t) &= \tau_1x^{\mathrm{T}}(t)Z_2x(t) - \int\limits_{t-\tau_1}^{t} x^{\mathrm{T}}(s)Z_1x(s)\,\mathrm{d}s,\\
LV_4(t) &= 2\int\limits_{t-\tau_2}^{t} (\theta - t + \tau_2)x^{\mathrm{T}}(t)Z_3x(\theta)\,\mathrm{d}\theta - \int\limits_{t-\tau_2}^{t} x^{\mathrm{T}}(\theta)\,\mathrm{d}\theta\ Z_3\int\limits_{t-\tau_2}^{t} x(\theta)\,\mathrm{d}\theta\\
&\leqslant \frac{\tau_2^2}{2}x^{\mathrm{T}}(t)Z_3x(t) + \int\limits_{t-\tau_2}^{t} (\theta - t + \tau_2)x^{\mathrm{T}}(\theta)Z_3x(\theta)\,\mathrm{d}\theta\\
&\quad - \int\limits_{t-\tau_2}^{t} x^{\mathrm{T}}(\theta)\,\mathrm{d}\theta\ Z_3\int\limits_{t-\tau_2}^{t} x(\theta)\,\mathrm{d}\theta,\\
LV_5(t) &= \frac{\tau_2^2}{2}x^{\mathrm{T}}(t)Z_3x(t) - \int\limits_{t-\tau_2}^{t} (\theta - t + \tau_2)x^{\mathrm{T}}(\theta)Z_3x(\theta)\,\mathrm{d}\theta,
\end{aligned}
$$

where $\eta = [x^{\mathrm{T}}(t)\ x^{\mathrm{T}}(t-\tau_1)\ \int_{t-\tau_2}^{t} x^{\mathrm{T}}(s)\,\mathrm{d}s]^{\mathrm{T}}$, $D = [D_i\ D_{1i}\ D_{2i}]$. Therefore,

$$
\begin{aligned}
LV(t) &= \sum_{k=1}^{5} LV_k(t)\\
&\leqslant 2\sum_{i=1}^{l}\sum_{j=1}^{l} x^{\mathrm{T}}P\Bigg[A_ix(t) + A_{1i}x(t-\tau_1) + A_{2i}\int\limits_{t-\tau_2}^{t} x(s) + B_iK_jx(t)\,\mathrm{d}s\\
&\quad + C_iv(t)\Bigg] + x^{\mathrm{T}}(t)\big[\varepsilon_2^{-1}x^{\mathrm{T}}(t)PB_iM_{1j}(PB_iM_{1j})^{\mathrm{T}} + \varepsilon_2N_{1j}^{\mathrm{T}}N_{1j}\big]x(t)\\
&\quad + \eta^{\mathrm{T}}D^{\mathrm{T}}PD\eta + x^{\mathrm{T}}(t)Z_1x(t) - x^{\mathrm{T}}(t-\tau_1)Z_1x(t-\tau_1)\\
&\quad + \tau_1x^{\mathrm{T}}(t)Z_2x(t) - \int\limits_{t-\tau_1}^{t} x^{\mathrm{T}}(s)Z_1x(s)\,\mathrm{d}s\\
&\quad + \tau_2^2x^{\mathrm{T}}(t)Z_3x(t) - \int\limits_{t-\tau_2}^{t} x^{\mathrm{T}}(\theta)\,\mathrm{d}\theta\ Z_3\int\limits_{t-\tau_2}^{t} x(\theta)\,\mathrm{d}\theta.
\end{aligned}
\tag{17}
$$

In order to get $J_2(u(t)) \leqslant \alpha$ and $J_\infty(u(t)) \leqslant \beta$, first, we prove the sufficient conditions for $J_2(u(t)) \leqslant \alpha$ of the MOP in (8).

$$
J_2\big(u(t)\big) = \mathbf{E}\Bigg\{\int\limits_0^\infty \big(x^{\mathrm{T}}(t)Q_1x(t) + u^{\mathrm{T}}(t)R_1u(t)\big)\,\mathrm{d}t\Bigg\},
$$

$$\begin{aligned}
J_2\big(u(t)\big) &\leqslant \mathbf{E}\big\{V(0)\big\} + \mathbf{E}\Bigg\{\int_0^\infty \big(x^{\mathrm{T}}(t)Q_1x(t) + u^{\mathrm{T}}(t)R_1u(t)\big)\,\mathrm{d}t + dV(t)\Bigg\} \\
&= \mathbf{E}\big\{V(0)\big\} + \mathbf{E}\Bigg\{\int_0^\infty \big(x^{\mathrm{T}}(t)Q_1x(t) + u^{\mathrm{T}}(t)R_1u(t)\big)\,\mathrm{d}t + LV(t)\,\mathrm{d}t\Bigg\} \\
&\leqslant \mathbf{E}\big\{V(0)\big\} + \mathbf{E}\Bigg\{\int_0^\infty \Bigg\{x^{\mathrm{T}}(t)Q_1x(t) + u^{\mathrm{T}}(t)R_1u(t) \\
&\quad + 2\sum_{i=1}^{l}\sum_{j=1}^{l} x^{\mathrm{T}}P\Bigg[A_ix(t) + A_{1i}x(t-\tau_1) + A_{2i}\int_{t-\tau_2}^{t} x(s)\,\mathrm{d}s + B_iK_jx(t)\Bigg] \\
&\quad + x^{\mathrm{T}}(t)\big[\varepsilon_2^{-1}x^{\mathrm{T}}(t)PB_iM_{1j}(PB_iM_{1j})^{\mathrm{T}} + \varepsilon_2N_{1j}^{\mathrm{T}}N_{1j}\big]x(t) \\
&\quad + \eta^{\mathrm{T}}D^{\mathrm{T}}PD\eta + x^{\mathrm{T}}(t)Z_1x(t) - x^{\mathrm{T}}(t-\tau_1)Z_1x(t-\tau_1) \\
&\quad + \tau_1x^{\mathrm{T}}(t)Z_2x(t) - \int_{t-\tau_1}^{t} x^{\mathrm{T}}(s)Z_1x(s)\,\mathrm{d}s \\
&\quad + \tau_2^2x^{\mathrm{T}}(t)Z_3x(t) - \int_{t-\tau_2}^{t} x^{\mathrm{T}}(\theta)\,\mathrm{d}\theta\ Z_3\int_{t-\tau_2}^{t} x(\theta)\,\mathrm{d}\theta\Bigg\}\,\mathrm{d}t\Bigg\}.
\end{aligned}$$

Now we show that the following two inequalities hold. On the one hand,

$$\mathbf{E}\big\{V(0)\big\} \leqslant \bar\lambda(P)\mathbf{E}\big\{(x_0^{\mathrm{T}}x_0)\big\} + \bar\lambda(Z_1)\tau_1\gamma + \bar\lambda(Z_2)\tau_1^2\gamma + \frac{3}{2}\bar\lambda(Z_3)\tau_2^3\gamma \leqslant \alpha, \tag{18}$$

where $\bar\lambda(P)$, $\bar\lambda(Z_1)$, $\bar\lambda(Z_2)$ and $\bar\lambda(Z_3)$ are maximum eigenvalue of P, Z_1, Z_2 and Z_3, respectively, $\gamma = \max_{-\bar\tau\leqslant s\leqslant 0}\{\mathbf{E}\{x^{\mathrm{T}}(s)x(s)\}\}$, $\bar\tau = \max\{\tau_1,\tau_2\}$, i.e., $P + \tau_1Z_1 + \tau_1^2Z_2 + (3/2)\tau_2^3Z_3 \leqslant \alpha\gamma^{-1}I$, which is (12). Then, by using Lemma 3, we have

$$\begin{aligned}
&u^{\mathrm{T}}(t)R_1u(t) \\
&\quad= \sum_{j=1}^{l} h_j(z)x^{\mathrm{T}}(t)(K_j + \varDelta K_j)^{\mathrm{T}}R_1(K_j + \varDelta K_j)x(t) \\
&\quad= \sum_{j=1}^{l} h_j(z)x^{\mathrm{T}}(t)\big(K_j + M_{1j}F_{1j}(t)N_{1j}\big)^{\mathrm{T}}R_1\big(K_j + M_{1j}F_{1j}(t)N_{1j}\big)x(t) \\
&\quad\leqslant \sum_{j=1}^{l} h_j(z)x^{\mathrm{T}}(t)\big[K_j^{\mathrm{T}}\big(R^{-1} - \varepsilon_1M_{1j}M_{1j}^{\mathrm{T}}\big)^{-1}K_j + \varepsilon_1^{-1}N_{1j}N_{1j}^{\mathrm{T}}\big].
\end{aligned} \tag{19}$$

Therefore,

$$
\begin{aligned}
& x^{\mathrm{T}}(t)Q_1x(t) + u^{\mathrm{T}}(t)R_1u(t) + 2\sum_{i=1}^{l}\sum_{j=1}^{l} x^{\mathrm{T}}P\Bigg[A_ix(t) + A_{1i}x(t-\tau_1) \\
&\quad + A_{2i}\int\limits_{t-\tau_2}^{t} x(s)\,\mathrm{d}s + B_iK_jx(t) + C_iv(t)\Bigg] \\
&\quad + x^{\mathrm{T}}(t)\big[\varepsilon_2^{-1}x^{\mathrm{T}}(t)PB_iM_{1j}(PB_iM_{1j})^{\mathrm{T}} + \varepsilon_2N_{1j}^{\mathrm{T}}N_{1j}\big]x(t) + \eta^{\mathrm{T}}D^{\mathrm{T}}PD\eta \\
&\quad + x^{\mathrm{T}}(t)Z_1x(t) - x^{\mathrm{T}}(t-\tau_1)Z_1x(t-\tau_1) + \tau_1x^{\mathrm{T}}(t)Z_2x(t) \\
&\quad - \int\limits_{t-\tau_1}^{t} x^{\mathrm{T}}(s)Z_1x(s)\,\mathrm{d}s + \tau_2^2x^{\mathrm{T}}(t)Z_3x(t) - \int\limits_{t-\tau_2}^{t} x^{\mathrm{T}}(\theta)\,\mathrm{d}\theta\ Z_3\int\limits_{t-\tau_2}^{t} x(\theta)\,\mathrm{d}\theta \\
&\leqslant \sum_{i=1}^{l}\sum_{j=1}^{l}\vartheta^{\mathrm{T}}(t)\varXi_1\vartheta(t),
\end{aligned} \tag{20}
$$

where $\vartheta(t) = [x^{\mathrm{T}}(t),\ x^{\mathrm{T}}(t-\tau_1),\ \int_{t-\tau_2}^{t} x^{\mathrm{T}}(s)\,\mathrm{d}s,\ \int_{t-\tau_1}^{t} x^{\mathrm{T}}(s)\,\mathrm{d}s]^{\mathrm{T}}$ and

$$
\varXi_1 = \begin{bmatrix} \varPi_1 & PA_{1i} & PA_{2i} & 0 \\ * & -Z_1 & 0 & 0 \\ * & * & -Z_3 & 0 \\ * & * & * & -Z_2 \end{bmatrix} + \begin{bmatrix} D_i^{\mathrm{T}} \\ D_{1i}^{\mathrm{T}} \\ D_{2i}^{\mathrm{T}} \\ 0 \end{bmatrix} P\big[D_i\ \ D_{1i}\ \ D_{2i}\ \ 0\big], \tag{21}
$$

$$
\begin{aligned}
\varPi_1 = {} & Q_1 + K_j^{\mathrm{T}}\big(R_1^{-1} - \varepsilon_1M_{1j}M_{1j}^{\mathrm{T}}\big)^{-1}K_j + \varepsilon_1^{-1}N_{1j}^{\mathrm{T}}N_{1j} + A_i^{\mathrm{T}}P \\
& + PA_i + PB_iK_j + (PB_iK_j)^{\mathrm{T}} + \varepsilon_2PB_iM_{1j}\big(PB_iM_{1j}^{\mathrm{T}}\big) \\
& + \varepsilon_2^{-1}N_{1j}^{\mathrm{T}}N_{1j} + Z_1 + \tau_1^2Z_2 + \tau_2^2Z_3.
\end{aligned}
$$

Let $W = P^{-1}$, $Z_1 = V^{-1}$, $Z_2 = U^{-1}$, $Z_3 = T^{-1}$, $Y_j = K_jW$, and pre-multiplying and post-multiplying (21) by matrix $\varLambda = \operatorname{diag}\{W, V, T, U\}$, we are easy to get

$$
\varLambda\varXi_1\varLambda = \begin{bmatrix} W\varPi_1W & A_{1i}V & A_{2i}T & 0 \\ * & -V & 0 & 0 \\ * & * & -T & 0 \\ * & * & * & -U \end{bmatrix} + \begin{bmatrix} WD_i^{\mathrm{T}} \\ VD_{1i}^{\mathrm{T}} \\ TD_{2i}^{\mathrm{T}} \\ 0 \end{bmatrix} W^{-1}\big[D_iW\ \ D_{1i}V\ \ D_{2i}T\ \ 0\big],
$$

where

$$
\begin{aligned}
W\varPi_1W = {} & WQ_1W + Y_j^{\mathrm{T}}\big(R_1^{-1} - \varepsilon_1M_{1j}M_{1j}^{\mathrm{T}}\big)^{-1}Y_j + \varepsilon_1^{-1}WN_{1j}^{\mathrm{T}}N_{1j}W \\
& + WA_i^{\mathrm{T}} + A_iW + B_iY_j + (B_iY_j)^{\mathrm{T}} + \varepsilon_2B_iM_{1j}\big(B_iM_{1j}^{\mathrm{T}}\big) \\
& + \varepsilon_2^{-1}WN_{1j}^{\mathrm{T}}N_{1j}W + WZ_1W + \tau_1^2WZ_2W + \tau_2^2WZ_3W.
\end{aligned}
$$

On the other hand, applying the Schur complement formula to the LMI in (14) results in $\varLambda\varXi_1\varLambda \leqslant 0$, which together with (18) gives $J_2(u(t)) \leqslant \alpha$.

Next, we prove the sufficient conditions for $J_\infty(u(t)) \leqslant \beta$ of the MOP in (8).

$$\begin{aligned}
&\mathbf{E}\Biggl\{\int_0^\infty \bigl(x^{\mathrm{T}}(t)Q_2x(t) + u^{\mathrm{T}}(t)R_2u(t)\bigr)\,\mathrm{d}t\Biggr\}\\
&\leqslant \mathbf{E}\bigl\{V(0)\bigr\} + \mathbf{E}\Biggl\{\int_0^\infty \bigl(x^{\mathrm{T}}(t)Q_2x(t) + u^{\mathrm{T}}(t)R_2u(t)\bigr)\,\mathrm{d}t + dV(t)\Biggr\}\\
&= \mathbf{E}\bigl\{V(0)\bigr\} + \mathbf{E}\Biggl\{\int_0^\infty \bigl(x^{\mathrm{T}}(t)Q_2x(t) + u^{\mathrm{T}}(t)R_2u(t)\bigr)\,\mathrm{d}t + LV(t)\,\mathrm{d}t\Biggr\}\\
&\leqslant \mathbf{E}\bigl\{V(0)\bigr\} + \mathbf{E}\Biggl\{\int_0^\infty \Biggl\{x^{\mathrm{T}}(t)Q_2x(t) + u^{\mathrm{T}}(t)R_2u(t)\\
&\quad + 2\sum_{i=1}^{l}\sum_{j=1}^{l} x^{\mathrm{T}}P\Biggl[A_ix(t) + A_{1i}x(t-\tau_1) + A_{2i}\int_{t-\tau_2}^{t} x(s)\,\mathrm{d}s + B_iK_jx(t)\Biggr]\\
&\quad + x^{\mathrm{T}}(t)\bigl[\varepsilon_2^{-1}x^{\mathrm{T}}(t)PB_iM_{1j}(PB_iM_{1j})^{\mathrm{T}} + \varepsilon_2N_{1j}^{\mathrm{T}}N_{1j}\bigr]x(t)\\
&\quad + \frac{1}{\beta}x^{\mathrm{T}}(t)PC_iC_i^{\mathrm{T}}Px(t) + \beta v^{\mathrm{T}}(t)v(t) + \eta^{\mathrm{T}}D^{\mathrm{T}}PD\eta + x^{\mathrm{T}}(t)Z_1x(t)\\
&\quad - x^{\mathrm{T}}(t-\tau_1)Z_1x(t-\tau_1) + \tau_1x^{\mathrm{T}}(t)Z_2x(t) - \int_{t-\tau_1}^{t} x^{\mathrm{T}}(s)Z_1x(s)\,\mathrm{d}s\\
&\quad + \tau_2^2x^{\mathrm{T}}(t)Z_3x(t) - \int_{t-\tau_2}^{t} x^{\mathrm{T}}(\theta)\,\mathrm{d}\theta\ Z_3\int_{t-\tau_2}^{t} x(\theta)\,\mathrm{d}\theta\Biggr\}\,\mathrm{d}t\Biggr\}.
\end{aligned} \tag{22}$$

From (19) we can obtain

$$\begin{aligned}
&x^{\mathrm{T}}(t)Q_2x(t) + u^{\mathrm{T}}(t)R_2u(t)\\
&\quad + 2\sum_{i=1}^{l}\sum_{j=1}^{l} x^{\mathrm{T}}P\Biggl[A_ix(t) + A_{1i}x(t-\tau_1) + A_{2i}\int_{t-\tau_2}^{t} x(s)\,\mathrm{d}s + B_iK_jx(t)\Biggr]\\
&\quad + x^{\mathrm{T}}(t)\bigl[\varepsilon_2^{-1}x^{\mathrm{T}}(t)PB_iM_{1j}(PB_iM_{1j})^{\mathrm{T}} + \varepsilon_2N_{1j}^{\mathrm{T}}N_{1j}\bigr]x(t)\\
&\quad + \frac{1}{\beta}x^{\mathrm{T}}(t)PC_iC_i^{\mathrm{T}}Px(t) + \eta^{\mathrm{T}}D^{\mathrm{T}}PD\eta + x^{\mathrm{T}}(t)Z_1x(t) + \tau_1x^{\mathrm{T}}(t)Z_2x(t)
\end{aligned}$$

$$
\begin{aligned}
&- \int_{t-\tau_1}^{t} x^{\mathrm{T}}(s) Z_1 x(s)\,\mathrm{d}s + \tau_2^2 x^{\mathrm{T}}(t) Z_3 x(t) - x^{\mathrm{T}}(t-\tau_1) Z_1 x(t-\tau_1) \\
&- \int_{t-\tau_2}^{t} x^{\mathrm{T}}(\theta)\,\mathrm{d}\theta \ Z_3 \int_{t-\tau_2}^{t} x(\theta)\,\mathrm{d}\theta \\
&\leqslant \sum_{i=1}^{l}\sum_{j=1}^{l} \vartheta^{\mathrm{T}}(t) \Xi_2 \vartheta(t),
\end{aligned}
$$

where

$$
\Xi_2 = \begin{bmatrix} \Pi_2 & PA_{1i} & PA_{2i} & 0 \\ * & -Z_1 & 0 & 0 \\ * & * & -Z_3 & 0 \\ * & * & * & -Z_2 \end{bmatrix} + \begin{bmatrix} D_i^{\mathrm{T}} \\ D_{1i}^{\mathrm{T}} \\ D_{2i}^{\mathrm{T}} \\ 0 \end{bmatrix} P \begin{bmatrix} D_i & D_{1i} & D_{2i} & 0 \end{bmatrix}, \tag{23}
$$

$$
\begin{aligned}
\Pi_2 = {}& Q_2 + K_j^{\mathrm{T}} \left(R_2^{-1} - \varepsilon_1 M_{1j} M_{1j}^{\mathrm{T}}\right)^{-1} K_j + \varepsilon_1^{-1} N_{1j}^{\mathrm{T}} N_{1j} + A_i^{\mathrm{T}} P + PA_i \\
&+ PB_i K_j + (PB_i K_j)^{\mathrm{T}} + \frac{1}{\beta} P C_i C_i^{\mathrm{T}} P + \varepsilon_2 PBM_{1j}\left(PBM_{1j}^{\mathrm{T}}\right) \\
&+ \varepsilon_2^{-1} N_{1j}^{\mathrm{T}} N_{1j} + Z_1 + \tau_1^2 Z_2 + \tau_2^2 Z_3.
\end{aligned}
$$

Pre-multiplying and post-multiplying (23) by matrix Λ, we have

$$
\begin{aligned}
\Lambda \Xi_2 \Lambda = {}& \begin{bmatrix} W\Pi_2 W & A_{1i} V & A_{2i} T & 0 \\ * & -V & 0 & 0 \\ * & * & -T & 0 \\ * & * & * & -U \end{bmatrix} \\
&+ \begin{bmatrix} WD_i^{\mathrm{T}} \\ VD_{1i}^{\mathrm{T}} \\ TD_{2i}^{\mathrm{T}} \\ 0 \end{bmatrix} W^{-1} \begin{bmatrix} D_i W & D_{1i} V & D_{2i} T & 0 \end{bmatrix},
\end{aligned} \tag{24}
$$

where

$$
\begin{aligned}
W\Pi_2 W = {}& WQ_2 W + Y_j^{\mathrm{T}} \left(R_2^{-1} - \varepsilon_1 M_{1j} M_{1j}^{\mathrm{T}}\right)^{-1} Y_j + \varepsilon_1^{-1} W N_{1j}^{\mathrm{T}} N_{1j} W + W A_i^{\mathrm{T}} \\
&+ A_i W + B_i Y_j + (B_i Y_j)^{\mathrm{T}} + \frac{1}{\beta} C_i C_i^{\mathrm{T}} + \varepsilon_2 B_i M_{1j}\left(B_i M_{1j}^{\mathrm{T}}\right) \\
&+ \varepsilon_2^{-1} W N_{1j}^{\mathrm{T}} N_{1j} W + W Z_1 W + \tau_1^2 W Z_2 W + \tau_2^2 W Z_3 W.
\end{aligned}
$$

Now, from the LMI in (15) it is easy is see, as shown in (24), which by the Schur complement formula implies that $\Lambda \Xi_2 \Lambda \leqslant 0$. From this and (22) we have $J_2(u(t)) \leqslant \beta$ for all $v(t) \neq 0$. This completes the proof. □

Case 2. For the multiplicative gain variation model $\Delta K_j = M_{2j} F_{2j}(t) N_{2j} K_j$, we have the following design condition.

Theorem 2. *For given scalars τ_1 and τ_2, if there exist positive scalars ε_3, ε_4 and positive definite symmetric matrices P, Z_1, Z_2 and Z_3 with appropriate dimensions such that the following LMIs-constrained MOP can be solved*:

$$\min_{P,Z_1,Z_2,Z_3,K_1,\dots,K_l}(\alpha,\beta) \quad \text{s.t. the following LMIs for all } i,j=1,2,\dots,l, \tag{25}$$

$$P+\tau_1 Z_1+\tau_1^2 Z_2+\frac{3}{2}\tau_2^3 Z_3 \leqslant \alpha\gamma^{-1}I, \tag{26}$$

$$\begin{bmatrix} \bar{\Pi}_{11}^{(1)} & \bar{\Pi}_{12}^{(1)} \\ * & \bar{\Pi}_{22}^{(1)} \end{bmatrix} \leqslant 0, \tag{27}$$

$$\begin{bmatrix} \bar{\Pi}_{11}^{(2)} & \bar{\Pi}_{12}^{(2)} \\ * & \bar{\Pi}_{22}^{(2)} \end{bmatrix} \leqslant 0, \tag{28}$$

where

$$\begin{aligned}
&W=P^{-1}, \qquad Z_1=V^{-1}, \qquad Z_2=U^{-1}, \qquad Z_3=T^{-1}, \qquad Y_j=K_jW,\\
&\bar{\Psi}_1=A_iW+WA_i^{\mathrm{T}}+B_iY_j+(B_iY_j)^{\mathrm{T}}+\varepsilon_4B_iM_{2j}M_{2j}^{\mathrm{T}}B_i,\\
&\bar{\Psi}_2=A_iW+WA_i^{\mathrm{T}}+B_iY_j+(B_iY_j)^{\mathrm{T}}+\varepsilon_4B_iM_{2j}M_{2j}^{\mathrm{T}}B_i+\frac{1}{\beta}C_iC_i^{\mathrm{T}},
\end{aligned}$$

$$\bar{\Pi}_{11}^{(1)}=\begin{bmatrix} \bar{\Psi}_1 & A_{1i}V & A_{2i}T & 0 \\ * & -V & 0 & 0 \\ * & * & -T & 0 \\ * & * & * & -U \end{bmatrix}, \qquad \bar{\Pi}_{11}^{(2)}=\begin{bmatrix} \bar{\Psi}_2 & A_{1i}V & A_{2i}T & 0 \\ * & -V & 0 & 0 \\ * & * & -T & 0 \\ * & * & * & -U \end{bmatrix},$$

$$\bar{\Pi}_{12}^{(1)}=\bar{\Pi}_{12}^{(2)}=\begin{bmatrix} WD_i^{\mathrm{T}} & W & Y_j^{\mathrm{T}} & Y_j^{\mathrm{T}}N_{2j}^{\mathrm{T}} \\ VD_{1i}^{\mathrm{T}} & 0 & 0 & 0 \\ TD_{2i}^{\mathrm{T}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{\Pi}_{22}^{(1)}=\operatorname{diag}\left\{-W,\ -Q_1^{-1}-V-\frac{1}{\tau_1^2}U-\frac{1}{\tau_2^2}T,\ -R_1^{-1}+\varepsilon_3M_{2j}M_{2j}^{\mathrm{T}},\ -(\varepsilon_1+\varepsilon_2)I\right\},$$

$$\bar{\Pi}_{22}^{(2)}=\operatorname{diag}\left\{-W,\ -Q_2^{-1}-V-\frac{1}{\tau_1^2}U-\frac{1}{\tau_2^2}T,\ -R_2^{-1}+\varepsilon_3M_{2j}M_{2j}^{\mathrm{T}},\ -(\varepsilon_1+\varepsilon_2)I\right\},$$

then the multiobjective H_2/H_∞ control $u(t)=\sum_{j=1}^{l}h_j(z)(K_j+M_{2j}F_{2j}(t)N_{2j}K_j)x(t)$ with $K_j=Y_jW^{-1}$ for the stochastic T–S fuzzy model with mixed delays in (8) *can be solved.*

Proof. It is similar with Theorem 1 by replacing the multiplicative gain with (10). The proof is omitted. □

Remark 3. Multiobjective H_2/H_∞ control can be regarded as the design of multiobjective H_2/H_∞ investment policy can be seen as how to search a management policy $u(t)$ to maximize higher return on investment (ROI) (H_2 management policy) and minimize investment risk (H_∞ management policy) the stochastic financial system with mixed time delays (2) or (4), simultaneously.

3.2 Stochastic stability analysis

In this section, assuming that the nonfragile control is known and we will study the conditions under which the closed-loop system is stochastically exponentially stable in the mean square. The following theorem will play a key role in the stability analysis of closed-loop system and design of the expected nonfragile control.

Theorem 3. *The trivial solution* $x(t) \equiv 0$ *of the nonlinear stochastic T–S fuzzy system with mixed delays* (4) *is said to be exponentially 2-stable (stability in mean square) if the external noise* $v(t) = 0$, *and* $u(t)$ *is a feasible solution of the MOP in* (12).

Proof. Since the given Lyapunov function (16) is satisfied with the following two inequalities:

$$k_1 \|x(t)\|^2 \leqslant V(t) \leqslant k_2 \|x(t)\|^2, \tag{29}$$

where $k_1 > 0$ and $k_2 > 0$, Expressing the difference $V(t) - V(0)$ by means of Itô formula, calculating its expectations, we get

$$\mathbf{E}\{V(t) - V(0)\} = \int_0^t \mathbf{E}\{LV(s)\}\,\mathrm{d}s, \tag{30}$$

from (20) and (29) we have

$$LV(t) \leqslant -x^{\mathrm{T}}(t) Q_1 x(t) \leqslant -k_3 \|x(t)\|^2 \leqslant -\frac{k_3}{k_2} V\big(x(t)\big), \tag{31}$$

differentiating equality (30) with respect to t and using (31), we see that

$$\frac{\mathrm{d}}{\mathrm{d}t} \mathbf{E}\{V(t)\} = \mathbf{E}\{LV(t)\} \leqslant -\frac{k_3}{k_2} \mathbf{E}\{V(t)\}.$$

This implies the estimate

$$\mathbf{E}\{\|x(t)\|^2\} \leqslant \mathbf{E}\frac{V(t)}{k_1} \leqslant \mathbf{E}\frac{V(0)}{k_1} \exp\left(-\frac{k_3 t}{k_2}\right),$$

it is obvious that

$$\lim_{t\to\infty} \mathbf{E}\{\|x(t)\|^2\} = 0,$$

i.e., $\lim_{t\to\infty} x(t) = 0$, so $x(t) \equiv 0$ exponentially stability in the mean square sense. The proof is completed. □

4 LMI-constrained MOEA

In this section, an LMI-constrained MOEA searching algorithm is developed to help us solve the MOP in Theorem 1 or Theorem 2. Before giving the design steps, three definitions for solutions of the MOP in (12) or (25) are provided to guarantee the domination of candidate solutions (α, β) in the searching process as follows:

Definition 3 [Pareto dominance]. (See [7].) Consider the LMI-constrained MOP in (13)–(15) or (26)–(28). A feasible objective vector (α^1, β^1) is said to dominate another feasible objective vector (α^2, β^2) if and only if $\alpha^1 \leqslant \alpha^2$ and $\beta^1 \leqslant \beta^2$ for at least one inequality being a strict inequality.

Definition 4 [Pareto optimality]. (See [7].) Let $(W^1, V^1, U^1, T^1, K_1^1, \dots, K_l^1)$ and $(W^2, V^2, U^2, T^2, K_1^2, \dots, K_l^2)$ be the feasible solution corresponding to the objective value (α^1, β^1) and (α^2, β^2) subject to the LMIs in (13)–(15) or (26)–(28) for all $i, j = 1, \dots, l$, respectively. $(W^1, V^1, U^1, T^1, K_1^1, \dots, K_l^1)$ is said to dominate $(W^2, V^2, U^2, T^2, K_1^2, \dots, K_l^2)$ if $\alpha^1 \leqslant \alpha^2$ and $\beta^1 \leqslant \beta^2$ for at least one inequality being a strict inequality.

Definition 5 [Pareto front]. (See [7].) For the MOP in (19), the Pareto front P_F^* is defined as $P_F^* = \{(\alpha^*, \beta^*) \,|\, (W^*, V^*, U^*, T^*, K_1^*, \dots, K_l^*)\}$, and (α^*, β^*) is generated by $(W^*, V^*, U^*, T^*, K_1^*, \dots, K_l^*)\}$ subject to the LMIs in (13)–(15) or (26)–(28).

In this paper, the LMI-constrained MOEA of multiobjective nonfragile fuzzy control design for nonlinear stochastic T–S fuzzy systems with mixed time delays is similar to the literatures [1] and [28] as follows:

Step 1. Select the searching range $(\alpha_0, \beta_0) \times (\bar{\alpha}, \bar{\beta})$ for the feasible objective vector (α, β) and set the iteration number $\bar{N}$, the population number N_p, the crossover ration N_c, and the mutation ratioN_m in the LMI-constrained MOEA.

Step 2. Select N_p feasible chromosomes from the feasible chromosome set randomly to be the initial population P_1.

Step 3. Set iteration index $N_i = 1$.

Step 4. Operate the EA with the crossover ratio N_c, the mutation ratio N_m, and generate $2N_p$ number feasible chromosomes by examining whether their corresponding objective vectors are feasible objective vectors for the LMIs in (13)–(15) or (26)–(28).

Step 5. Set the iteration index $N_i = N_i + 1$ and select N_p chromosomes from the $2N_p$ feasible chromosomes in Step 4 through nondominated sorting method to be the population P_{N_i}.

Step 6. Repeat Steps 4 and 5 until the iteration number $\bar{N}$ is reached. If the iteration number $\bar{N}$ is satisfied, then we set $P_{N_i} = P_F$.

Step 7. Select a preferable feasible objective individual $(\alpha^\dagger, \beta^\dagger) \in P_F$ according to designer own preference. Once the preferable feasible objective individual is selected, the corresponding Pareto optimal solution $\varsigma^\dagger = \{W^\dagger, V^\dagger, U^\dagger, T^\dagger, K_1^\dagger, \dots, K_l^\dagger\}$ is obtained. By using $\varsigma^\dagger$, the proposed multiobjective H_2/H_∞ fuzzy control design problem $u(t)$

can be constructed, and the multiobjective H_2/H_∞ control design problem in (4) can be solved with $J_2(u(t)) = \alpha^\dagger$ and $J_\infty(u(t)) = \beta^\dagger$, simultaneously.

5 Illustrative example

In this section, we shall give an example to demonstrate the effectiveness of the proposed approach. The related parameters and disturbance are given as follows:

$$a = 1.5, \quad b = 0.2, \quad c = 0.25, \quad \mu = 0.2, \quad \tau_1 = 0.1, \quad \tau_2 = 0.2,$$
$$h_1 = h_2 = 0.1, \quad x_0 = [0.85, -1.93, 0.7], \quad \mathbf{x}_d = [0.1, 4.5, -0.2],$$
$$\bar{g}_1 = 0.03\big[z(t) + (y(t) - a)x(t)\big], \quad \bar{g}_2 = 0.01\big[1 - by(t) - (x(t))^2\big],$$
$$\bar{g}_3 = 0.02\big[-x(t) - cz(t)\big], \quad v(t) = \big[0.01\sin(2t), -0.02\sin(2t), -0.01\sin(2t)\big],$$

$$A_1 = \begin{bmatrix} 1.2300 & 0.6000 & 1.0000 \\ -0.2391 & 1.3000 & 0 \\ -0.9667 & 0 & -0.2500 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.9600 & 0.6099 & 1.0000 \\ -0.2391 & 0.9100 & 0 \\ -0.9550 & 0 & -0.2500 \end{bmatrix},$$

$$A_{11} = \begin{bmatrix} 1.1935 & 0.3000 & 1.0000 \\ -0.2273 & 1.5000 & 0 \\ -0.9866 & 0 & -0.2500 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0.9243 & 0.3015 & 1.0000 \\ -0.2273 & 1.1000 & 0 \\ -0.9627 & 0 & -0.2500 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} 0.9467 & 0.1000 & 1.0000 \\ -0.2058 & 1.700 & 0 \\ -0.9950 & 0 & -0.2500 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} 0.8901 & 0.1000 & 1.0000 \\ -0.2058 & 1.400 & 0 \\ -0.9860 & 0 & -0.2500 \end{bmatrix},$$

$$M_{11} = M_{12} = \begin{bmatrix} 0.01 & 0.05 & 0 \\ 0.02 & 0.03 & 0 \\ 0 & 0.01 & -0.01 \end{bmatrix}, \quad N_{11} = N_{12} = \begin{bmatrix} 0.01 & 0 & 0.05 \\ 0.02 & 0.01 & 0 \\ 0.03 & 0 & 0.01 \end{bmatrix},$$

$$D_1 = D_2 = \begin{bmatrix} 0.2800 & 0.1800 & 0.3000 \\ -0.0239 & 0.1300 & 0 \\ -0.1933 & 0 & -0.0500 \end{bmatrix}, \quad F_{11} = F_{12} = \begin{bmatrix} \sin t & 0 & 0 \\ 0 & \cos t & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$D_{11} = D_{12} = \begin{bmatrix} 0.238 & 0.0300 & 0.3000 \\ -0.0227 & 0.1300 & 0 \\ -0.1933 & 0 & -0.0500 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix},$$

$$D_{21} = D_{22} = \begin{bmatrix} 0.194 & 0.0300 & 0.3000 \\ -0.0227 & 0.0910 & 0 \\ -0.1910 & 0 & -0.0500 \end{bmatrix}, \quad B_1 = C_1 = C_2 = I_{3\times3},$$

$$Q_1 = R_1 = Q_2 = I_{3\times3}, \quad R_2 = 0.5I_{3\times3}.$$

Before considering the investment policy $u(t)$, the three states $x(t)$, $y(t)$ and $z(t)$ of the nonlinear stochastic system in (1) are described in Fig. 1.

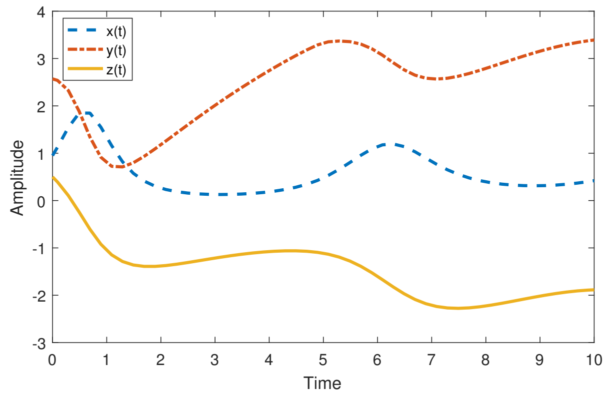

Figure 1. Trajectories of the interest rate $x(t)$, the investment demand $y(t)$ and the price index $z(t)$ for system in (1).

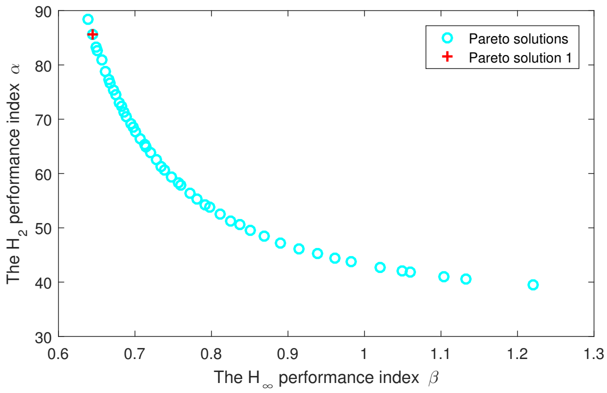

Figure 2. Pareto front for 50 optimal solutions.

It is easy to see the fluctuations in the real environment from Fig. 1. Now we will consider introducing introducing the investment policy $u(t)$. For the proposed LMI-based MOEA to solve the MOP (12), the selection range $\Gamma = [30, 90] \times [0.6, 1.2]$, the iteration number $\bar{N} = 100$, population number $N_p = 50$, crossover ratio $N_c = 0.8$, and mutation ratio $N_m = 0.15$, based on the nonfragile state feedback controller nonfragile feedback controller with additive form and Theorem 1, there are 50 Pareto optimal solutions as the Pareto front in Fig. 2.

We select one of the Pareto objective vectors $(85.6301, 0.6447)$. By using the toolbox of LMI, K_1, K_2 are obtained respectively:

$$K_1 = \begin{bmatrix} -21.3067 & -19.4730 & -11.7339 \\ -9.2337 & -27.1337 & -6.5989 \\ -5.9666 & -4.1976 & -5.2869 \end{bmatrix},$$

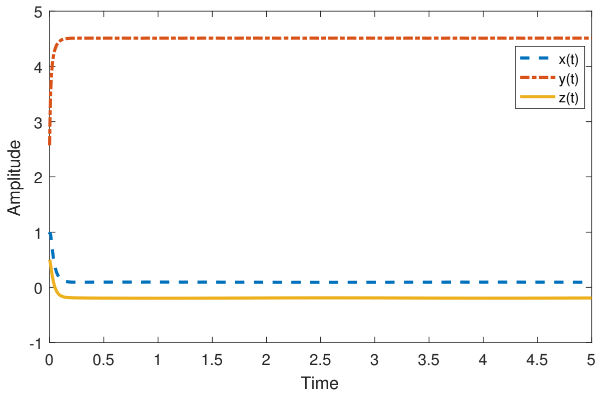

Figure 3. The trajectories $x(t)$, $y(t)$ and $z(t)$ of the chosen Pareto solution.

$$K_2 = \begin{bmatrix} -21.4217 & -19.5645 & -11.8055 \\ -9.2576 & -27.1576 & -6.6092 \\ -5.9575 & -4.2253 & -5.2854 \end{bmatrix}.$$

Then we get interest rate $x(t)$, investment demend $y(t)$ and price index $z(t)$ trajectories in Fig. 3. From Fig. 3 we can see that $x(t)$, $y(t)$ and $z(t)$ have reached the desired state of investors, respectively.

6 Conclusion

This study has investigated the multiobjective nonfragile fuzzy control design for a class of nonlinear dynamic systems with mixed time delays to guarantee the optimal H_2 and H_∞ performance simultaneously. T–S fuzzy model has been used to approximate the nonlinear dynamic system. By the help of the T–S fuzzy model, two form of nonfragile state feedback controllers has been designed to stabilize the nonlinear dynamic system, and the multiobjective H_2/H_∞ nonfragile fuzzy control problem has been transformed into LMI-constrained MOP. Furthermore, we have efficiently solved 50 Pareto optimal solutions of the MOP. The designers can freely choose multiobjective H_2/H_∞ control strategy according to their own preferences. Finally, an example has been provided to illustrate the effectiveness of the proposed approach.

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