Fuzzy Probabilistic Analysis of Steel Structure Focused on Reliability Design Concept of Eurocodes*

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Abstract. The modern unification of the European standards EUROCODE requires securing a constant quality of metallurgical production in the EU countries. In this paper, experimentally found statistical characteristics of yield stress, ultimate tensile strength and ductility of Czech and Austrian steel are presented. In the probabilistic reliability analysis, the experimentally found yield stress histograms of structural steel S235 of both Czech and Austrian manufacturing processes are considered as basic parameters. The reliability of steel members designed according to EUROCODE 3 is investigated. The objective of the studies is the verification of partial safety factors of load-carrying capacity, and of load action given in the standard EN1990. Differences in failure probabilities of steel members of Czech and Austrian production are studied in connection with the influence of model fuzzy uncertainties in the determination of load action and load-carrying capacity values.

Keywords: Eurocode, stability, constructional steel, stochastic yield stress, structural reliability, fuzzy sets, fuzzy reliability.

1 Introduction

In practical design, the primary reliability of structural systems and objects is ensured by unified standard design prescriptions – EUROCODE. Besides the design standards, the production quality of load-carrying members also plays an important role in the resulting steel structure reliability; in individual EU countries, this quality may vary in dependence on different production technologies.

One of the primary problems in the EU is the definition of the optimal reliability level of structural systems. Securing the optimal reliability level requires control of the optimal variability of material properties and tolerances on shape and dimensions of metallurgical production in individual EU countries. The definition of the optimal

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reliability level of structural systems, in particular securing it from the point of view of the optimal variability of quality parameters (size and dispersion of material properties) and of tolerances on shape and dimensions of metallurgical production in individual EU countries is among the primary problems. In the Czech Republic, the quality and reliability of materials and steel products are controlled both by manufacturers and at independent scientific workplaces.

As in many probabilistic reliability studies, material properties are input data identified, at scientific workplaces, with maximum objectivity [1]. Although the statistical data are satisfactory, the quality and reliability of Czech materials and steel products are sometimes considered to be unsatisfactory in western countries. In this context, the unfinished elaboration state of probabilistic studies is of topical significance with aim at investigating to what extent differences in manufacturing quality can influence the reliability from the point of view of transparency and the verification of processes applied in practice [2]. The material properties obtained in the Czech Republic [1] were compared with material properties of Austrian steels [3, 4]. Evaluations were carried out independently at an Austrian workplace in Vienna and at a Czech workplace in Brno; this guaranteed maximum objectivity of results and conclusions drawn from them. Comparison of statistical characteristics of yield stress, ultimate tensile strength and ductility provide satisfactory evidence that, on the European market, Czech products are fully competitive [4]. The topic of the presented studies is the application of this knowledge to probabilistic studies focused at reliability design concepts of Eurocodes.

The paper is aimed at the probability study of the ultimate limit state of a hot-rolled beam IPE220 of steel grade S235 designed according to [5] with maximum load-carrying capacity. The misalignment of the design failure probability according to [6] is studied applying statistical yield stress characteristics of Austrian and Czech steel. Discrepancies between failure probabilities of members from Czech and Austrian production are compared. Numerous uncertainties, which are not of random character, exist during the evaluation of the failure probability [7]. With the aim to analyse the effect of these uncertainties, the probability calculation is supplemented with fuzzy analysis. The fuzzy inputs were considered to be model uncertainties in determining the load action and load-carrying capacity effects. The fuzzy analysis of output failure probabilities was evaluated according to the general extension principle [8]. The fuzzy numbers of failure probability are the outputs. The supports of fuzzy numbers are compared with crisp failure probability values of steel members of Austrian and Czech production.

2 Parametric probabilistic study of the steel member

The reliability of a bar under permanent (G) and long-time variable (Q) load actions was analysed in the parametric study. The standard design reliability condition according to [6] can be written in the form:

$$\gamma_G G_k + \gamma_Q Q_k \leq R_{A\chi} f_{y,k} / \gamma_M, \tag{1}$$

where $R_{A\chi} = \chi A$ is the product of buckling coefficient χ and cross sectional area A, values G_k, Q_k represent the characteristic load action values, $f_{y,k}$ is the yield stress characteristic value. The design reliability is secured by partial safety factors γ. The standard design reliability condition (1) can be rewritten as an inequality of the design load action S_d and the design load-carrying capacity R_d:

$$S_d \leq R_d. \tag{2}$$

In the probabilistic analysis, the function of ultimate limit state is expressed by the inequality:

$$G + Q \leq R. \tag{3}$$

Failure occurs if condition (3) is not fulfilled, i.e., if the random load action effect $S = G + Q$ is higher than the random load-carrying capacity R. The random load-carrying capacity R is a function of material and geometrical imperfections, which can be determined from experimental research [1, 4]. The member load-carrying capacity is calculated from the relation:

$$R = f_y A. \tag{4}$$

According to [6], it can be presumed during the determination of the statistical characteristics of random load actions G and Q that the characteristic values G_k and Q_k are quantiles of a Gaussian probability distribution and of a Gumbel distribution, respectively. It can be assumed for the permanent load action G that the characteristic value G_k represents the mean value of the Gaussian probability density function, and that the variation coefficient equals 0.1. Gumbel distribution with mean value $0.6\,Q_k$ and standard deviation $0.21Q_k$ was considered for the long-time random load action. When defining characteristic load action values G_k, Q_k, it is assumed that in (1), the design value of load action effects S_d is equal to the design value of load-carrying capacity R_d determined according to [5]. The aim of the study is the analysis of the failure probability in dependence on the parameter δ, which expresses the ratio of variable load action Q_k to the general load action $G_k + Q_k$.

$$\delta = \frac{Q_k}{G_k + Q_k}. \tag{5}$$

3 Experimental results of mechanical and geometrical characteristics

The yield stress is the basic mechanical characteristic of structural steels. The yield stress is controlled in metallurgical works, and is utilized for the determination of design resistance values and the partial safety factor values connected with steel structural design standards EUROCODES.

Statistical characteristics of yield stress, ultimate tensile strength and ductility of steel grade S235 produced both in Bohemia and Austria are given in Table 1. Sample elements of 20 mm thick metal sheets were analysed. The results pertaining to Austrian production were obtained from measurements on 1123 samples. Czech steel results were obtained from measurements on 5293 samples. The nominal yield stress of tested samples for element of thickness $t \leq 40$ mm is 235 MPa according to [5].

Table 1. Mechanical characteristics of Austrian and Czech steel

	Austrian steel S235			Czech steel S235		
	Yield stress	Ultimate strength	Ductility [%]	Yield stress	Ultimate strength	Ductility [%]
Mean value	289.01 MPa	408.85 MPa	38.28 %	284.43 MPa	421.77 MPa	37.902 %
Standard deviation	19.82 MPa	18.83 MPa	2.99 %	21.59 MPa	19.322 MPa	3.057 %
Coef. of variation	0.0697	0.0461	0.0781	0.0759	0.0458	0.0806
Stand. skewness	0.61083	1.4547	−0.65823	0.61429	0.84048	−0.41169
Stand. kurtosis	1.3312	7.078	1.4492	1.6107	5.4322	0.7847

Comparison of results illustrate that the Austrian steel yields a slightly higher yield stress and lower standard deviation, which results in their higher reliability. As will be illustrated later, this difference is negligible, i.e., yield stress statistical characteristics of both manufacturers are in very good agreement from the technical point of view. The yield stress histograms are presented in Fig. 1.

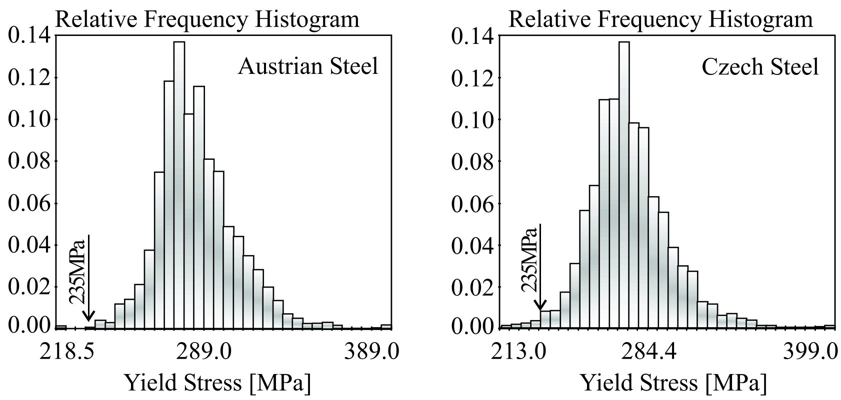

Fig. 1. Histograms of yield stress of Austrian and Czech steel S235.

The cross section of the IPE220 is defined by the parameters h, b, t_1, t_2, which represent further input random quantities. The nominal geometrical cross sectional dimensions of IPE220 are presented in Fig. 2.

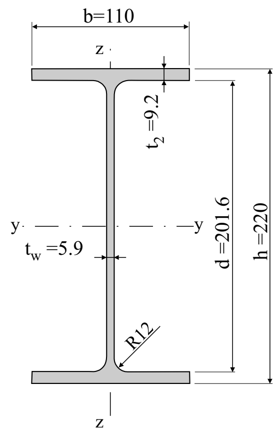

Fig. 2. Geometry of hot-rolled profile IPE220.

The cross sectional area calculated from three rectangles using nominal values $h = 220$ mm, $b = 110$ mm, $t_1 = 5.9$ mm, $t_2 = 9.2$ mm is 3213 mm^2. The tabular nominal value (taking into account rounding off in corners) is 3340 mm^2; this value is higher by 127 mm^2. As only random quantities h, b, t_1, t_2, have been measured and statistically evaluated by experimental research [1], the area random function will be written in the form:

$$A = 2bt_2 + (h - 2t_2)t_1 + 127. \qquad (6)$$

The area of 127 mm^2 was considered as deterministic. According to comparison studies, it does not actually influence the probabilistic analysis results. The input random quantities are specified in Table 2.

Table 2. Input statistical characteristics

Symbol	Quantity		Density function	Mean value	Standard deviation
h	Cross sectional height		Histogram	220.22 mm	0.975 mm
b	Flange width		Histogram	111.49 mm	1.093 mm
t_1	Web thickness		Histogram	6.225 mm	0.247 mm
t_2	Flange thickness		Histogram	9.136 mm	0.421 mm
G	Permanent action		Gauss	G_k	$0.1G_k$
Q	Variable action		Gumbel	$0.6Q_k$	$0.21Q_k$
f_y	Yield stress	Austrian	Histogram	289.01 MPa	19.82 MPa
		Czech	Histogram	284.43 MPa	21.59 MPa

4 Probabilistic analysis

The probabilistic analysis results were obtained utilizing the Monte Carlo simulation. The probability that condition (3) is not fulfilled was evaluated. Random load-carrying capacity R was calculated by (4) and (6) using random quantities from Table 2. Input statistical characteristics of load actions G and Q in (3) are defined in Table 2 by characteristic values G_k and Q_k. Characteristic values G_k and Q_k are calculated according to the relation:

$$1.35G_k + 1.5Q_k = 784.9\text{ kN}. \tag{7}$$

Equation (7) is derived from (1) for partial safety factors $\gamma_G = 1.35$; $\gamma_Q = 1.5$ and $\gamma_M = 1.0$ [6]. The value 784.9 kN on the right side of equation (7) represents the design load-carrying capacity of profile IPE220, determined by the partial safety factors method according to [5]:

$$R_d = \frac{Af_y}{\gamma_M} = \frac{3.34 \cdot 10^{-3} \cdot 235 \cdot 10^6}{1.0} = 784.9 \cdot 10^3\text{ N}. \tag{8}$$

While determining the characteristic values of dead G_k and variable Q_k load it is necessary in (7) to choose the ratio δ (5). The δ value is stepwise increased and sampling is repeated in order to get a dependency between δ and failure probability P_f. G_k and Q_k (for selected δ) are fixed for each simulation run.

Practically: for selected value of parameter δ (e.g. $\delta = 0.1$) the characteristic values G_k and Q_k were evaluated according to (7). All input random variable in Table 2, which are necessary for the evaluation of the failure probability according to (3), are known upon the evaluation of G_k and Q_k.

Sufficient runs of the Monte Carlo simulation were used for determining the failure probability P_f, so that condition (3) was not fulfilled minimally 200 times. This guarantees a balanced probability assessment error of approximately 7 %. This problem was analysed for $\delta = 0, 0.1, \ldots, 1$. The probabilistic study results are depicted in Fig. 3.

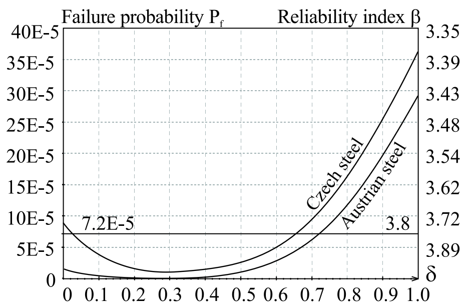

Fig. 3. Misalignment of failure probability by [6].

It is apparent from Fig. 3 that higher failure probability was obtained for the Czech steel, which has a lower mean value and higher standard deviation of yield stress than the Austrian steel. The design of the member from Czech steel is satisfactory (failure probability is lower than the reference value $7.2 \cdot 10^{-5}$) for $\delta \in \langle 0.03; 0.66 \rangle$. For the

Austrian steel the interval $\delta \in \langle 0.0; 0.72\rangle$ is reliable, i.e., the structural stress is relatively unsafe for high values of long-time variable load action (Q). Differences between both results are low, and as will be shown below, they can be clearly transcended by the fuzzy uncertainty of the stochastic calculation model.

5 Fuzzy probabilistic analysis

The apparatus of mathematical statistics provides the classical form of representation of uncertainty. The theoretical value of failure probability of structures may be evaluated provided sufficient information on input random variables and their correlation is available. Limitation of stochastic models rests mainly in the ability to reflect only uncertainty of the stochastic nature. In the event that sufficient information on input random variables is unavailable, a further source of uncertainty is of fuzzy (vague) origin. The notion “fuzzy”was firstly used by Prof. Lotfi Zadeh in 1962 [9]. In 1965, L. Zadeh published his pioneer, today still classical paper entitled “Fuzzy sets” [10]. Commonly encountered problems may be characterised by both fuzzy and stochastic uncertainty. This “combined” fuzzy-random uncertainty can be modelled by applying fuzzy random variables and fuzzy random functions only [7, 11]. Newer mathematical approaches, which extend or depart from the probability theory, are also available in [12–16].

The combined fuzzy-random uncertainty is also encountered during the analysis of failure probability according to (3). The source of fuzzy uncertainty is for e.g. density functions and statistical characteristics of random variables G and Q. Precise statistical information on loading is not generally known during structural design. Further uncertainty may occur due to human involvement during the realization of experiments, evaluation of results of experimental research, etc.

The aim of further studies is not the elaborate analysis of the origin of model uncertainties, but rather the theoretical quantification of their influence on the behaviour of failure probability P_f in dependence on parameter δ. Model uncertainties can be quantified utilizing the so-called coefficients of model uncertainties K_S, K_R and modified reliability conditions (3):

$$K_S(G + Q) \leq K_R R. \tag{9}$$

The influence of coefficients K_S and K_R on the failure probability P_f may generally be either linear or non-linear. For this purpose coefficients K_S, K_R were chosen as fuzzy numbers with linear triangular symmetrical membership functions, see Fig. 4 and Fig. 5.

The graphical representations of uncertainty in Figs. 4 and 5 assign to K_R and K_S uncertainty by means of a degree of membership into the set on the vertical axis [8]. The membership function has nothing in common with probability. In the case of probability, we examine the frequency of occurrences of a given phenomenon that occurred.

The fuzzy analysis of failure probability according to (9) was evaluated according to the general extension principle for 10α-cuts [8], see Fig. 4 and Fig. 5.

$$\mu_{P_f}(K_R, K_S) = \bigvee_{P_f} \big(\mu_1(K_R) \wedge \mu_2(K_S)\big). \tag{10}$$

The output of (10) is the fuzzy number of failure probability P_f. An illustration of output fuzzy number of failure probability P_f evaluated according to (10) for Austrian steel for $\delta = 1$ is depicted in Fig. 6. Equation (10) requires the evaluation of minimal and maximal P_f for all realization combinations of coefficients K_S, K_R on each α-cut.

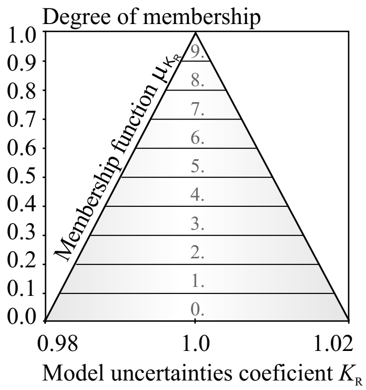

Fig. 4. Fuzzy number of resistance uncertainty.

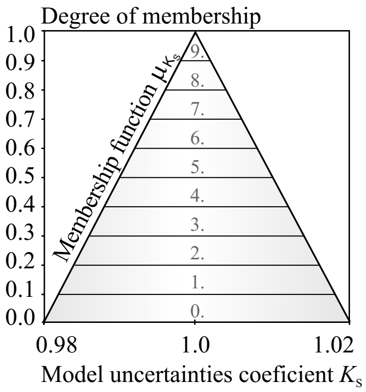

Fig. 5. Fuzzy number of load action uncertainty.

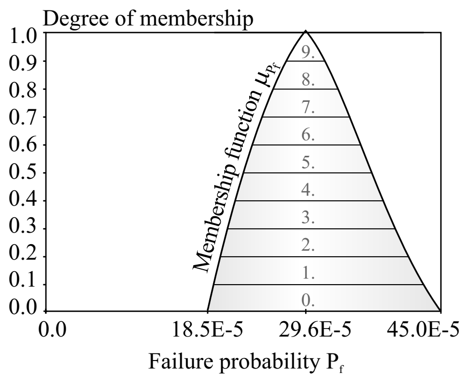

Fig. 6. Fuzzy number of P_f – Austrian steel, $\delta = 1.0$.

The fuzzy analysis procedure according to (10) may be explained on one α-cut. Let us consider the zero α-cut (the so-called support) of input fuzzy numbers K_S, K_R and output fuzzy number P_f. The minimum $P_{f,min} = 18.5 \cdot 10^{-5}$ is evaluated for $K_S = 0.98$, $K_R = 1.02$ and maximum $P_{f,max} = 5 \cdot 10^{-5}$ is evaluated for $K_S = 1.02$, $K_R = 0.98$, see Fig. 6. The procedure is analogical for other α-cuts. The non-linear membership function evaluated for 10 α-cuts is apparent from Fig. 6. Results depicted in Fig. 6 quantify the dependence of P_f on the change in coefficients K_S, K_R (sensitivity analysis of the influence of coefficients K_S, K_R on P_f). The failure probability P_f is non-linearly dependent on coefficients K_S, K_R for all considered δ values. Fuzzy analysis results of failure probability for Austrian and Czech steel are depicted in Fig. 7 and Fig. 8.

The fuzzy analysis results in Fig. 7 and Fig. 8 supplement the information given in Fig. 3 with the influence of coefficients of model uncertainties K_R and K_S. Membership functions of failure probability were calculated for $\delta \in \{0, 0.2, 0.4, 0.6, 0.8, 1.0\}$. Degree of membership 1.0 on the vertical axis means that the failure probability belongs to the set fully (kernel). The so-called support limiting the set of all failure probability values with positive membership function is a further characteristic of fuzzy numbers. In Fig. 7 and Fig. 8, boundaries of the support interval are marked by the dashed line. The defuzzified "crisp" failure probability value is drawn by dot-and-dash line as the last value; it can be compared with the reference value $7.2 \cdot 10^{-5}$ [6]. The defuzzification was evaluated utilizing the centre of gravity method [8]. The major characteristics of the failure probability fuzzy analysis from Fig. 7 and Fig. 8 are clearly depicted in Fig. 9 and Fig. 10.

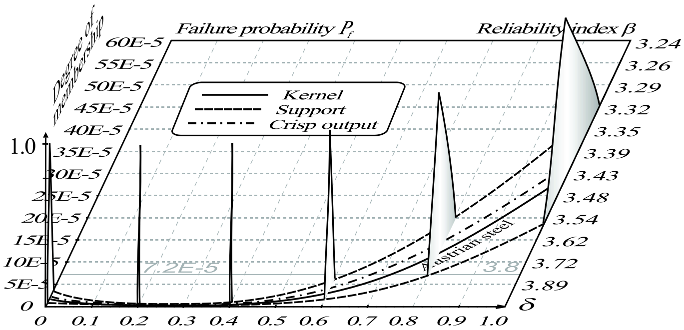

Fig. 7. Fuzzy analysis of failure probability – Austrian steel.

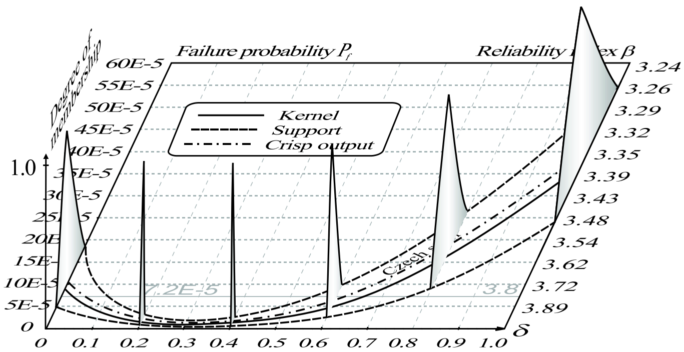

Fig. 8. Fuzzy analysis of failure probability – Czech steel.

It is apparent from Fig. 9 and Fig. 10 that defuzzified values are higher than the kernel ones – obtained by crisp stochastic solution in Fig. 3. This is due to the non-linear and asymmetrical membership functions of failure probability P_f. The courses of defuzzified values presenting a crisp controllable output are clearly shown in Fig. 11.

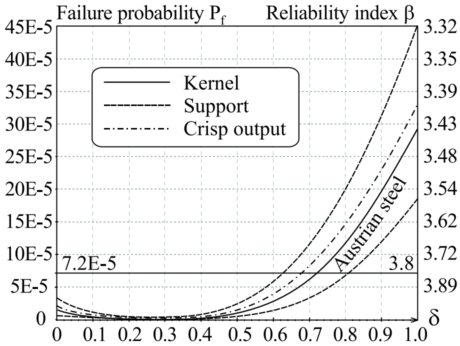

Fig. 9. Fuzzy output of failure probability – Austrian steel.

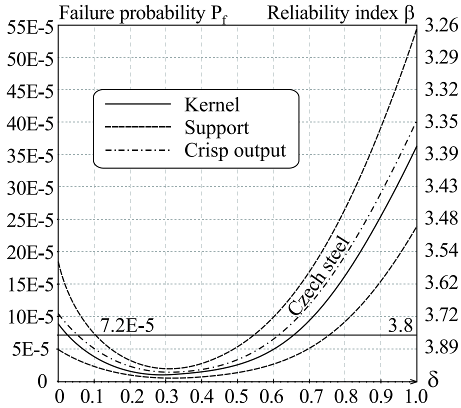

Fig. 10. Fuzzy output of failure probability – Austrian steel.

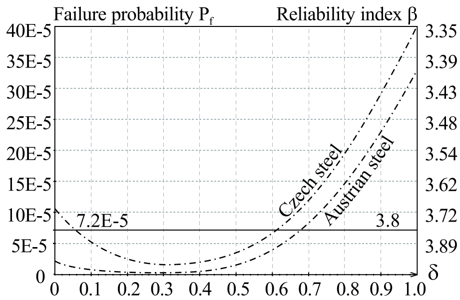

Fig. 11. Results of fuzzy analysis of failure probability.

6 Conclusion

Basic information on the misalignment and discrepancies of failure probability of a steel member produced from Austrian and Czech steel are presented in Fig. 3. The failure probability of the steel bar made from Austrian steel is lower than that of Czech steel for the same bar. The higher reliability of the member produced from Austrian steel is due to the moderately higher mean value and lower standard deviation of yield stress. The mentioned differences, however, are not significant in comparison with the effect of further uncertainties inevitably met when calculating the load-carrying capacity and load action effects.

The output membership functions are non-linear and pronouncedly asymmetric, triangular symmetric membership functions having been selected as input fuzzy numbers of coefficients K_R and K_S. This information is very valuable because it quantifies the non-linear dependence between the coefficients of model uncertainties K_R and K_S and the failure probability.

It is apparent from results depicted in Fig. 3 and Figs. 9 to 11 that design of steel bars is satisfactory for reasonable ratios of the variable load action to the total load action. High failure probability values were obtained for $\delta = 0$ (purely permanent load) and $\delta = 1$ (purely variable load), which represent limit unrealistic cases. The reliability analysis of bars under buckling is planned in the future. It can be expected that imperfections will present a significant source of uncertainty. Whilst information on statistical characteristics of initial strut curvature is available, there is an insufficiency of experimental information on system imperfections of steel frames. Further analytical studies are planned for additional values of partial safety factors $\gamma_G,\ \gamma_Q,\ \gamma_M$. The reliability analysis of the design of steel structures according to the allowable stress design method for $\gamma_G = 1.0$; $\gamma_Q = 1.0$ a $\gamma_M = 1.5$ will be performed. Probabilistic analysis results of the limit state method and the allowable stress method will be compared.

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