Motion of a Self-Propelling Micro-Organism in a Channel Under Peristalsis: Effects of Viscosity Variation

G. Radhakrishnamacharya[1], **R. Sharma**[2]

[1]Department of Mathematics and Humanities, National Institute of Technology
Warangal-506 004 (A.P.), India
grk.nitw@yahoo.com

[2]Department of Science and Technology, Technology Bhawan
New Delhi-110 016, India

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Abstract. The motion of a self propelling micro-organism symmetrically located in a rectangular channel containing viscous fluid has been studied by considering the peristaltic and longitudinal waves travelling along the walls of the channel. The expressions for the velocity of the micro-organism and time average flux have been obtained under long wavelength approximation by taking into account the viscosity variation of the fluid across the channel. Particular cases for constant viscosity and when it is represented by a step function have been discussed. It has been observed that the velocity of the micro-organism decreases as the viscosity of the peripheral layer increases and its thickness decreases.

Keywords: peristalsis, micro-organism, peripheral layer, wavelength.

1 Introduction

The study of the self propelling micro-organism in a biofluid was initiated by Sir Taylor [1], who modelled it as a two dimensional sheet of zero thickness with sinusoidal wave travelling down its length. Since then, many researchers have studied the problem under various conditions, Hancock [2], Shack et al. [3], Shukla et al. [4]. It may be noted that the motion of micro-organism is affected by the nature of biofluid, dynamical interaction of the duct walls, and the cilia motion, if any, in the lumen of the duct. In some physiological situations, e.g. mid period of menstrual cycle, James et al. [5], it is known that the biofluid viscosity shows variation across the channel cross-section. Further, in situations such as oviduct, it is observed that the muscular activity of walls and the action of cilia generate peristaltic and longitudinal waves on the duct walls, Guha et al. [6], Blake et al. [7], Shukla et al. [8]. Philip and Chandra [9] analyzed the self-propulsion of spermatozoa through mucus filling a channel with flexible boundaries.

Keeping this in view, a mathematical model is presented here to study the effect of viscosity variation of biofluid on the motion of micro-organism. Further, peristaltic and longitudinal wave motion travelling along the channel wall is considered to account for the dynamical interaction of the walls. The micro-organism is taken as a two dimensional sheet of finite thickness with transverse wave motion along its surface. The expressions for propulsion velocity of micro-organism and the time averaged flow flux have been obtained under long wavelength approximation.

2 Mathematical model

Consider the swimming motion of a thin flexible sheet of finite thickness $2\delta'$ in a Newtonian incompressible fluid flowing through a two-dimensional channel having flexible boundaries. It is assumed that the fluid filled in the channel has varying viscosity and that the sheet, while swimming, sends down lateral waves of finite amplitude along its length. Further, peristaltic waves of finite amplitude are imposed along the flexible walls of the channel in the direction opposite to the motion of the sheet. The sheet is considered to be swimming with a propulsive velocity V_p' in the negative axial direction (Fig. 1). It

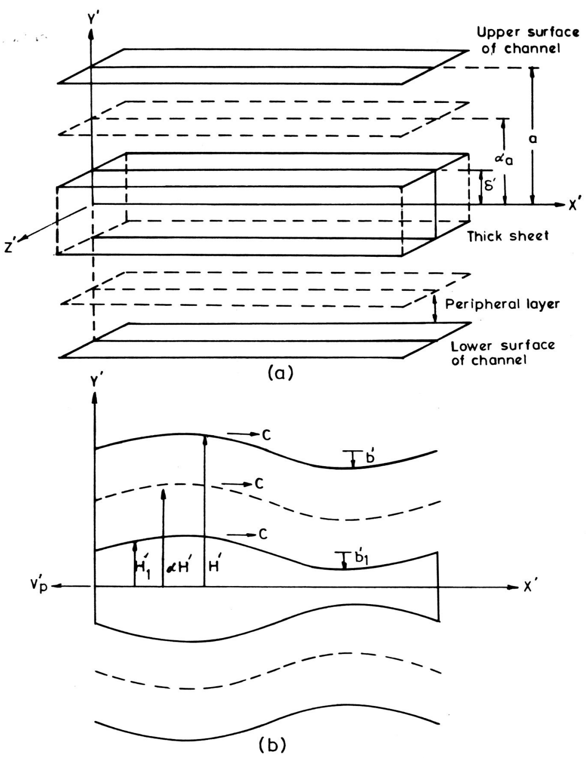

Fig. 1. Flow of fluid in channel with peripheral layer: (a) without peristalsis; (b) with peristalsis.

is assumed that the waves travelling along the channel walls and along the sheet are in synchronization under steady state and thus have the same wave speed c (along positive axial direction) and the wavelength λ. In a fixed frame of reference, (X', Y', t'), the wall of the channel ($Y' = \pm H'$) and the sheet ($Y' = \pm H_1'$) at an instant t' are given by

$$H'(X', t') = a + b' \sin \frac{2\pi}{\lambda}(X' - ct' + V_p' t'), \tag{1}$$

$$H_1'(X', t') = \delta' + b_1' \sin \frac{2\pi}{\lambda}(X' - ct' + V_p' t'). \tag{2}$$

As the sheet is self propelling, the forces exerted by the fluid on its surface must balance its motion. This force equilibrium condition on the surface of the organism can be written under symmetrical situation as

$$\int_S T' ds = \delta' \Delta p', \tag{3}$$

where T' is the resultant of the forces acting on the surface of the micro-organism, $\Delta p'$ is the pressure rise over a wavelength and S is the surface of the micro-organism.

The Reynolds number of the flow in such situations is of the order 10^{-3} and hence the inertia terms can be neglected. Further, in a frame moving with velocity $c - V_p'$ in the positive axial direction, the boundaries of the channel and the micro-organism appear stationary. Thus, transforming various quantities from the stationary frame (X', Y', t') to the corresponding quantities in the moving frame and using the following non-dimensionalization scheme,

$$\begin{aligned} &x = (X' - ct' + V_p' t')/\lambda, \quad y = Y'/a, \quad t = ct'/\lambda, \\ &u = (U' - c + V_p')/c, \quad (c_1, V_p) = (c_1', V_p')/c, \quad p = p' a^2/(\lambda c \mu), \\ &(\varepsilon, \varepsilon_1, \delta) = (b', b_1', \delta')/a, \quad \mu(y) = \mu'(y')/\mu_0, \quad \text{where } \mu_0 = \mu'(0), \end{aligned} \tag{4}$$

the equations of motion under long wavelength approximation reduce to the following simple form

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left[\mu(y)\frac{\partial u}{\partial y}\right], \tag{5}$$

$$0 = -\frac{\partial p}{\partial y}. \tag{6}$$

The boundary conditions for u in moving frame can be written as

$$\begin{aligned} &u = g(x) + V_p - 1 \quad \text{at } y = \pm h(x), \\ &u = \pm 1 \quad \text{at } y = \pm h_1(x), \end{aligned} \tag{7}$$

where $g(x) = c_1 \sin 2\pi x$ gives the longitudinal wall motion due to the presence of cilia in the lumen of the channel and

$$h(x) = 1 + \varepsilon \sin 2\pi x, \quad h_1(x) = \delta + \varepsilon_1 \sin 2\pi x.$$

The force equilibrium condition, in this case, becomes

$$\int_0^1 \left[\left(\mu \frac{\partial u}{\partial y} \right)_{y=h_1(x)} + \frac{dh_1(x)}{dx} p \right] dx = \delta \Delta p. \tag{8}$$

Now using the symmetry, we solve these equations for the region $y \geq 0$ only.

3 Analysis

The equation (5) can be solved, using boundary conditions (7) to give

$$u = -1 + \left(\frac{\partial p}{\partial x} \right) \left[I_2(y) - \frac{I_2(h)}{I_1(h)} I_1(y) \right] + (g + V_p) \frac{I_1(y)}{I_1(h)}, \tag{9}$$

where

$$I_1(y) = \int_{h_1}^{y} \frac{dy}{\mu(y)}, \quad I_2(y) = \int_{h_1}^{y} \frac{y}{\mu(y)} dy, \quad I_3(y) = \int_{h_1}^{y} \frac{y^2}{\mu(y)} dy.$$

The flux q, in the moving frame is constant and is obtained from,

$$q = \int_{h_1}^{h} u dy \tag{10}$$

which on using the expression for u, gives $\frac{\partial p}{\partial x}$ as

$$-\left(\frac{\partial p}{\partial x} \right) = \frac{\lfloor \{ q + (h - h_1) \} I_1(h) + (g + V_p) \{ I_2(h) - h I_1(h) \} \rfloor}{I_3(h) I_1(h) - I_2^2(h)}. \tag{11}$$

Integrating (11) we get

$$-\Delta p = F_{11} q + F_{12} V_p + F_{13} c_1 + F_{14}, \tag{12}$$

where

$$F_{11} = \int_0^1 I_1(h) G(h) dx, \qquad F_{12} = \int_0^1 G_1(h) dx,$$

$$F_{13} = \int_0^1 G_1(h) \sin 2\pi x dx, \quad F_{14} = \int_0^1 (h - h_1) I_1(h) G(h) dx,$$

$$G(h) = 1/\lfloor I_1(h) I_3(h) - I_2^2(h) \rfloor, \quad G_1(h) = \left[I_2(h) - h I(h) \right] G(h).$$

The force equilibrium condition on using (9) and (10) gives

$$0 = F_{21}q + F_{22}V_p + F_{23}c_1 + F_{24}, \tag{13}$$

$$F_{21} = \int_0^1 I_2(h)G(h)dx, \qquad F_{22} = \int_0^1 G_2(h)dx,$$

$$F_{23} = \int_0^1 G_2(h)\sin 2\pi x dx, \quad F_{24} = \int_0^1 (h-h_1)I_2(h)G(h)dx,$$

and $G_2(h) = [I_3(h) - hI_2(h)]G(h)$.

Further, the fluxes in the stationary and wave frames are related by

$$Q = q - (V_p - 1)(H - H_1).$$

Hence the time averaged flux $\overline{Q}$ in stationary frame can be obtained from the relation

$$\overline{Q} = q + (1 - V_p)(1 - \delta). \tag{14}$$

By eliminating q from equations (12)–(14), the expressions for the propulsive velocity V_p and the time averaged flux $\overline{Q}$ in stationary frame can be obtained in terms of Δp and $\mu(y)$.

It may be noted that for the case of thin sheet ($\varepsilon_1 = 0,\ \delta = 0$) the expressions for V_p and $\overline{Q}$ can be obtained by putting $h_1 = 0$ in the equations (12)–(14).

4 Results and discussion

To see the effects of various parameters explicitly, we consider here two particular cases.

1. Constant viscosity i.e. $\mu(y) = 1$.

2. Step function viscosity (i.e. for peripheral layer).

$$\mu(y) = \begin{cases} \overline{\mu}, & \alpha h \le y \le h, \\ 1, & h_1 \le y \le \alpha h, \end{cases}$$

where $(1-\alpha)$ gives the thickness of the peripheral layer near the wall of the channel.

The various integrals occurring in equations (12) and (13) are numerically evaluated for these two cases, and the values of V_p and $\overline{Q}$ are calculated for different values of Δp, ε_1 and ε_2.

For the case of constant viscosity, the effects of various parameters on V_p and $\overline{Q}$ are shown in Figs. 2–7. We notice from Figs. 2, 3 and 4 that for given Δp

$$V_p(c_1, \varepsilon_1, \varepsilon) = V_p(-c_1, -\varepsilon_1, -\varepsilon)$$

and that V_p increases as $|\varepsilon_1|$ increases. This implies that the lateral peristaltic waves on the surface of the micro-organism increases the speed of micro-organism.

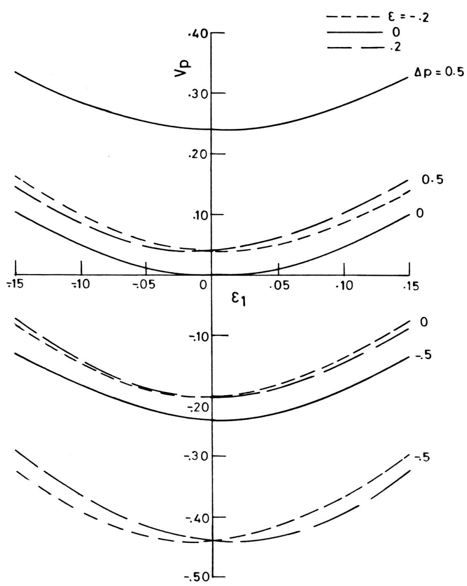

Fig. 2. Effect of Δp and ε on V_p ($C_1 = 0$, $\delta = 0.2$).

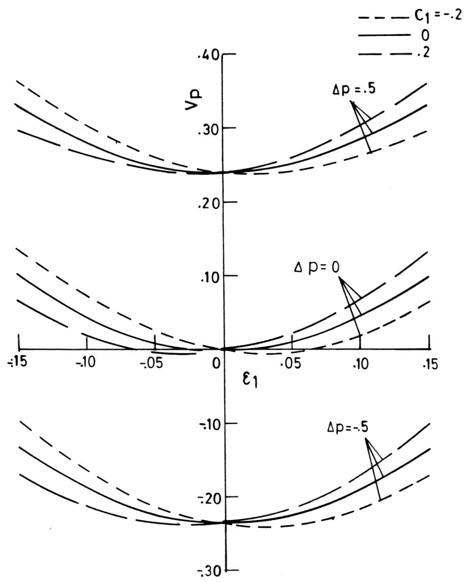

Fig. 3. Effect of Δp and C_1 on V_p ($\varepsilon = 0$, $\delta = 0.2$).

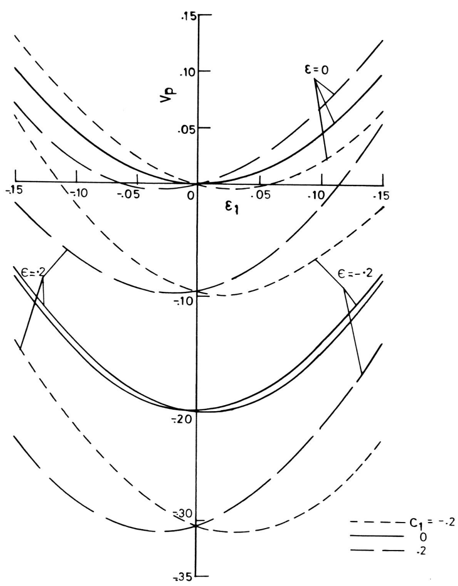

Fig. 4. Effect of ε and C_1 on V_p ($\Delta p = 0$, $\delta = 0.2$).

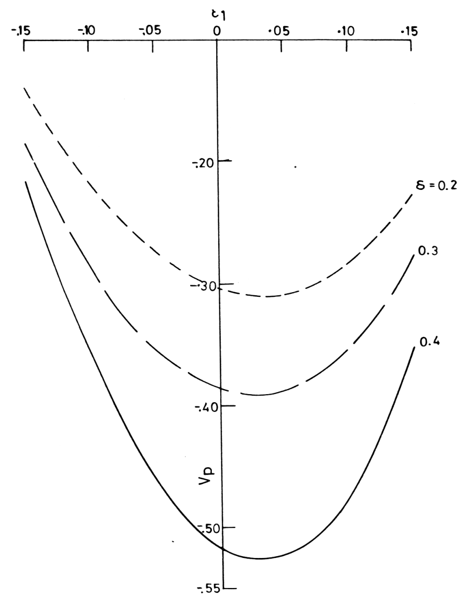

Fig. 5. Effect of δ on V_p ($\Delta p = 0$, $\varepsilon = 0$, $C_1 = -0.2$).

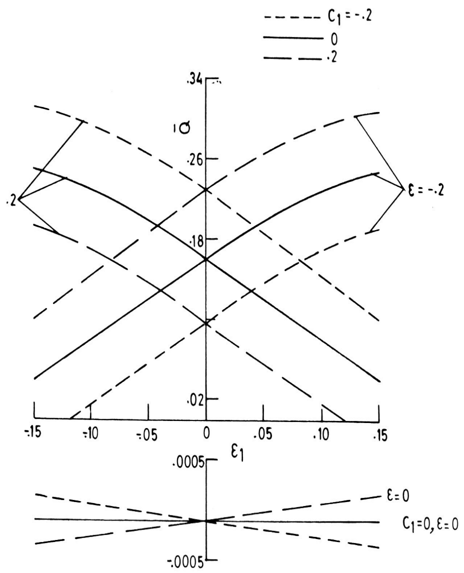

Fig. 6. Effect of C_1 and ε on $\overline{Q}$ ($\Delta p = 0$, $\delta = 0.2$).

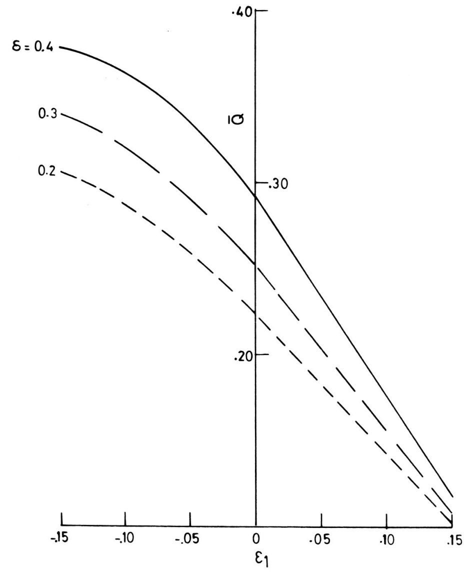

Fig. 7. Effect of δ on $\overline{Q}$ ($\Delta p = 0$, $\varepsilon = 0.2$, $C_1 = -0.2$).

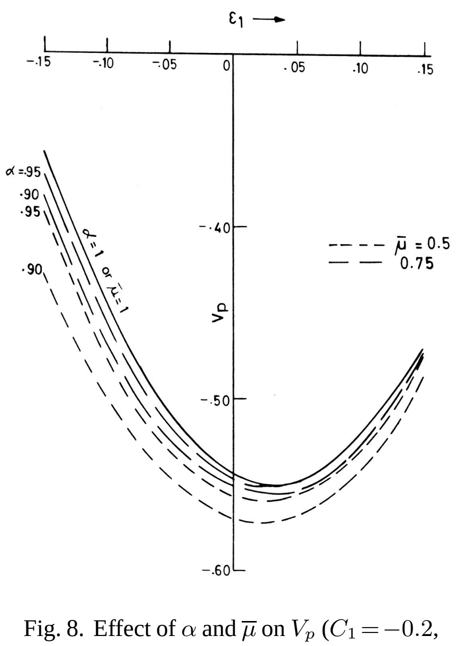

Fig. 8. Effect of α and $\overline{\mu}$ on V_p ($C_1 = -0.2$, $\varepsilon = 0.2$, $\Delta p = -0.5$, $\delta = 0.2$).

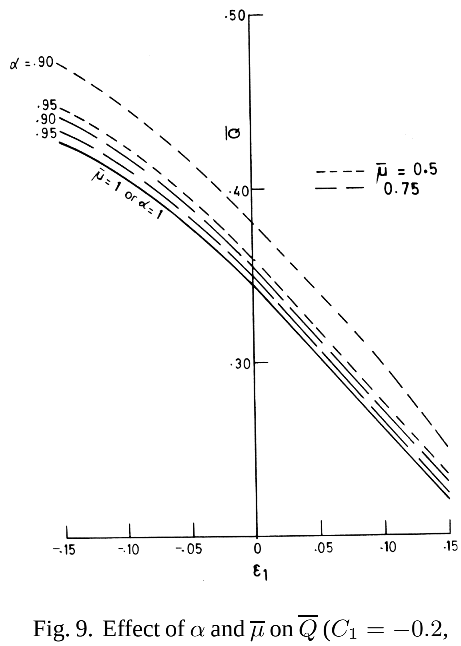

Fig. 9. Effect of α and $\overline{\mu}$ on $\overline{Q}$ ($C_1 = -0.2$, $\varepsilon = 0.2$, $\Delta p = -0.5$, $\delta = 0.2$).

The effects of ε and Δp on V_p are shown in Fig. 2 for $c_1 = 0$. It is observed here that V_p decreases as Δp decreases and it even becomes negative for negative Δp, i.e. the motion of the micro-organism can be reversed by suitable pressure drop. Also, V_p decreases as the magnitude of ε increases i.e., a peristaltic wave on channel wall does not facilitate the motion of the micro-organism. Fig. 3 shows that for $\varepsilon = 0$, V_p increases as c_1 increases for $\varepsilon_1 > 0$ while the reverse trend is observed for $\varepsilon_1 < 0$. It may also be seen here that V_p decreases as Δp decreases for all values of c_1. The Figs. 2 and 4 show that the behaviour of V_p with respect to ε depends upon the value of Δp, c_1 and ε_1. From Fig. 5, it is observed that V_p increases as δ decreases ($\Delta p = 0$, $\varepsilon = 0$, $c_1 = -0.2$).

The effects of c_1, ε and δ on $\overline{Q}$ are shown in Figs. 6 and 7 by taking $\Delta p = 0$. Fig. 6 shows that for $\varepsilon < 0$, $\overline{Q}$ decreases as either c_1 increases or as ε_1 increases. However, the opposite behaviour is observed for $\varepsilon > 0$. The decrease in the thickness, δ, of the sheet decreases $\overline{Q}$ (Fig. 7). The effect of Δp on $\overline{Q}$ has been found to be opposite to its effect on V_p.

For the case of step function viscosity, the effects of peripheral layer thickness (α) and viscosity step function $\overline{\mu}$ on V_p and $\overline{Q}$ are shown in Figs. 8–11 by taking $c_1 = -0.2$, $\varepsilon = 0.2$ and $\delta = 0.2$. Figs. 8 and 9 show that for $\Delta p = -0.5$, V_p decreases and $\overline{Q}$ increases with the increase in peripheral layer thickness $(1 - \alpha)$ as well as with the decrease in $\overline{\mu}$. This effect is observed for all the values of ε_1. Further, the effect of Δp on V_p is similar to the case of $\overline{\mu} = 1$ (Fig. 10). However, for $\Delta p > 0$, V_p decreases as

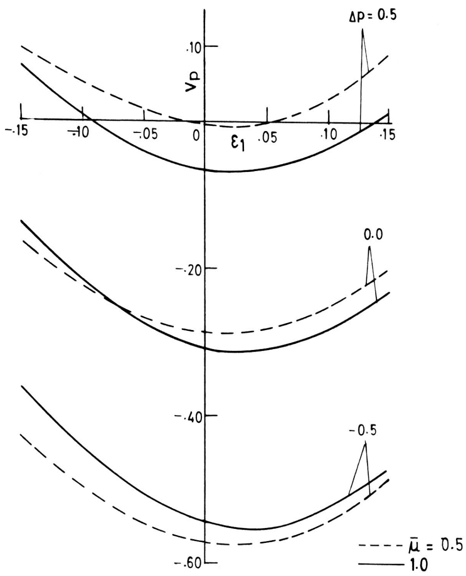

Fig. 10. Effect of $\overline{\mu}$ and Δp on variation of V_p with ε_1 ($\delta = 0.2, C_1 = -0.2, \varepsilon = -0.2, \alpha = 0.9$).

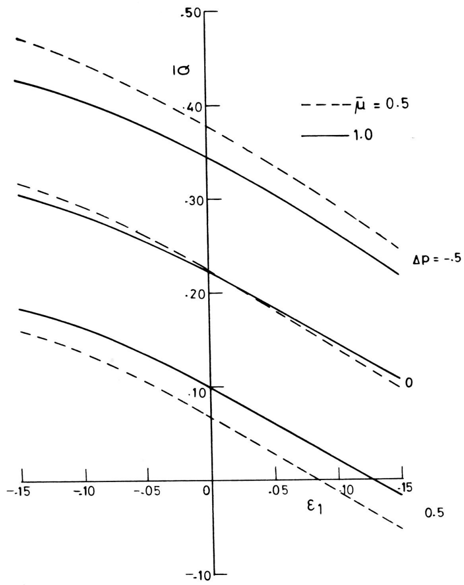

Fig. 11. Effect of $\overline{\mu}$ and Δp on variation of $\overline{Q}$ with ε_1 ($\delta = 0.2$, $C_1 = -0.2$, $\varepsilon = 0.2$, $\alpha = 0.9$.

$\overline{\mu}$ increases while reverse trend is observed in the case of $\Delta p < 0$. Fig. 11 shows that $\overline{Q}$ decreases with ε_1 and that $\overline{Q}$ increases as Δp decreases. The effect of $\overline{\mu}$ on $\overline{Q}$ depends upon the sign of Δp.

To compare our analysis with the available data, we take the following values of various parameters (Gupta and Seshadri [10], Shukla et al. [11], Guha et al. [6]):

$$\mu = 0.04 \text{ g/cm s}, \quad \overline{\mu} = 0.2, \quad \lambda = 8 \text{ cm}, \quad c = 8 \text{ mm/s}, \quad a = 0.05 \text{ mm},$$
$$\alpha = 0.9 \quad \varepsilon = 0.2, \quad c_1 = 0.2, \quad \Delta p = 0.25 \quad \text{(non-dimensional)}.$$

Using this data, the propulsion velocity V_p is calculated as 0.044 mm/s. This differs from the observed value by about 12 %. Further, the flux $\overline{Q}$ is calculated as 0.006 ml/s whereas the observed value is given by 0.007 ml/s.

5 Conclusion

Mathematical model to study the effect of the viscosity variation across the cross-section on the swimming of a micro-organism has been presented. It has been shown that the velocity of the micro-organism decreases as the viscosity of the peripheral layer increases and its thickness decreases. Further, the motion of the micro-organism can be reversed by applying peristaltic wave on the channel wall.

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