

Conjugate Effects of Heat and Mass Transfer on Natural Convection Flow Across an Isothermal Horizontal Circular Cylinder with Chemical Reaction

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Abstract. Natural convection flow across an isothermal cylinder immersed in a viscous incompressible fluid in the presence of species concentration and chemical reaction has been investigated. The governing boundary layer equations are transformed into a system of non-dimensional equations and the resulting nonlinear system of partial differential equations is reduced to a system of local non-similarity boundary layer equations, which is solved numerically by a very efficient implicit finite difference method together with the Keller-box scheme. Numerical results are presented by the velocity, temperature and species concentration profiles of the fluid as well as the local skin-friction coefficient, local heat transfer rate and local species concentration transfer rate for a wide range of chemical reaction parameter γ ($\gamma = 0.0, 0.5, 1.0, 2.0, 4.0$), buoyancy ratio parameter N ($N = -1.0, -0.5, 0.0, 0.5, 1.0$), Schmidt number Sc ($Sc = 0.7, 10.0, 50.0, 100.0$) and Prandtl number Pr ($Pr = 0.7, 7.0$).

Keywords: natural convection, chemical reaction, skin-friction, rate of heat transfer, rate of species concentration, cylinder.

Nomenclature

a	radius of the circular cylinder	g	acceleration due to gravity
C	species concentration in the fluid	Gr	Grashof number
C_∞	species concentration with fluid away from the cylinder	J_w	concentration flux
C_p	specific heat at constant pressure	K	thermal conductivity
C_w	species concentration at the surface of the cylinder	K_1	chemical reaction parameter
C_{fx}	local skin-friction	N	buoyancy ratio parameter
D	chemical molecular diffusivity	Nu_x	local Nusselt number
f	dimensionless stream function	Pr	Prandtl number
		q_w	heat flux at the surface
		Sc	Schmidt number

Sh_x	local Sherwood number	u, v	the dimensionless x and y component of the velocity
T	temperature of the fluid in the boundary layer	\hat{u}, \hat{v}	the dimensional \hat{x} and \hat{y} component of the velocity
T_∞	temperature of the ambient fluid	x, y	axis in the direction along and normal to the surface
T_w	temperature at the surface		

Greek symbols

β	volumetric coefficient of thermal expansion	ρ	density of the fluid
ψ	stream function	μ	viscosity of the fluid
τ_w	shearing stress	ϕ	dimensionless concentration function
γ	chemical reaction parameter	θ	dimensionless temperature function

1 Introduction

The application of boundary layer techniques to mass transfer has been of considerable assistance in developing the theory of separation processes and chemical kinetics. Some of the interesting problems that have been studied are mass transfer from droplets, free convection on electrolysis in non-isothermal boundary layer. Heat, mass and momentum transfer on a continuously moving or a stretching sheet has several applications in electro-chemistry and polymer processing [1–4].

Gebhart and Pera [5] investigated the nature of vertical natural convection flow resulting from the combined buoyancy effects of thermal and mass diffusion. Diffusion and chemical reaction in an isothermal laminar flow along a soluble flat plate was studied and an appropriate mass-transfer analogue to the flow along a flat plate that contains a species say, A slightly soluble in the fluid say, B has been discussed by Fairbanks and Wick [6]. Hossain and Rees [7] have investigated the combined effect of thermal and mass diffusion in natural convection flow along a vertical wavy surface. The effects of chemical reaction; heat and mass transfer on laminar flow along a semi-infinite horizontal plate have been studied by Anjalidevi and Kandasamy [8].

By taking advantage of the mathematical equivalence of the thermal boundary layer problem with the concentration analogue, results obtained for heat transfer characteristics can be carried directly over to the case of mass transfer by replacing the Prandtl number Pr by the Schmidt number Sc . However, the presence of a chemical reaction term in the mass diffusion equation generally destroys the formal equivalence with the thermal energy problem and moreover, generally prohibits the construction of the otherwise attractive similarity solutions. Takhar et al. [9] for example, considered diffusion of chemically reactive species from a stretching sheet.

The application of the boundary layer theory with chemical reaction has been applied to some problems of free and mixed convection flow from the surface of simple geometry by the above authors. Chiang et al. [10] investigated the laminar free convection from a horizontal cylinder. Sparrow and Lee [11] looked at the problem of vertical stream over a

heated horizontal circular cylinder. They have obtained a solution by expanding velocity and temperature profiles in powers of x , the co-ordinate measuring distance from the lowest point on the cylinder. The exact solution is still out of reach due to the non-linearity in the Navier-Stokes equations. It appears that Merkin [12, 13] was the first who presented a complete solution of this problem using Blasius and Görtler series expansion method along with an integral method and a finite-difference scheme. Ingham [14] investigated the free convection boundary layer flow on an isothermal horizontal cylinder. Recently, Nazar et al. [15] have considered the problem of natural convection flow from lower stagnation point to upper stagnation point of a horizontal circular cylinder immersed in a micropolar fluid.

In the present study the focus is given on the effects of heat and mass transfer on the natural convection boundary layer flow across a horizontal circular cylinder with chemical reaction when the surface is at a uniform temperature and a uniform mass diffusion. Here it has been assumed that the level of species concentration is very low and that the heat generated during chemical reaction can be neglected. The basic equations are transformed to local non-similarity boundary layer equations, which are solved numerically using a very efficient finite-difference scheme together with the Keller-box method [16]. To the best of our knowledge, this problem has not been considered before. Consideration is given to the situation where the buoyancy forces assist the natural convection flow for various combinations of the chemical reaction parameter γ , buoyancy ratio parameter N , Prandtl number Pr and Schmidt number Sc . The results allow us to predict the different kinds of behaviour that can be observed when the relevant parameters are varied.

2 Formulation of problem

Let us consider a steady two-dimensional laminar free convective flow that flows across a uniformly heated horizontal circular cylinder of radius a , which is immersed in a viscous and incompressible fluid. It is assumed that the surface temperature of the cylinder is T_w , where $T_w > T_\infty$. Here T_∞ is the ambient temperature of the fluid and T is the temperature of the fluid. The configuration considered is as shown in Fig. 1.

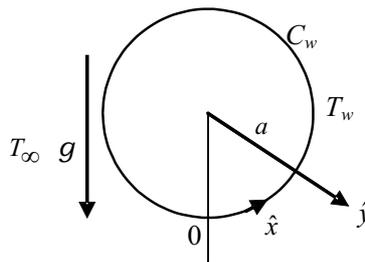


Fig. 1. Physical model and coordinate system.

An appropriate mass transfer analogue to the problem shown in Fig. 1 would be the flow across the surface of a horizontal circular cylinder that contains a species A slightly

soluble in the fluid B . The concentration of the reactant is maintained at a constant value C_w at the surface of the cylinder and the solubility of A in B is D and the concentration of A far away from the surface of the cylinder is assumed to be C_∞ . Let the reaction of a species A with B be the first order homogeneous chemical reaction with rate constant, K_1 . It is desired to analyse the system by a boundary layer method. It is assumed that the concentration of dissolved A is small enough and the physical properties ρ , μ and D are virtually constant throughout the fluid. Under the usual Boussinesq approximation, the governing equations of the flow are

$$\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} = 0, \quad (1)$$

$$\rho \left(\hat{u} \frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v} \frac{\partial \hat{u}}{\partial \hat{y}} \right) = \mu \frac{\partial^2 \hat{u}}{\partial \hat{y}^2} + \rho g \beta_T (T - T_\infty) \sin \left(\frac{\hat{x}}{a} \right) + \rho g \beta_C (C - C_\infty) \sin \left(\frac{\hat{x}}{a} \right), \quad (2)$$

$$\hat{u} \frac{\partial T}{\partial \hat{x}} + \hat{v} \frac{\partial T}{\partial \hat{y}} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial \hat{y}^2}, \quad (3)$$

$$\hat{u} \frac{\partial C}{\partial \hat{x}} + \hat{v} \frac{\partial C}{\partial \hat{y}} = D \frac{\partial^2 C}{\partial \hat{y}^2} - K_1 C. \quad (4)$$

The boundary conditions of equation (1) to (4) are

$$\hat{u} = \hat{v} = 0, \quad T = T_w, \quad C = C_w \quad \text{at} \quad \hat{y} = 0, \quad (5a)$$

$$\hat{u} \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as} \quad \hat{y} \rightarrow \infty, \quad (5b)$$

where (\hat{u}, \hat{v}) are velocity components along the (\hat{x}, \hat{y}) axes, g is the acceleration due to gravity, ρ is the density, k is the thermal conductivity, β_T is the coefficient of thermal expansion, β_c is the coefficient of concentration expansion, μ is the viscosity of the fluid, C_p is the specific heat due to constant pressure and D is the molecular diffusivity of the species concentration. We now introduce the following non-dimensional variables:

$$\begin{aligned} x &= \frac{\hat{x}}{a}, \quad y = Gr^{1/4} \frac{\hat{y}}{a}, \quad u = \frac{a}{\nu} Gr^{-1/2} \hat{u}, \quad v = \frac{a}{\nu} Gr^{-1/4} \hat{v}, \\ \theta &= \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}, \quad Gr = \frac{g \beta_T (T_w - T_\infty) a^3}{\nu^2}, \end{aligned} \quad (6)$$

where $\nu = (\mu/\rho)$ is the reference kinematic viscosity and Gr is the Grashof number, θ is the non-dimensional temperature and ϕ is the non-dimensional species concentration function.

Substituting the variables (6) into equation (1)–(4) leads to the following non-dimensional equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (7)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + (\theta + N\phi) \sin x, \quad (8)$$

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2}, \quad (9)$$

$$u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - \gamma \phi. \quad (10)$$

The boundary conditions (5) become

$$u = v = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad y = 0, \quad (11a)$$

$$u \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \quad \text{as} \quad y \rightarrow \infty, \quad (11b)$$

where N is the ratio of the buoyancy forces due to the temperature and concentration, γ is the chemical reaction parameter are defined respectively as

$$N = \frac{\beta_C(C_w - C_\infty)}{\beta_T(T_w - T_\infty)} \quad \text{and} \quad \gamma = \frac{K_1 a^2}{\nu Gr^{1/2}}. \quad (12)$$

To solve equations (7)–(10), subject to the boundary conditions (11), we assume the following variables

$$\psi = xf(x, y), \quad \theta = \theta(x, y), \quad \phi = \phi(x, y), \quad (13)$$

where ψ is the non-dimensional stream function defined in the usual way as

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}. \quad (14)$$

Substituting (13) into equations (8)–(10) we get, after some algebra the following transformed equations

$$\frac{\partial^3 f}{\partial y^3} + f \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial f}{\partial y} \right)^2 + (\theta + N\phi) \frac{\sin x}{x} = x \left(\frac{\partial f}{\partial y} \frac{\partial^2 f}{\partial x \partial y} - \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial y^2} \right), \quad (15)$$

$$\frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + f \frac{\partial \theta}{\partial y} = x \left(\frac{\partial f}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \theta}{\partial y} \frac{\partial f}{\partial x} \right), \quad (16)$$

$$\frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} + f \frac{\partial \phi}{\partial y} - \gamma \phi = x \left(\frac{\partial f}{\partial y} \frac{\partial \phi}{\partial x} - \frac{\partial \phi}{\partial y} \frac{\partial f}{\partial x} \right) \quad (17)$$

along with boundary conditions

$$f = \frac{\partial f}{\partial y} = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad y = 0, \quad (18a)$$

$$\frac{\partial f}{\partial y} \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \quad \text{as} \quad y \rightarrow \infty. \quad (18b)$$

It can be seen that near the lower stagnation point of the cylinder i.e. $x \approx 0$, equations (15)–(17) reduce to the following ordinary differential equations:

$$f''' + ff'' - f'^2 + (\theta + N\phi) = 0, \quad (19)$$

$$\frac{1}{Pr} \theta'' + f\theta' = 0, \quad (20)$$

$$\frac{1}{Sc} \phi'' + f\phi' - \gamma\phi = 0, \quad (21)$$

subject to the boundary conditions at

$$f(0) = f'(0) = 0, \quad \theta(0) = 1, \quad \phi(0) = 1 \quad \text{at } y = 0, \quad (22a)$$

$$f' \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \quad \text{as } y \rightarrow \infty. \quad (22b)$$

In the above equations primes denote differentiation with respect to y .

In practical applications, the physical quantities of main interest are the shearing stress, the rate heat transfer and the rate of species concentration transfer in terms of the skin-friction coefficients C_f , Nusselt number Nu and Sherwood number Sh respectively, which can be written as

$$C_f = \frac{Gr^{-3/4}a^2}{\mu\nu}\tau_w, \quad Nu = \frac{aGr^{-1/4}}{k(T_w - T_\infty)}q_w, \quad Sh = \frac{aGr^{-1/4}}{D(C_w - C_\infty)}J_w, \quad (23)$$

$$\tau_w = \mu\left(\frac{\partial \hat{u}}{\partial \hat{y}}\right)_{\hat{y}=0}, \quad q_w = -k\left(\frac{\partial T}{\partial \hat{y}}\right)_{\hat{y}=0} \quad \text{and} \quad J_w = -D\left(\frac{\partial C}{\partial \hat{y}}\right)_{\hat{y}=0}. \quad (24)$$

Using the variables (6), (13) and the boundary condition (18a) into (23)–(24), we get

$$C_{fx} = xf''(x, 0), \quad (25)$$

$$Nu_x = -\theta'(x, 0), \quad (26)$$

$$Sh_x = -\phi'(x, 0). \quad (27)$$

We also discuss the effect of the chemical reaction parameter γ and buoyancy ratio parameter N on the velocity, temperature and concentration distribution. The values of the velocity, temperature and concentration distribution are calculated respectively from the following relations:

$$u = \frac{\partial f}{\partial y}, \quad \theta = \theta(x, y), \quad \phi = \phi(x, y). \quad (28)$$

3 Results and discussion

Equations (15)–(17) subject to the boundary conditions were solved numerically using a very efficient implicit finite difference method together with the Keller-box scheme, which is described by Cebeci and Bradshaw [17]. The numerical solutions start at the lower stagnation point of the cylinder i.e. at $x \approx 0.0$, with initial profiles given by the equations (19)–(21) along with boundary conditions (22) and proceed round the cylinder up to the upper stagnation point $x \approx \pi$. Solutions are obtained for fluid having Prandtl number Pr ($Pr = 0.7, 7.0$), Schmidt number Sc ($Sc = 0.7, 10.0, 50.0, 100.0$), buoyancy ratio parameter N ($N = -1.0, -0.5, 0.0, 0.5, 1.0$) and for a wide range of the values of chemical reaction parameter γ ($\gamma = 0.0, 0.5, 1.0, 2.0, 4.0$). Since the values of $f''(x, 0)$, $[-\theta'(x, 0)]$ and $[-\phi'(x, 0)]$ are known from the solutions of the equations (15)–(17), numerical values of the local skin-friction coefficient C_{fx} , the local Nusselt number Nu_x and the local Sherwood number Sh_x are calculated respectively from the equations

(25)–(27) for the surface of the cylinder from lower stagnation point to upper stagnation point. Numerical values of C_{fx} , Nu_x and Sh_x are depicted in Tables 1, 2 and Figs. 2–5. A comparison of the local Nusselt number Nu_x and C_{fx} for different values of curvature parameter x while $Pr = 1.0$ and $N = \gamma = 0.0$, obtained in the present work and obtained earlier by Merkin [12] and Nazar et al. [15] has been made in Table 1. It is clearly seen that there is an excellent agreement among the respective results.

Table 1. Numerical values of C_{fx} and Nu_x for different values of curvature parameter x while $Pr = 1.0$ and $N = \gamma = 0.0$

x	Nu_x			C_{fx}		
	Merkin	Nazar et al.	Present	Merkin	Nazar et al.	Present
	[12]	[15]		[12]	[15]	
0.0	0.4214	0.4214	0.4241	0.0000	0.0000	0.0000
$\pi/6$	0.4161	0.4161	0.4161	0.4151	0.4148	0.4145
$\pi/3$	0.4007	0.4005	0.4005	0.7558	0.7542	0.7539
$\pi/2$	0.3745	0.3741	0.3740	0.9579	0.9545	0.9541
$2\pi/3$	0.3364	0.3355	0.3355	0.9756	0.9698	0.9696
$5\pi/6$	0.2825	0.2811	0.2812	0.7822	0.7740	0.7739
π	0.1945	0.1916	0.1917	0.3391	0.3265	0.3264

The effect of the chemical reaction parameter γ , on the reduced local skin-friction coefficient C_{fx} , local Nusselt number Nu_x and the local Sherwood number Sh_x is shown in Figs. 2(a)–(c) respectively for $\gamma = 0.0, 0.5, 1.0, 2.0, 4.0$, while $Pr = Sc = 0.7$ and $N = 0.5$. It is seen that an increase in the chemical reaction parameter, γ ($\gamma = 0.0, 0.5, 1.0, 2.0, 4.0$), leads to a decrease in the local skin-friction coefficient and the local Nusselt number and an increase in the local Sherwood number. This may be attributed to the fact that the increase in the values of γ implies more interaction of species concentration with the momentum boundary layer and less interaction with the thermal boundary layer.

Figs. 3(a)–(c) illustrate the effect of varying values of chemical reaction parameter γ ($\gamma = 0.0, 0.5, 1.0, 2.0, 4.0$) on the velocity, temperature and species concentration profiles at $x = \pi/2$ while $Pr = Sc = 0.7$, $N = 0.5$. Here it is found that both the velocity and concentration profiles decrease significantly and the non-dimensional temperature profile increases slightly with the increase of chemical reaction parameter.

In Table 2 we have entered the numerical values C_{fx} , Nu_x and Sh_x for different values of Pr ($Pr = 0.7, 7.0$) while $N = \gamma = 0.5$ and $Sc = 0.7$. It is observed that for increasing axial distance parameter x , the values of skin-friction increase and the values of rate of heat transfer and that of the rate of species concentration decrease for both the values of Prandtl number Pr . On the other hand both the C_{fx} and Sh_x decrease and Nu_x increases accordingly as Prandtl number Pr increases. And these changes in C_{fx} , Nu_x and Sh_x due to an increase in Pr are consistent with a free convection boundary layer.

Figs. 4(a)–(c) show how variations in N ($N = -1.0, -0.5, 0.0, 1.0$) affect the flow. When $N = 0.0$, the flow is induced entirely by thermal effects and the detailed

concentration field is computed as a forced convection problem, however, when $N = 1.0$, it is the concentration field which induce the boundary flow. It can be stated that an increase in the values of N leads to an increase in the values of the local skin-friction coefficients C_{fx} , local Nusselt number Nu_x and the local Sherwood number Sh_x .

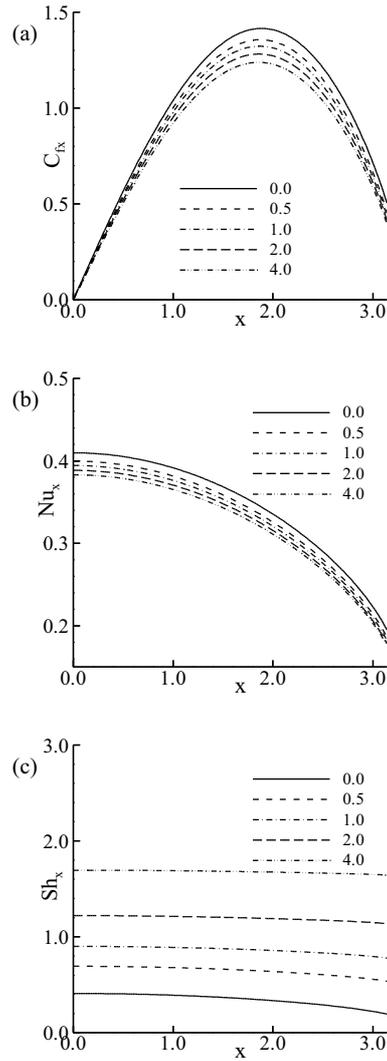


Fig. 2. (a) Skin-friction coefficient, (b) rate of heat transfer and (c) rate of species concentration for different values of γ while $Pr = Sc = 0.7$ and $N = 0.5$.

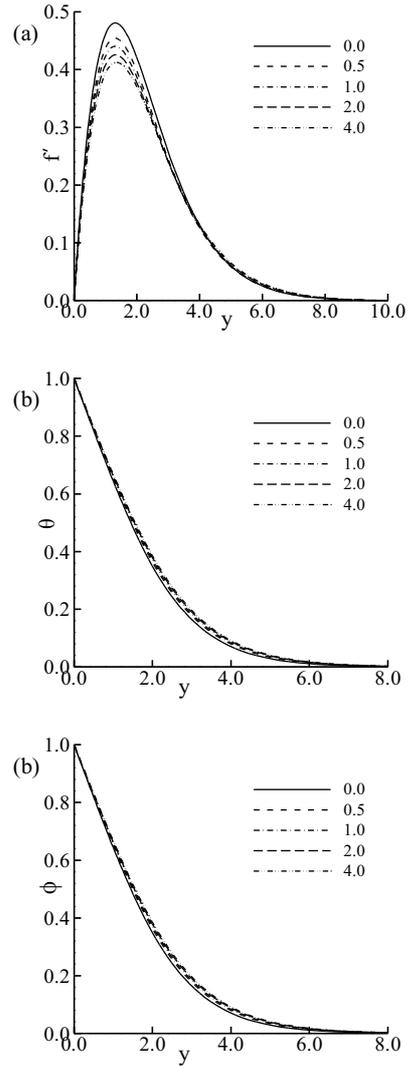


Fig. 3. (a) Velocity, (b) temperature and (c) concentration distribution for different values of γ while $Pr = Sc = 0.7$ and $N = 0.5$ at $x = \pi/2$.

Variation in the Schmidt number Sc are considered in Figs. 5(a)–(c) while $Pr = 0.7$ and $\gamma = N = 0.5$. The local skin-friction coefficient C_{fx} and the local Nusselt number Nu_x decrease sharply and the local Sherwood number Sh_x increases significantly with the increasing values of Sc , which is expected.

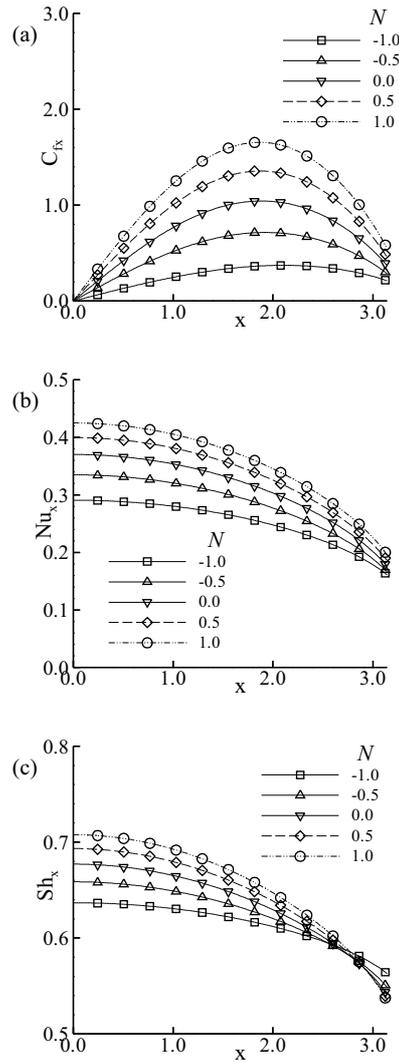


Fig. 4. (a) Skin-friction coefficient, (b) rate of heat transfer and (c) rate of species concentration for different values of N while $Pr = Sc = 0.7$ and $\gamma = 0.5$.

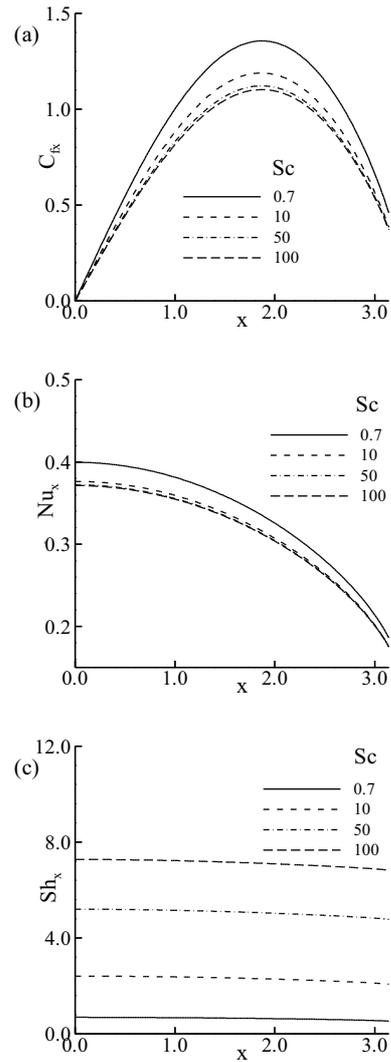


Fig. 5. (a) Skin-friction coefficient, (b) rate of heat transfer and (c) rate of species concentration for different values of Sc while $Pr = 0.7$ and $N = \gamma = 0.5$.

Table 2. The values of C_{fx} , Nu_x and Sh_x while $N = \gamma = 0.5$ and $Sc = 0.7$ for different values of Prandtl number Pr

x	$Pr = 0.7$			$Pr = 7.0$		
	C_{fx}	Nu_x	Sh_x	C_{fx}	Nu_x	Sh_x
0.0	0.00000	0.39966	0.69327	0.00000	0.93647	0.65935
$\pi/6$	0.57051	0.39461	0.68947	0.43927	0.92362	0.65640
$\pi/3$	1.03685	0.37969	0.67828	0.79521	0.88555	0.64772
$\pi/2$	1.31095	0.35454	0.65962	0.99739	0.82093	0.63320
$2\pi/3$	1.33074	0.31811	0.63302	0.99606	0.72603	0.61240
$5\pi/6$	1.06220	0.26722	0.59649	0.76233	0.58917	0.58302
π	0.46033	0.18616	0.53651	0.23597	0.33747	0.52354

4 Conclusions

The effect of chemical reaction, heat and mass diffusion in natural convection flow from an isothermal horizontal cylinder has been investigated numerically. New variables to transform the complex geometry into a simple shape have been introduced and the resulting non-similarity boundary layer equations are solved by a very efficient implicit finite difference method together with the Keller-box scheme [16]. From the present investigation the following conclusions may be drawn:

- Both the local skin-friction coefficient C_{fx} and the local Nusselt number Nu_x decrease and the local Sherwood number Sh_x increases when the value of the chemical reaction parameter γ increases.
- As γ increases both the velocity and concentration distribution decrease significantly and the temperature profile increases slightly at $x = \pi/2$ of the surface.
- An increase in the values of N leads to an increase in the values of the local skin-friction coefficients C_{fx} , local Nusselt number Nu_x and the local Sherwood number Sh_x .
- The skin-friction coefficient C_{fx} and local Nusselt number Nu_x decrease and the rate of species concentration Sh_x increases within the boundary layer as the value of Sc increases.
- The skin-friction coefficient C_{fx} and the rate of species concentration Sh_x decrease and local Nusselt number Nu_x increases for increasing values of Pr .

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