Free Convection from a Vertical Permeable Circular Cone with Pressure Work and Non-Uniform Surface Temperature

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Abstract. Laminar free convection from a vertical permeable circular cone maintained at non-uniform surface temperature with pressure work term is considered. Non-similarity solutions for boundary layer equations are found to exist when the surface temperature follows the power law variations with the distance measured from the leading edge. The numerical solutions of the transformed non-similar boundary layer equations are obtained by using two methodologies, namely, (i) a finite difference method and (ii) a series solution or perturbation method for small values of ξ, the dimensionless suction parameter. Solutions obtained in terms of skin-friction coefficient, local rate of heat transfer, velocity and temperature profiles for the values of Prandtl number, pressure work parameter and temperature gradient are displayed in both graphical and tabular forms. Finite difference solutions are compared with the solutions obtained by perturbation technique and found to be in excellent agreement near the leading edge.

Keywords: free convection, transpiration velocity, temperature gradient, pressure work.

Nomenclature

C_p	specific heat at constant pressure
C_{fx}	local skin friction
f	dimensionless stream function
g	acceleration due to gravity
Gr_x	local Grashof number
k	thermal conductivity of the fluid
n	temperature gradient parameter
Nu_x	local Nusselt number
Pr	Prandtl number
p	fluid pressure
q_w	surface heat flux
T	temperature of the fluid
T_w	temperature at the surface
T_∞	temperature of the ambient fluid
u	velocity component in the x-direction
v	velocity component in the y-direction
V	transpiration velocity
x	measured from the leading edge
y	distance normal to the surface

Greek symbols

α thermal diffusivity
β coefficient of volume expansion
γ the cone apex half-angle
ξ the dimensionless suction parameter
η the pseudo-similarity variable
ν kinematic viscosity
μ viscosity of the fluid
θ dimensionless temperature
ρ density of the fluid inside the boundary layer
ψ stream function
ϵ pressure work parameter

1 Introduction

Theoretical studies on laminar free convection flow on axisymmetric bodies have received wider attention, especially in case of non-uniform surface temperature and surface heat flux distributions. Mark and Prins [1] developed the general relations for similar solutions on isothermal axisymmetric forms and showed that for the flow past a vertical cone has such a solution. Approximate boundary layer techniques were utilized to arrive at an expression for the dimensionless heat transfer. Braun et al. [2] contributed two more isothermal axisymmetric bodies for which similar solutions exist, and used an integral method to provide heat transfer results for these and the cone over a wide range of Prandtl number. In the above investigation, the authors obtained the results by numerical integration of the differential equations for fluid having Prandtl number 0.72. The similarity solutions for free convection from the vertical cone have been exhausted by Hering and Grosh [3]. They showed that the similarity solutions to the boundary layer equations for a cone exist when the wall temperature distribution is a power function of distance along a cone ray. In their paper they presented the results for isothermal surface as well as for the surface maintained at the temperature varying linearly with the distance measured from the apex of the cone for Prandtl number 0.7. Latter, Hering [4] extended the analysis to investigate for low Prandtl number fluids. On the other hand, Roy [5] has studied the same problem for the high values of the Prandtl number. Na and Chiou [6] studied the effect of slenderness on the natural convection flow over a slender frustum of a cone. The problem of natural convection flow over a frustum of a cone without transverse curvature effect (i.e., large cone angles when the boundary layer thickness is small compared with the local radius of the cone) has been treated in the literature, even though the problem for a full cone has been considered quite extensively by Sparrow and Guinle [7], Lin [8], Kuiken [9] and Oosthuizen and Donaldson [10]. Latter, Na and Chiou [6] studied the laminar natural convection flow over a frustum of a cone. In the above investigations the wall temperature as well as the wall heat flux had been considered constant. On the other hand, Alamgir [11] investigated the overall heat transfer in laminar natural convection flow from vertical cones by using the integral method. Hassain and Paul [12] investigate the non-uniform surface temperature over a free convection from a vertical permeable circular cone.

In the present analysis, we propose to investigate the laminar free convection flow from a vertical permeable circular cone maintained at non-uniform surface temperature with pressure work term that follows the power law variations with the distance measured from the apex of the cone. Under the usual Boussinesq approximation, the governing partial differential equations are reduced to locally non-similar partial differential equations. The transformed equations are solved numerically by using finite difference with Keller Box method. The solutions are obtained in terms of skin-friction coefficient and rate of heat transfer for various values of Prandtl number Pr, pressure work parameter ϵ and temperature gradient parameter n, are displayed in tabular form as well as graphically. Also the effect of varying the Prandtl number Pr, the surface temperature gradient n and the suction parameter ξ, on velocity and temperature distributions are shown graphically.

2 Mathematical formalism

A steady two-dimensional laminar free convection flow past a non-isothermal vertical porous cone with variable surface temperature is considered. The physical coordinates (x, y) are chosen such that x is measured from the leading edge, O, in the stream wise direction and y is measured normal to the surface of the cone. The coordinate system and the flow configuration are shown in Fig. 1.

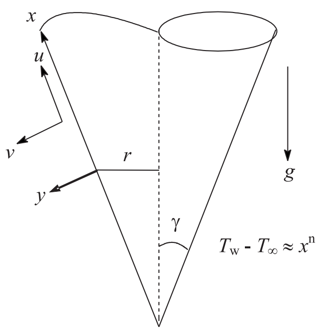

Fig. 1. Physical model and coordinates system.

The boundary layer equations for steady, axisymmetric, non-dissipative and constant property flow is

$$\frac{\partial(ur)}{\partial x} + \frac{\partial(vr)}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \nu\frac{\partial^2 v}{\partial y^2} + g\beta\cos\gamma(T - T_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{T\beta u}{\rho C_p}\frac{\partial p}{\partial x}, \tag{3}$$

where u, v are the fluid velocity components in the x-directions and y-directions respectively, ν is the kinematics coefficient of viscosity, g is the acceleration due to gravity, β is the coefficient of volume expansion, α is the thermal diffusivity, γ is the cone apex half-angle and T is the temperature of the fluid.

The boundary conditions are as follows

$$\begin{aligned} &u = 0, \quad v = -V, \quad T = T_w(x) \quad \text{at} \quad y = 0, \\ &u = 0, \quad T = T_\infty \quad \text{at} \quad y \to \infty, \end{aligned} \tag{4}$$

where V represent the transpiration velocity of the fluid through the surface of the cone, T_∞ is the ambient fluid temperature, T_w is the surface temperature with $T_w > T_\infty$. When V is positive, it stands for suction or withdrawal and V is negative for injection or blowing of fluid through the surface of the cone. In the present case, only suction is considered and therefore, V is taken as positive throughout.

Since near the apex of the cone, the boundary layer is much similar that the free-convection boundary layer in the absence of suction, we can introduce the following transformations

$$\begin{aligned} &\psi = \nu r (Gr_x)^{1/4}\Big(f(\xi,\eta) + \frac{1}{2}\xi\Big), \quad T - T_\infty = (T_w - T_\infty)\theta(\xi,\eta), \\ &\eta = \frac{y}{x}(Gr_x)^{1/4}, \quad \xi = \frac{Vx}{\nu}(Gr_x)^{1/4}, \quad r = x\sin\gamma, \\ &Gr_x = \frac{g\beta\cos\gamma(T - T_\infty)x^3}{\nu^2}, \quad T_w - T_\infty \approx x^n, \end{aligned} \tag{5}$$

where Gr_x is the local Grashof number, ξ is the dimensionless suction parameter, η is the pseudo-similarity variable and ψ is the stream function defined by

$$u = \frac{1}{r}\frac{\partial\psi}{\partial y} \quad \text{and} \quad v = -\frac{1}{r}\frac{\partial\psi}{\partial x}.$$

Finally, the functions $f(\xi, 0)$ and $\theta(\xi, 0)$ are, respectively, the dimensionless stream function and the temperature function of the fluid in the boundary layer region. Hence

$$\begin{aligned} &u = \nu\frac{(Gr_x)^{1/2}}{x}\frac{\partial f}{\partial\eta}, \quad \frac{\partial u}{\partial y} = \nu\frac{(Gr_x)^{3/4}}{x^2}\frac{\partial^2 f}{\partial\eta^2}, \quad \frac{\partial^2 u}{\partial y^2}u = \nu\frac{Gr_x}{x^3}\frac{\partial^3 f}{\partial\eta^3}, \\ &v = \frac{\nu(n+7)}{4x}(Gr_x)^{1/4}f(\xi,\eta) - \frac{\nu\xi(n+7)}{4x}(Gr_x)^{1/4} \\ &\quad - \frac{\nu\eta(n-1)}{4x}(Gr_x)^{1/4}\frac{\partial f}{\partial\eta} - \frac{\nu\xi(n+7)}{4x}(Gr_x)^{1/4}\frac{\partial f}{\partial\xi}, \\ &\frac{\beta T}{\rho C_p}u\frac{\partial p}{\partial x} = -\frac{\rho g\beta T}{\rho C_p}u = -\frac{\epsilon\big(T_\infty - (T_w - T_\infty)\big)}{x^2}\theta(\xi,\eta)(Gr_x)^{1/2}\frac{\partial f}{\partial\eta}. \end{aligned}$$

Here $g\beta x/C_p = \epsilon$, is pressure work parameter which is first used by Gebhart [13] as dissipation parameter. Therefore the modified momentum equation can be re-written as

$$f''' + \frac{n+7}{4}ff'' - \frac{n+1}{2}f'^2 + \theta = \frac{n+7}{4}\xi\Big(f'\frac{\partial f'}{\partial\xi} - f''\frac{\partial f}{\partial\xi} - f''\Big), \tag{6}$$

where $T_w - T_\infty = x^n$.

Also the energy equation can be written as

$$\frac{1}{Pr}\theta'' + \frac{n+7}{4}f\theta' - (n+\epsilon)\theta f' = \frac{n+7}{4}\xi\Big(f'\frac{\partial\theta}{\partial\xi} - \theta'\frac{\partial f}{\partial\xi}\Big). \tag{7}$$

The corresponding boundary conditions to be satisfied are:

$$f = f' = 0, \quad \theta = 1 \quad \text{at} \quad \eta = 0, \tag{8}$$

$$f' = 0, \quad \theta = 0 \quad \text{as} \quad \eta \to \infty, \tag{9}$$

where $Pr = \nu/\alpha$ is Prandlt number and primes denoting differentiation with respect to η For the flow from an impermeable surface (i.e., $\xi = 0$), the equations (6) and (7) subjected to the boundary conditions (8) and (9) have been solved by Hering and Grosh [3] for non-isothermal surface. Solutions of the local non-similar partial differential equations (6) to (7) subjected to the boundary conditions (8) and (9) are obtained by using the implicit finite difference method, which has been used, by Hossain [14] and Hossain et al. [15].

Once we know the values of the functions f and θ and also their derivaties, it becomes important to calculate the values of the local skin-friction coefficient, C_{fx} and the local Nusselt number Nu_x, from the following relations.

$$C_{fx} = \frac{\tau_w}{\rho U^2} \quad \text{and} \quad Nu_x = -\frac{q_w x}{\kappa(T_w - T_\infty)},$$

where $\tau_w = \mu\Big(\frac{\partial u}{\partial y}\Big)_{y=0}$ and $q_w = -\kappa\Big(\frac{\partial T}{\partial y}\Big)_{y=0}$ are, respectively, the shear stress and rate of heat-flux at the surface and $U = \nu(Gr)^{1/2}/x$ is the reference velocity. Now

$$\tau_w = \mu\Big(\frac{\partial u}{\partial y}\Big)_{y=0} = \mu\nu\frac{(Gr_x)^{3/4}}{x^2}\,\frac{\partial^2 f(\xi,0)}{\partial\eta^2},$$

$$q_w = -\kappa\Big(\frac{\partial T}{\partial y}\Big)_{y=0} = -\kappa\frac{x^n(Gr_x)^{1/4}}{x}\,\frac{\partial\eta(\xi,0)}{\partial\eta}.$$

Therefore, we have

$$C_{fx} = \frac{\tau_w}{\rho U^2} = \mu\nu\,\frac{(Gr_x)^{3/4}}{x^2}\,\frac{\partial^2 f(\xi,0)}{\partial\eta^2}\Big/\frac{\rho\nu^2 Gr_x}{x^2} = \frac{\partial^2 f(\xi,0)}{\partial\eta^2}\Big/(Gr_x)^{1/4}.$$

Which implies that

$$(Gr_x)^{1/4}C_{fx} = \frac{\partial^2 f(\xi,0)}{\partial\eta^2}. \tag{10}$$

Again

$$Nu_x = \frac{q_w x}{\kappa(T_w - T_\infty)} = -\kappa\,\frac{x^n(Gr_x)^{1/4}}{x}\,\frac{\partial\theta(\xi,0)}{\partial\eta}\Big/\kappa(T_w - T_\infty) = -(Gr_x)^{1/4}\,\frac{\partial\theta(\xi,0)}{\partial\eta}.$$

Therefore,

$$(Gr_x)^{1/4}Nu_x = -\frac{\partial\theta(\xi,0)}{\partial\eta}. \tag{11}$$

3 Perturbation solutions for small ξ

Since near apex of the cone, ξ is small for small x or small V or both, series solution of the equations (6) to (7) may be obtained by using perturbation method treating ξ as a perturbation parameter. Hence, we expand the functions $f(\xi,\eta)$ and $\theta(0,\xi)$ in powers of ξ, yield expansions of

$$f(\xi,\eta)=\sum_{i=0}^{\infty}\xi^i f(\eta) \quad \text{and} \quad \theta(\xi,\eta)=\sum_{i=0}^{\infty}\xi^i\theta^i(\eta). \tag{12}$$

Substituting the above expansion (12) into equations (6) and (7) and equating the various power of ξ up to $O(\xi^2)$, we get the following sets of equation:

$$f_0{}''' + \theta_0 + \frac{n+7}{4} f_0 f_0{}'' - \frac{n+1}{2}(f_0{}')^2 = 0, \tag{13}$$

$$\frac{1}{Pr}\theta_0{}'' + \frac{n+7}{4} f_0\theta_0{}' - n f_0{}'\theta_0 - \epsilon f_0{}'\theta_0 = 0 \tag{14}$$

and the boundary conditions

$$f_0(0) = f_0{}'(0) = 0, \quad \theta_0(0) = 1, \quad f_0{}'(\infty) = \theta_0{}'(\infty) = 0,$$

$$f_1{}''' + \frac{n+7}{4} f_0 f_1{}'' + \frac{n+7}{2} f_1 f_0{}'' - \frac{5n+11}{4} f_0{}' f_1{}' + \frac{n+7}{4} f_0{}'' + \theta_1 = 0, \tag{15}$$

$$\frac{1}{Pr}\theta_1{}'' + \frac{n+7}{4}\theta_1{}' f_0 + \frac{n+7}{2} f_1\theta_0{}' - (n+\epsilon)(\theta_0 f_1{}' + \theta_1 f_0{}') - \frac{n+7}{4} f_0{}'\theta_1 = 0 \tag{16}$$

and the boundary conditions

$$f_1(0) = f_1{}'(0) = 0, \quad \theta_1(0) = 0, \quad f_1{}'(\infty) = \theta_1{}'(\infty) = 0,$$

$$\begin{aligned} f_2{}''' &+ \frac{n+7}{4} f_0 f_2{}'' + \frac{n+7}{2} f_1 f_1{}'' + \frac{3n+21}{4} f_2 f_0{}'' - \frac{3n+9}{4} f_1{}'^2 \\ &- (3n+9) f_0{}' f_2{}' + \frac{n+7}{4} f_1{}'' + \theta_2 = 0, \end{aligned} \tag{17}$$

$$\begin{aligned} \frac{1}{Pr}\theta_2{}'' &+ \frac{n+7}{4}\theta_2{}' f_0 + \frac{n+7}{2} f_1\theta_1{}' + \frac{3n+21}{4} f_2\theta_0{}' \\ &- (n+\epsilon)(\theta_0 f_2{}' + \theta_1 f_1{}' + \theta_2 f_0{}') - \frac{n+7}{2} f_0{}'\theta_2 - \frac{n+7}{4} f_1{}'\theta_1 = 0 \end{aligned} \tag{18}$$

and the boundary conditions

$$f_2(0) = f_2{}'(0) = 0, \quad \theta_2(0) = 0, \quad f_2{}'(\infty) = \theta_2{}'(\infty) = 0.$$

The coupled equations (13) and (14) are non-linear, whereas (15) to (18) are linear, and these may be solved pair-wise one after another. The implicit Runge-Kutta-Butcher

(Butcher [16]) initial value solver together with the Nachtsheim-Swigert iteration scheme of Nachtsheim and Swigert [17] is employed to solve the above equations pair-wise up to $O(\xi^2)$. Thus solutions are obtained for f_i and θ_i for $i = 0, 1, 2$ and their derivatives. The local skin-friction coefficient and the local Nusselt number are calculated from the following expressions

$$(Gr)^{1/4} C_{fx} = \frac{\partial^2 f(\xi, 0)}{\partial \eta^2} = f_0''(0) + \xi f_1'(0) + \xi^2 f_2''(0) + O(\xi^3), \tag{19}$$

$$(Gr)^{1/4} Nu_x = -\frac{\partial \theta(\xi, 0)}{\partial \eta} = -\big(\theta_0'(0) + \xi \theta_1'(0) + \xi^2 \theta_2'(0) + O(\xi^3)\big). \tag{20}$$

4 Results and discussion

In this paper we have investigated the problem of laminar free convective flow and heat transfer from a vertical permeable circular cone with pressure work and non-uniform surface temperature. The solutions of the momentum and energy equations with the appropriate boundary condition are obtained by the finite difference method together with the Keller-Box scheme and by perturbation methods. Results are obtained in terms the local skin-friction, the rate of heat transfer, velocity and temperature profiles and presented in graphical as well as in tabular form.

The effects of varying Prandtl number Pr ($Pr = 0.72,\ 1.0,\ 3.0,\ 5.0,\ 7.0$), pressure work parameter ϵ ($\epsilon = -0.6,\ 0.1,\ 0.3,\ 0.6,\ 0.9$) and temperature gradient parameter n ($n = -0.45,\ -0.1,\ 0.5,\ 0.9,\ 1.20$) on the dimensionless velocity profile $f'(\xi, \eta)$ and the dimensionless temperature profiles $\theta(\xi, \eta)$ are shown in Figs. 2–4. The skin friction and the surface heat transfer coefficients are shown in Figs. 5–7.

Fig. 2(a), (b) shows the velocity and temperature profiles for pressure work parameter ϵ ($\epsilon = -0.6,\ 0.1,\ 0.3,\ 0.6,\ 0.9$) with temperature gradient parameter $n = 0.5$ and Prandtl number $Pr = 0.72$, we see that in Fig. 2(a), (b), the velocity and temperature

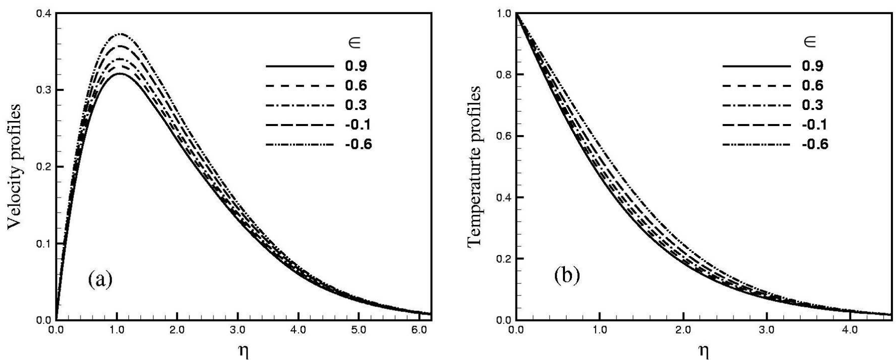

Fig. 2. (a) Velocity and (b) temperature profiles against η for different values of pressure work parameter ϵ with $n = 0.5$, $Pr = 0.72$.

profiles are decreases with the effect of pressure work parameter. From Fig. 3(a), (b), we observe that the velocity and temperature profiles are decreases with the increasing of temperature gradient parameter n ($n = -0.45,\ -0.1,\ 0.5,\ 0.9,\ 1.20$) with other controlling parameter $Pr = 0.72$ and $\epsilon = 0.2$. The effects of varying Prandtl number Pr ($Pr = 0.72,\ 1.0,\ 3.0,\ 5.0,\ 7.0$) on the dimensionless velocity, $f'(\xi, \eta)$ and the dimensionless temperature, $\theta(\xi, \eta)$ distributions against η for the pressure work parameter $\epsilon = 0.5$ and the temperature gradient parameter $n = 0.4$ are shown in Fig. 4(a),(b). From these figures, it is seen that the velocity and temperature profiles are decreases with the increasing values of Pr. In case of water at 20° C ($Pr = 7.0$), the free laminar boundary shows a sharp decrease compared to the effects in electrolyte solution such as salt water ($Pr = 1.0$) and air ($Pr = 0.72$) at 20° C and $Pr = 3.0,\ 5.0$ have been used theoretically.

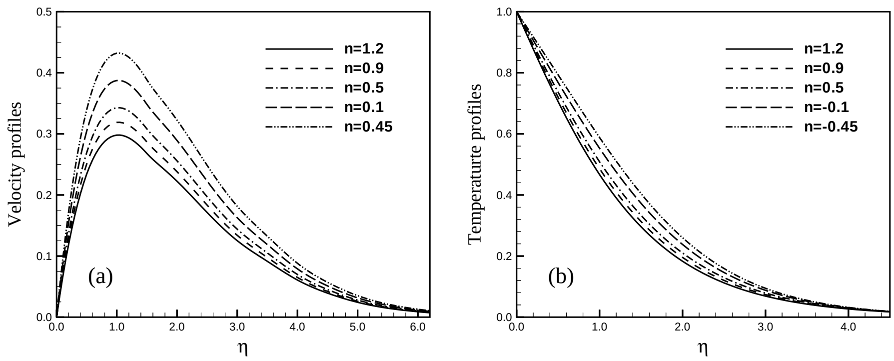

Fig. 3. (a) Velocity and (b) temperature profiles against η for different values of temperature gradient parameter n with $\epsilon = 0.2$, $Pr = 0.72$.

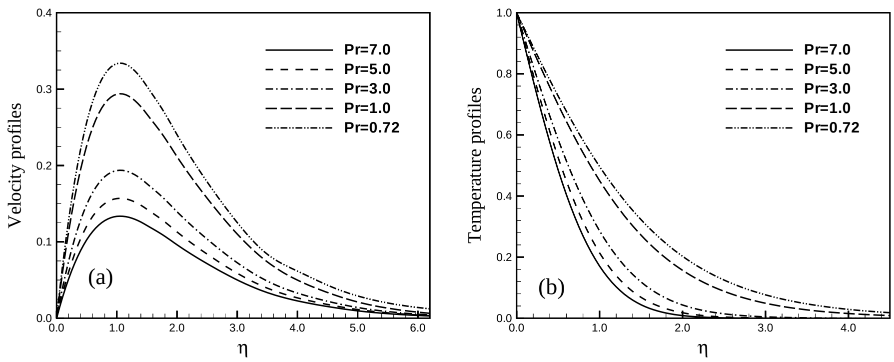

Fig. 4. (a) Velocity and (b) temperature profiles against η for different values of Prandtl number Pr with $\epsilon = 0.5$, temperature gradient parameter $n = 0.4$.

In Figs. 5–7 numerical values of local skin-friction coefficient and the local heat transfer coefficients obtained by finite difference as well as perturbation methods have been presented for different values of pressure work parameter ϵ, temperature gradient parameter n and Prandtl number Pr. In all these figures we observe that there are excellent agreements between perturbation solutions and finite difference solutions for $0 \leq \xi \leq 1$ as expected. We also observed that the values of local skin friction coefficients increase with the increase in ξ near the apex of the cone and its values decrease to the asymptotic value as ξ increase.

In Fig. 5(a), (b) the effects of the pressure work parameter ϵ on the local skin-friction coefficient and the local heat transfer coefficient are observed. From this Fig. 5(a), it can be seen that an increase in the values of ϵ ($\epsilon = -0.6,\ 0.1,\ 0.3,\ 0.6,\ 0.9$) leads to a decrease in the values of skin-friction coefficient for all ξ and small ξ. Also in Fig. 5(b) we see that the heat transfer coefficient decrease with the increase of pressure work parameter ϵ. Perturbation solutions are in very good agreement with finite difference solution.

The numerical values of local skin-friction coefficient, $C_{fx}(Gr_x)^{1/4}$ and local heat

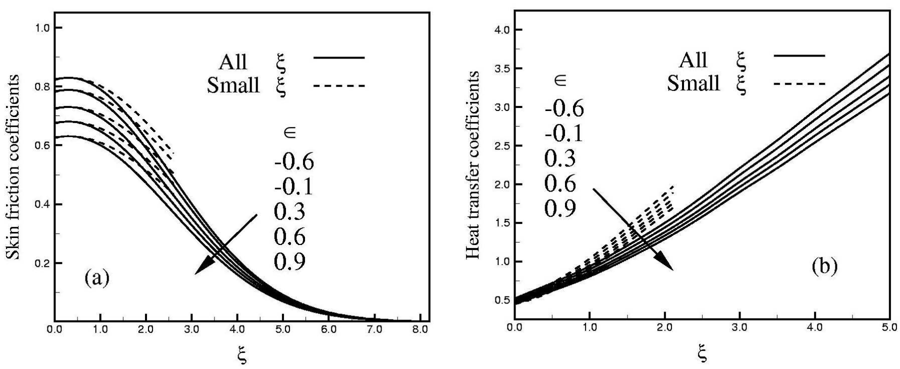

Fig. 5. (a) Skin friction and (b) heat transfer coefficients against ξ for different values of pressure work parameter ϵ with $n = 0.5$, $Pr = 0.72$.

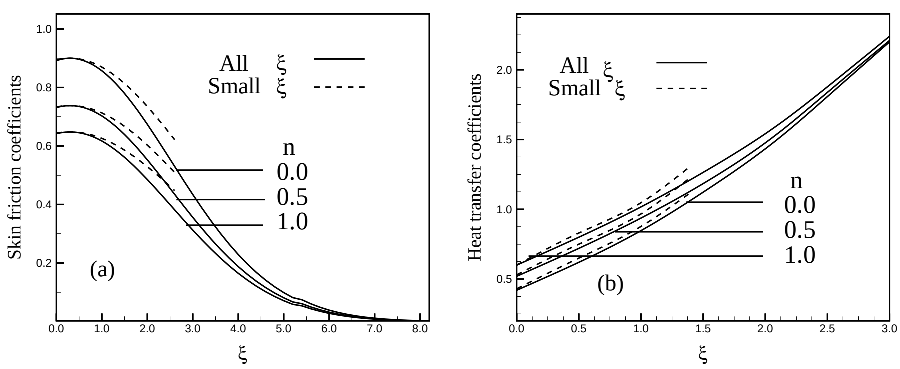

Fig. 6. Skin friction and (b) heat transfer coefficients against ξ for different values of temperature gradient parameter n with $\epsilon = 0.2$, $Pr = 0.72$.

transfer coefficient, $Nu_x/Gr_x^{1/4}$, against suction parameter ξ for different values of temperature gradient parameter n ($n = 0.0, 0.5, 1.0$) with Prandtl number $Pr = 0.72$ and the pressure work parameter $\epsilon = 0.2$ are displaced in Fig. 6(a), (b) respectively. From Fig. 6(a) we observe that the value of local skin friction coefficients decreases with the increase of temperature gradient parameter n against suction parameter ξ. Also from the Fig. 6(b), here we observed that the heat transfer coefficients are increasing with the increase of temperature gradient n. This indicates that the heat transfer coefficient does wholly depend on temperature gradient parameter.

In Fig. 7(a), (b), it is depicted that the local skin friction and local heat transfer coefficient are decreasing with the increasing values of Prandtl number Pr ($Pr = 0.72,\ 1.0,\ 3.0,\ 5.0,\ 7.0$) against suction parameter x and other controlling parameters $\epsilon = 0.3$ and $n = 0.4$.

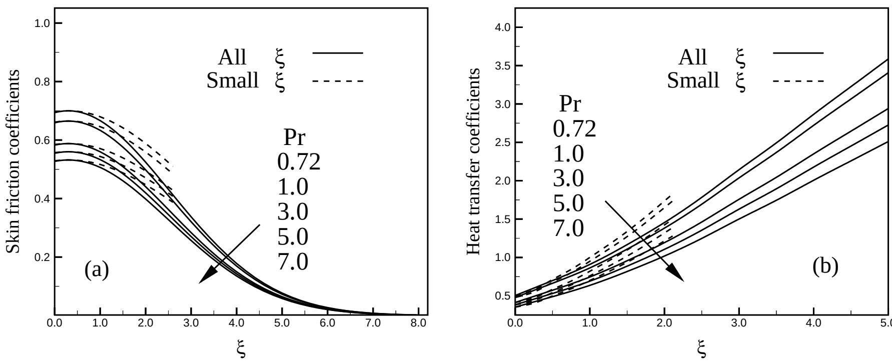

Fig. 7. (a) Skin friction and (b) heat transfer coefficients against ξ for different values of Prandtl number Pr with $\epsilon = 0.3$, temperature gradient parameter $n = 0.4$.

Tabulated values of the skin friction and heat transfer coefficients against ξ are shown in Table 1 for different values of temperature gradient n ($n = 0.0,\ 0.5,\ 1.0$). In this table, we see that the values of skin friction coefficients decrease with the increasing values of temperature gradient parameter n and the local heat transfer coefficients increases with the increasing values of the temperature gradient parameter n.

Table 1. Numerical values of skin-friction and heat transfer coefficient against ξ for the different values of temperature gradient n ($n = 0.0,\ 0.5,\ 1.0$) while $Pr = 0.72$, $\epsilon = 0.6$

n	0.0		0.5		1.0	
ξ	$f''(\xi,0)$	$-\theta'(\xi,0)$	$f''(\xi,0)$	$-\theta'(\xi,0)$	$f''(\xi,0)$	$-\theta'(\xi,0)$
0.0	0.891936	0.420508	0.7313878	0.520629	0.642194	0.600726
0.5	0.896407	0.620748	0.735054	0.720871	0.645413	0.800968
1.0	0.856963	0.848025	0.702710	0.938134	0.617013	1.018231
1.5	0.779210	1.129365	0.639025	1.196446	0.561095	1.276543
2.0	0.674111	1.441742	0.552771	1.501815	0.485351	1.576906
3.0	0.434152	2.202662	0.356004	2.212674	0.312589	2.242712

5 Conclusions

The present paper deals with the effect of transpiration velocity on laminar free convection boundary layer flow from a vertical non-isothermal cone. Numerical solutions of the equations governing the flow in the all ξ and small ξ (the scaled stream wise variable for the transpiration velocity) have been obtained by using the implicit finite difference method together with Keller-Box scheme and by perturbation method. From the present investigation, the following conclusions may be drawn:

- The value of skin friction coefficients increase with the increase in ξ near the apex of the cone and its values decrease to the asymptotic value as ξ increase.
- The local rate of heat transfer coefficient increases due to the increasing values of ξ.
- For increasing values of temperature gradient n, the skin friction coefficients decrease but the rate of heat transfer coefficients increase as the value of the temperature gradient parameter increases.
- An increase in the value of Prandtl number, Pr, leads to decrease in the values of skin-friction coefficient and the local rate of heat transfer respectively.
- For increase values of pressure work parameter ϵ, the skin friction coefficient and the rate of heat transfer are decreases.
- Due to the increase in temperature gradient parameter n, the velocity as well as the surface temperature decreases.
- The velocity and temperature distribution decreases with the increasing values the dissipation parameter ϵ.
- The fluid velocity as well as the temperature profiles decreases owing to the increase in the values of Prandtl number, Pr.
- There are excellent agreements between perturbation solutions and finite difference solutions for $0 \leq \xi \leq 1$ as expected.

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