Influence of Yield Strength Variability over Cross-Section to Steel Beam Load-Carrying Capacity*

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Abstract. Authors of article analysed influence of variability of yield strength over cross-section of hot rolled steel member to its load-carrying capacity. In calculation models, the yield strength is usually taken as constant. But yield strength of a steel hot-rolled beam is generally a random quantity. Not only the whole beam but also its parts have slightly different material characteristics. According to the results of more accurate measurements, the statistical characteristics of the material taken from various cross-section points (e.g. from a web and a flange) are, however, more or less different. This variation is described by one dimensional random field. The load-carrying capacity of the beam IPE300 under bending moment at its ends with the lateral buckling influence included is analysed, nondimensional slenderness according to EC3 is $\bar{\lambda} = 0.6$. For this relatively low slender beam the influence of the yield strength on the load-carrying capacity is large. Also the influence of all the other imperfections as accurately as possible, the load-carrying capacity was determined by geometrically and materially nonlinear solution of very accurate FEM model by the ANSYS programme.

Keywords: steel, yield strength, random fields, imperfections, nonlinear.

1 Introduction

This article is focused on the influence of the yield strength variation over the cross-section. Analyzing this influence by experimental manner is very com-

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plicated. The authors of the experimental research usually concentrate on the microstructure of the material, not on its characteristics as seen by engineers – these are to be found rarely [1]. Here, the steel profile characteristics are enumerated, the variability of the yield stress, however, is not commonly analyzed. The results of the experimental research were extrapolated by numerical studies analyzing influence of other characteristics distribution over cross-section, for example residual stress [2]. They showed an important impact of different diagrams of residual stress over cross-section on the load-carrying capacity. Taking these diagrams into consideration means to create a complicated computational model based on shell finite elements. For this reason, the material properties of the whole member tend to be diminished. The development of numerical simulations and increasing computer performance make it possible to look into so far neglected phenomena. The yield stress considerably influences on the load-carrying capacity. Since measuring its distribution over cross-section is very complicated (usually one web specimen and one flange specimen are being extracted), the authors of this article decided to analyze the impact of a random yield stress distribution on the load-carrying capacity of a bended beam by using a numerical model. The spatial variability of mechanical and geometrical properties of a system can be conveniently represented by means of random fields [3]. Besides other possibilities, the randomness can be taken into account directly in the material model [4]. For the study described in this article, a discrete approach was opted for. Because of the discrete nature of the finite element formulation, the random field must also be discredited into random variables [5]. Both approaches are amplified in a number of areas; no references were found about their usage it for the description of yield stress distribution over cross-section was not found.

It can be expected that, due to manufacturing technology in general, the yield strength of each cross-section segment will be different. Further, it can be anticipated that the yield strength of neighbouring segments will show strong correlation. By evaluating on larger number of bodies (cross-sections) of two neighbouring segments, we will obtain values the correlation of which will be higher than that of more distant segments. The decrease in the correlation among the segment i, the neighbouring segments $i + 1$, $i + 2$ etc. can be described by the auto-correlation function. The determination of this auto-correlation function based on the results of real material tests would be very valuable but at the same time, also highly demanding both from the economic and time aspects.

Therefore it is advantageous to study these problems using simulation methods. In this connection, several fundamental questions arise:

1. In what way does the yield strength variability over the cross-section influence the load-carrying capacity?
2. How does this influence change with the beam slenderness?
3. What is the real correlation function among individual cross-section segments?

Note: The last question can be answered only by means of experiments.

2 Random fields

Only the decreasing character of the auto-correlation function can be stated with certainty. If a sufficient number of experimental data from material tests was obtained, this function could be determined based on measurements results. The majority of real random phenomena occurring in the nature can be described by a correlation function, which decreases approximately exponentially. In our studies, it will be supposed that the numbers of a correlation matrix can be determined according to the relation (2), given in the paper [6], considering the auto-correlation function in Gaussian form:

$$c_{i,j} = S^2 \cdot e^{-\left(\frac{\xi_{i,j}}{L_{cor}}\right)^2}. \tag{1}$$

In the relation (1), the coefficient $c_{i,j}$ is dependent both on the so-called correlation length L_{cor}, standard deviation S of a random field, and further on, on the distance $\zeta_{i,j}$ between individual segments, see Fig. 1.
The correlation coefficient $\rho_{i,j}$ of the correlation matrix can be determined as:

$$\rho_{i,j} = \frac{c_{i,j}}{\sqrt{c_{i,i} \cdot c_{j,j}}}. \tag{2}$$

The random field was introduced for the yield strength of segments in one half of a flange and an adjacent half of the web. So, the problem was defined

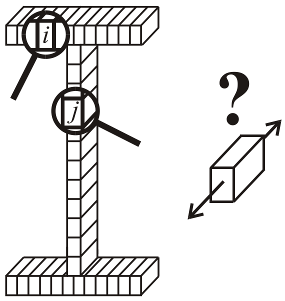

Fig. 1. Segments of IPE profiles.

by the one-dimensional correlation function (1) with variable parameter L_{cr}. The yield strength for the other elements was introduced in a symmetrical manner (as in the case of the biaxially symmetrical cross-section). The yield strength realizations were determined by means of the numerical simulation method LHS [3], elaborated by improving the standard Monte Carlo method (see, e.g., [7]). An example of one yield strength realization in dependence on the correlation length L_{cor} can be seen in Fig. 2.

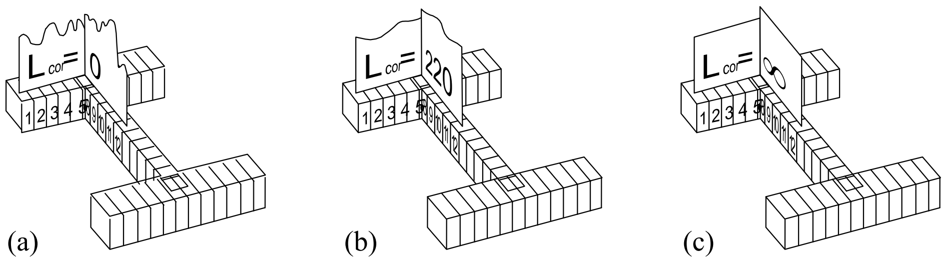

Fig. 2. Virtual segments of the IPE300 profile – yield strength variability over cross section.

3 Initial random parameters

Not only yield strength was calculated as random characteristic. From author's recent studies [2] is known influence of random distribution of material and geometrical characteristics, namely: geometry of the section (tolerances of plate elements and section shape), geometry of the beam (initial curvature and twist), residual stresses, yield strength and modulus of elasticity. The initial geometry of the beam section was considered according to Fig. 3. Apart from tolerances of elements thickness and section dimensions, also the cross-section shape imperfections characterized by parameters k_1, k_2, f and m (see Fig. 3(b), (c)) were taken into account. Statistical characteristics of quantities k_1, k_2, f, m were determined on the assumption that, at the same time, nominal values were also mean values, and that 95% of all the values measured lay within the tolerance limits of the standard [8].

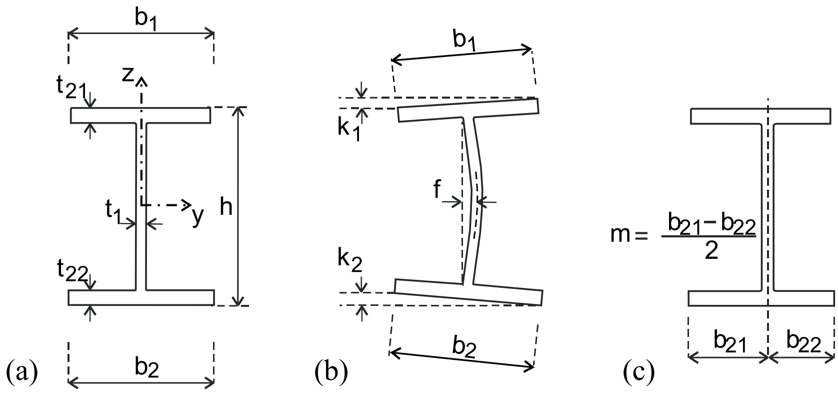

Fig. 3. Initial geometrical imperfections of the beam section.

Statistical characteristics both of the yield strength and the quantities h, b, t_1, t_2 were considered according to the histograms determined experimentally [9], see also [6], Table 1. Geometrical deviations of the initial beam curvature were introduced in form of sinusoids both for initial course in the bending plane defined by axes x-z and for plane x-y. In both cases, the amplitudes of the maximal initial deflection were supposed to be random quantities, distributed in the interval -2.3 mm to 2.3 mm in a uniform manner. Young's modulus E was considered, as based on two independent experimental results [10, 11], to be a random quantity with mean value $m_E = 210$ GPa and with standard deviation $S_E = 12.6$ GPa. An ideally elastic-plastic material was supposed.

The statistic independence was supposed for all the random quantities given in Table 2. A simplified correlation 1.0 was considered between the thickness and width of the lower and upper flanges. The residual stress was introduced with the mean value $m_{rs} = 60$ MPa and with standard deviation S_{rs} =20 MPa, with triangular distribution both on the flanges and on the web [12].

With statistical characteristics of the yield stress given in Table 2, the web

Table 1. Values of statistical characteristics of the output load-carrying capacity

		Correlation length L_{cor} [m]				
		0 mm	110 mm	220 mm	∞ mm	Trend
Statistical moments	Mean [kNm]	153.880	154.320	154.270	154.320	–
	S. deviation [kNm]	7.724	9.691	10.021	10.193	↗
	S. skewness	0.424	0.391	0.252	0.201	–
	S. kurtosis	3.481	3.339	3.254	3.208	–
0.1 percentile	Gauss [kNm]	130.01	124.37	123.30	122.82	↘
	Lognormal [kNm]	131.62	126.87	125.97	125.58	↘
	Hermite [kNm]	131.64	127.02	124.78	123.97	↘

Table 2. Statistical characteristics of input random quantities

No.	Quantity	Name of random quantity	Type of distribution	Dimensions	Mean value	Standard deviation
1.	f_y	Flange yield strength	Gauss	MPa	297.30	16.80
2.	f_y	Web yield strength	Gauss	MPa	307.30	16.80
3.	E	Young's modulus	Gauss	GPa	210.00	12.60
4.	h	Cross-section height	Gauss	mm	300.30	1.33
5.	b	Flange width	Gauss	mm	151.80	1.54
6.	t_1	Web thickness	Gauss	mm	7.49	0.30
7.	t_2	Flange thickness	Gauss	mm	10.60	0.47
8.	k_1	Upper flange displace	Gauss	mm	0	1.00
9.	k_2	Lower flange displace	Gauss	mm	0	1.00
10.	f	Initial web deflection	Gauss	mm	0	0.75
11.	m	Web out of symmetry	Gauss	mm	0	1.75
12.	rs	Residual stress	Gauss	MPa	60.00	20.00
13.	e_0	Amplitude of curvature	Rectangular in the interval $\langle -2.3; 2.3\,\text{mm}\rangle$			

yield stress with a higher mean value – compared to the flange yield stress – was considered. This corresponds to the results of an experimental research [9, 12]. When covering this section by finite element mesh, for each cross-section element, the yield stress was assumed as a random quantity. The correlation according to (2) was given among the elements. The correlation among the random field elements is given by the correlation length in (1). This is the main parameter of the random field used for modelling the variability of the yield stress over cross-section. Using three different correlation length values can be seen in Fig. 2. The main objective of this article is the analysis of the influence of the yield stress variability over cross-section on the load-carrying capacity.

4 Nonlinear computation model

The fine shell computation model was elaborated by means of the ANSYS programme [13]. The beam under in web plane bending moment M_y at its ends, which was made of hot rolled steel profile IPE300 with length $L = 2.3\,\mathrm{m}$, with nondimensional slenderness $\bar{\lambda} = 0.6$, was analysed.

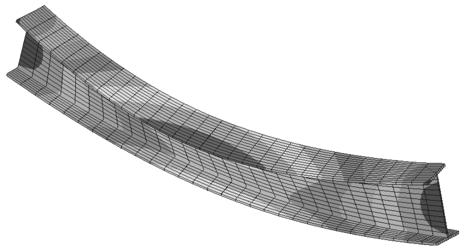

Fig. 4. Nonlinear finite shell element model – lateral beam buckling.

Geometry was formed using 540 elements SHELL181. It is a four-node element with twenty Gauss points (while four being located in an area, and five through the thickness of an element). This model is not limited by a beam theory (i.e., cross-section planarity). The symmetry along the beam axis was used with regard to the very exacting character of the problem solved. At the end of the beam on the web, zero displacement in the direction perpendicular to the web was prescribed. On the lower flange vertical displacement was fixed. The upper flange was left free.

It was supposed that the steel plastification occurred when the Misses stress exceeded the yield strength. Bilinear kinematic hardening of material was considered for steel S235. The Euler method based on incremental loading combined with the Newton Raphson method was applied for the nonlinear problem solution.

In each run of the LHS method, we searched for such a loading level at which the determinant of the tangential stiffness matrix K_t of the structure would approach zero with satisfactory exactness. The number of the LHS method runs applied was 200.

In each run there was solved the load-carrying capacity by a geometrically and materially nonlinear FEM computation which factually was the simulation of results of real 200 ultimate loading tests. The realizations of the yield strength input values by the LHS method are near the experimentally found material characteristics of the steel beams of steel S235, and the other geometrical characteristics correspond to available statistic results of measured quantities, as well.

5 Statistic analysis of load-carrying capacity

The results of a statistical analysis are given in Table 1. The design value was determined as the 0.1 percentile, based on design reliability conditions of EC1 having been presented, e.g., in [14]. The statistical characteristics of the load-carrying capacity are influenced by all random quantities given in Table 2. The variability of the yield stress over cross-section (see Fig. 2) changes in accordance with the change of the correlation length. The correlation length infinity means that the flange yield stress and the web yield stress are two independent random quantities, see Fig. 2(c). The correlation length zero, on the contrary, means that the yield stress of each random field element is an independent random quantity, see Fig. 2(a). The values $L_{cr} = 0$ and $L_{cr} = \infty$ represent two limit states of an analyzed phenomenon. It is obvious that the change in the correlation length influences esp. on a considerable deviation of the load-carrying capacity.

6 Conclusion

It is evident from the results presented in Table 1 that the standard deviation increases with increasing the correlation length which influences the load-carrying capacity design value. Opposite to this, the mean value is sensitive to the correlation length change only very little.

The results obtained from the theoretical analysis refer to the fact that for hot-rolled beams the influence of material characteristics variability along the cross-section cannot be neglected. The yield strength determination on the samples taken from one cross section point (mostly from one third of the flange) is not sufficient. Real correlation lengths can be determined, however, only by experimental research. According to [12], the yield strength on flanges was, on average, by till 26% lower than on the web. For hot rolled cross sections, more samples to be analysed should be taken both from the flange and the web.

References

1. M. Fajkus, M. Holický, L. Rozlívka, M. Vorlíček. Random properties of steel produced in Czech republic, in: *Proc. of 2nd International Conference EUROSTEEL'99, Prague, ČVUT,* pp. 657–60, 1999, ISBN 80-01-01963-2.

2. Z. Kala, J. Kala. The Statistical Correlation of Material Characteristics – Experimental and Theoretical Results of Hot-Rolled Steel Beam, in: *Proc. of the International Conference on Metal Structures, Miskolc, Hungary, April 3–5, 2003,* K. Jarmai, J. Farkas (Eds.), Millpress Science Publishers, Rotterdam, pp. 23–26, ISBN 90 77017 75 5.

3. D. Novák, W. Lawanwisut, C. Bucher. Simulation of random fields based on orthogonal transformation of covariance matrix and Latin Hypercube Sampling, in: *Proc. of the International conference on Monte Carlo Simulation, Monte Carlo, 18–21 June, 2000,* pp. 129-136.

4. N. Bilger, F. Auslender, M. Norbert, J.C. Michel, H. Moulinec, P. Suquet, A. Zakuj. Effect of a nonuniform distribution of voids on the plastic response of voided materials: a computational and statistical analysis, *International Journal of Solids and Structures,* **42**(2), pp. 517–538, 2005.

5. E. Vanmarcke, M. Shinozuka, S. Nakagiri, G. Schuëller, M. Grigoriu. Random fields and stochastic finite elements, *Journal Structural Safety,* **3**, pp. 143–166, 1986.

6. Z. Kala. The Influence of Initial Curvature of the Axis Upon the Member Ultimate Strength, *Journal of Structural Mechanics,* **36**(1), pp. 3–14, 2003, ISSN 0783-6104.

7. P. Marek, J.M. Brozzetti, M. Guštar, P. Tikalsky, Z. Kala, et al. *Probabilistic Assessment of Structures using Monte Carlo simulation. Background, Exercises and Software,* 2nd extended edition, ITAM, Czech Republic, ISBN 80-86246-19-1.

8. EN 10034: Bars with I and H cross-sections made of structural steels – Limit deviations of dimensions and shape tolerance, September 1995.

9. J. Melcher, Z. Kala, M. Holický, M. Fajkus, L. Rozlívka. Design Characteristics of Structural Steels Based on Statistical Analysis of Metallurgical Products, *Journal of Constructional Steel Research*, **60**, pp. 795–808, 2004, ISSN 0143-974X.

10. Y. Fukumoto, N. Kajina, T. Aoki. Evaluation of Column Curves Based on Probabilistic Concept, in: *Proc. of the International Conference on Stability,* Prelim. Rep., Gakujutsu Bunken Fukyu-Kai, Tokyo, 1976.

11. C. Guedes Soares. Uncertainty Modelling in Plate Buckling, *Journal Structural Safety,* **5**, pp.17–34, 1988.

12. I. Daddi, F.M. Mazzolani. Détermination expérimentale des imperfections structurales des profilés en acier, *Construction métallique*, **1**, pp. 24–45, 1974.

13. ANSYS, User's Guide for rev. 9.0, SAS IP Inc., 2004.

14. EN 1990: Eurocode. Basis of Structural Design, 2002.