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The Optimization Process of Elimination Sediment from the Pipe by Impact Load

V. Doroševas, V. Volkovas

Technological Systems Diagnostics Institute
Kaunas University of Technology, Kęstučio st. 27, Kaunas, Lithuania
vikdor@ktu.lt

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Abstract. The article deals with dynamics of a heterogeneous system of two cylindrical surfaces with different physical and mechanical qualities, which are impacted by shock load. An effective method was suggested for optimization process of elimination sediment from pipe as the result of theoretical studies. This method allows transforming a heterogeneous system of pipe into a homogeneous system, which preserves its functional purpose. Shock impact is applied for transformation, which is optimized according to the suggested qualitative criteria of the transformation process and removing sediment from pipe.

Keywords: optimization, impact loads, mathematical model, pipelines systems.

1 Introduction

While operating pipeline systems we often run into a situation when there is a layer of transported substance (sediment) formed on the internal surface of the pipe. Such system can be conditionally called "a pipe in a pipe" (see Fig. 1). An external pipe is made from steel and the internal one – from the sediment. Thus a heterogeneous system of two cylindrical surfaces with different physical and mechanical qualities is formed. Sediment hinders technological processes of transportation, thus various ways of its elimination are applied [1]. Unfortunately, very often traditional ways are ineffective, especially when such a heterogeneous system is formed in pipeline systems for oil processing. Shock loadings were suggested for the renovation of the pipeline system [2]. We have found a way of calculating the real displacements of solidified sediment in the pipe after an explosion of a single charge at a specific place in the pipe [3]. However, its efficiency depends on many factors and it is necessary to study the influence of various impact parameters and to evaluate local reactions of the heterogeneous system, to optimize the sediment elimination process in accordance with the quality criteria and quantitative parameters of impact that assure durability and functionality of the pipeline systems.

2 Problem formulation

The basic stage of optimization research is the formation and description of the real system of the model as it becomes possible to evaluate the chance of the solution and its realization practically. In our case, it is the process of the optimization of the elimination, the assurance of the effectiveness, of the solid sediment being affected by the impact load when the effect is minimal, i.e., while taking into account the criteria of the stability of offsets, impact load and the construction of the pipe. The structure of the problem of the optimization is reflected in the diagram (see Fig. 2).

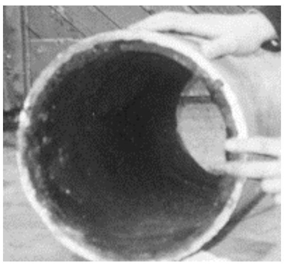

Fig. 1. Fragment of a pipe with sediment.

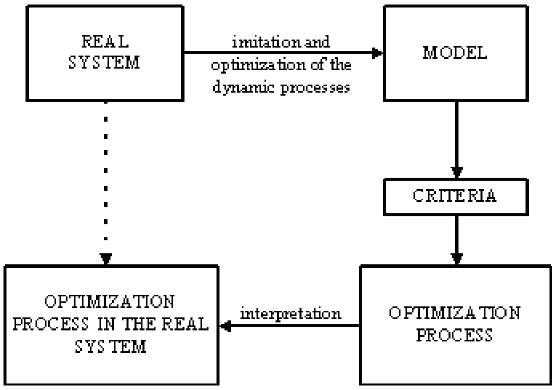

Fig. 2. The diagram of optimization.

As we can see, the direct way (dotted line) leading to the optimal process of the realization of the elimination of sediment is exchanged into the circuitous, which leads to the optimal pulverization process of the solid sediment in the real system through the formation of the model, the imitation and optimization of the dynamic processes in it. It is obvious that such attitude towards the optimization of the system needs some concrete easiness of the real system.

3 Problem formulation

For example (Fig. 1) we will form a transformed model (see Fig. 3) of the reflected real system and analyse its possibilities of optimization. There fore we have to formulate the optimization problem with multi-criteria where the function of the aim depends on the parameters of the optimization – the size of the impact effect, duration and its position in the pipe, the displacements of the sediment, which occurred because of the effect of the impact load and its influence on the increase of the pressure for the construction of the pipe. On the other hand, we should introduce the limitation of the optimization problem, i.e. the conditions describing the warrantable meanings of these parameters.

3.1 Model of the sediment

In order to reach this, we split the harden sediment in the pipe into the segments conditionally (see Fig. 4).Let us assume that the length of the side of the segment is equal to b and the angle φ. That means we will get some finite amount of segments and will be able to number them according to their position in the cylindrical system of r, θ, z coordinates. The numeration of segments allowing to identify them in the cavity of the pipe is expedient to be done while taking into account the specific process of the elimination of the sediment, indexing segments in the vertical slashes and after that indexing those slashes along the z axis. The position of segment in the system of cylindrical coordinates can be expressed in the following way: $x_i = r_i \cos\theta_j$, $y_j = r_i \sin\theta_j$, $z_k = z_k$.

According to Fig. 3 an Fig. 4, the amount $N^{(k)}$ of segments in the cross-section z_k is usually expressed mathematically in such a way:

$$N^{(k)} = I \cdot J, \tag{1}$$

where $I = \frac{R_2 - R_1}{b}$, $\quad J - \frac{180}{\varphi}$.

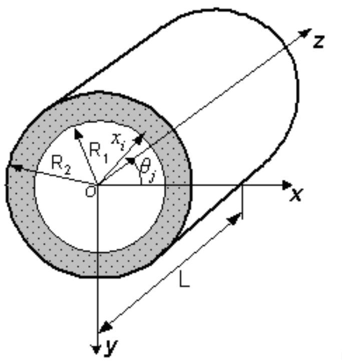

Fig. 3. Displacement counting diagram.

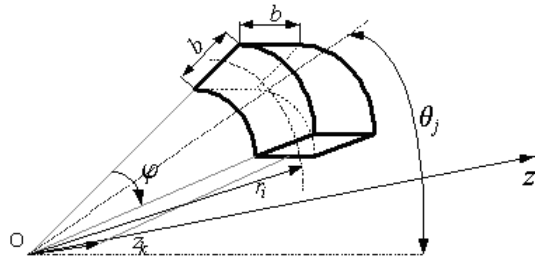

Fig. 4. The diagram of the segment of the sediment.

The sum N is the finite amount of the segments and in the whole volume it will be

$$N = K \cdot N^{(k)} = I \cdot J \cdot K, \quad K = \frac{L}{b}. \tag{2}$$

In equations (1) and (2) b and φ must be such, that I, J and K were be positive integers. Then in the cross-section z_k the coordinates of the i, j segment will be r_i, θ_j, z_k (see Fig. 5) and $i = 1, 2, \ldots, I, \;\; j = 1, 2, \ldots, J, \;\; k = 1, 2, \ldots, K$.

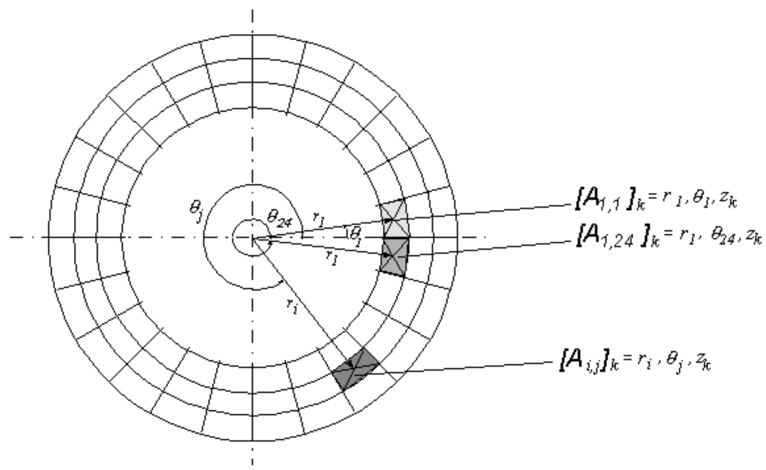

Fig. 5. The diagram of the segments in the cross-section z_k.

For theoretical investigations every i, j segment in cross-section z_k will be defined as segment n from cross-section k.

3.2 Optimization the theoretical displacements of segments

Let us suppose that displacements (see Fig. 3) towards Ox direction are $u = r\cos\theta$, towards Oy direction – $v = r\sin\theta$, towards Oz direction – w in cylindrical system of axis and that separation function are:

$$u = Uq, \quad v = Vq, \quad w = Wq, \tag{3}$$

where $U = U(r, \theta, z),\ V = V(r, \theta, z),\ W = W(r, \theta, z),\ q = q(t)$. Functions U, V and W are selected according to the boundary conditions, i.e. they should fit for a body presented in Fig. 3.

While knowing the physical-mechanical parameters of the material and the value of impact load we will be able to use the procedure of calculation of the theoretical displacements [3] and conventionally call them test displacements in order to evaluate the calculation test displacement of the n segment in the cross-section k:

$$u_{n,k}^{(t)} = U_{n,k}\, q_{n,k}^{(t)}, \tag{4}$$
$$v_{n,k}^{(t)} = V_{n,k}\, q_{n,k}^{(t)}, \tag{5}$$
$$w_{n,k}^{(t)} = W_{n,k}\, q_{n,k}^{(t)}, \tag{6}$$

where accordingly n, k are the indexes of the identification of the n segment and the cross-section z_k.

Let us form the test matrixes of the displacements for each cross-section, which we can mark as follows:

$$\lfloor u_n^{(t)} \rfloor_k, \quad \lfloor v_n^{(t)} \rfloor_k, \quad \lfloor w_n^{(t)} \rfloor_k. \tag{7}$$

While summing up the derived matrixes vectorially we can form the test summed matrix $s_n^{(t)}$ for each cross-section:

$$\lfloor \vec{s}_n^{(t)} \rfloor_k = \big\lfloor \lfloor \vec{u}_n^{(t)} \rfloor_k + \lfloor \vec{v}_n^{(t)} \rfloor_k + \lfloor \vec{w}_n^{(t)} \rfloor_k \big\rfloor. \tag{8}$$

We can rewrite the derived form (8) while writing test summed displacements in the cross-section k into one matrix:

$$\begin{bmatrix} s_{11}^{(t)} & \dots & s_{1j}^{(t)} & \dots & s_{1J}^{(t)} \\ \vdots & & \vdots & & \vdots \\ s_{i1}^{(t)} & \dots & s_{ij}^{(t)} & \dots & s_{iJ}^{(t)} \\ \vdots & & \vdots & & \vdots \\ s_{I1}^{(t)} & \dots & s_{Ij}^{(t)} & \dots & s_{IJ}^{(t)} \end{bmatrix}_k . \tag{9}$$

Let us say that we know the critical summed size $\varepsilon_k^{(t)}$ of the test displacement in the cross-section z_k ($\varepsilon_k^{(t)} = \varepsilon^{(t)}$ if the physical-mechanical parameters of the sediment along the pipe are constant) the existence of which the breaking of the solid sediment is possible. After using this criterion we can write the necessary condition for the elimination of the sediment:

$$\lfloor s_{ij}^{(t)} \rfloor_k \geq \varepsilon_k^{(t)}. \tag{10}$$

That means, when we use the condition presented in the form (10), i.e. while checking its validity for the matrix (9) we can localize these segments in which it will not be done and state that the displacement of these segments is unsatisfactory for the breaking of the sediment and here the impact load needs to be increased. Hence, we define the most solid areas of the material for each cross-section k in which

$$\lfloor s_{lm}^{(t)} \rfloor_k < \varepsilon_k^{(t)}, \tag{11}$$

where $\lfloor s_{lm}^{(t)} \rfloor_k$ is the matrix of summed test displacements in the k cross-section which does not satisfy the condition (11) and these segments can be conditionally called defective. After performing this action we can consider, that all identified defective segments well known for us in the whole volume of the solid sediment in which the test displacements are not sufficient or are minimal and we can optimize impact load and perform the calculation procedure of real displacements [3].

3.3 Optimization the real displacements of segments

For this purpose we imitate the impact load in the k cross-section of the defective segment s_{lm} and calculate the real displacements by [3] in the whole cross-section:

$$u_{n,k} = U_{n,k}\, q_{n,k}, \tag{12}$$

$$v_{n,k} = V_{n,k}\, q_{n,k}, \tag{13}$$

$$w_{n,k} = W_{n,k}\, q_{n,k}. \tag{14}$$

Analogically, as in the case of test displacements, we form the matrixes of real summed displacements for each cross-section:

$$[\vec{s}_n^{(t)}]_k = \big[[\vec{u}_n^{(t)}]_k + [\vec{v}_n^{(t)}]_k + [\vec{w}_n^{(t)}]_k\big], \tag{15}$$

$$[s_n]_k = \begin{bmatrix} s_{11} & \dots & s_{1j} & \dots & s_{1J} \\ \vdots & & \vdots & & \vdots \\ s_{i1} & \dots & s_{ij} & \dots & s_{iJ} \\ \vdots & & \vdots & & \vdots \\ s_{I1} & \dots & s_{Ij} & \dots & s_{IJ} \end{bmatrix}_k . \tag{16}$$

We compare the derived meanings to the criterion of the elimination of the sediment:

$$\lfloor s_{ij} \rfloor_k \geq \varepsilon_k. \tag{17}$$

If the condition presented in the equation (17) does not corresponds to all segments of the real displacement matrix then after adjusting the parameters of the impact load it is necessary to calculate according the equations (12)–(16) until the condition (17) will be realized.

Let us say that we have got the matrix of solid sediment, where the real displacements of all segments satisfy the condition of the elimination of the sediment, i.e. we optimize the dynamic processes from the point of impact load and displacements. However, the pipe also belongs to a heterogeneous system. Thus, it is necessary to evaluate its endurance and functionality according to real sediment displacements and to perform a process of heterogeneous system's transformation into the homogeneous one. That is means that after initiating the impact load in the defined segments it is possible to get the eliminate sediment with minimal expenditure of energy. However, it is necessary to check the influence of the effect on the stability of the pipe construction.

3.4 Optimization the influence of solidified sediment displacements on the pipe structure

Referring to the earlier presented theoretical investigations, it's possible to produce a method of calculating displacements of a solidified sediments under the action of impact load. However, the dynamic system that is being considered includes a pipe of a cylindrical form. The influence of the dynamic processes on the structure of the pipe are therefore the assesses and conditions (under which the strength of the pipe structure is ensured) to be determined. Consequently, it's important to known how the pressure of the sediments on the walls of the pipe will increase when affected by the impact load. When formulating the assignment, the pipe was considered to be absolutely solid and in equilibrium. We can thus affirm that under the action of the impact load the displacements of the pipe walls are equal to zero. This is approximately true, assuming that the pressure increase is small. The pressure of external surface forces, p, and the reaction of elastic forces are defined by the following equalities in the cylindrical coordinates:

$$
\begin{aligned}
p_{rn} = {} & \lambda\beta\cos(nr) \\
& + G\left[\left(\frac{\partial u}{\partial r}\right)\cos(nr) + \left(\frac{\partial u}{r\partial\theta}\right)\cos(n\theta) + \left(\frac{\partial u}{\partial z}\right)\cos(nz)\right] \\
& + G\left[\left(\frac{\partial u}{\partial r}\right)\cos(nr) + \left(\frac{\partial v}{\partial r} - \frac{v}{r}\right)\cos(n\theta) + \left(\frac{\partial w}{\partial r}\right)\cos(nz)\right],
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
p_{\theta n} = {} & \lambda\beta\cos(n\theta) \\
& + G\left[\left(\frac{\partial v}{\partial r} - \frac{v}{r}\right)\cos(nr) + \left(\frac{\partial v}{r\partial\theta} + \frac{u}{r}\right)\cos(n\theta) + \left(\frac{\partial v}{\partial z}\right)\cos(nz)\right] \\
& + G\left[\left(\frac{\partial u}{r\partial\theta}\right)\cos(nr) + \left(\frac{\partial v}{r\partial\theta} + \frac{u}{r}\right)\cos(n\theta) + \left(\frac{\partial w}{r\partial\theta} - \frac{w}{r}\right)\cos(nz)\right],
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
p_{zn} = {} & \lambda\beta\cos(nz) \\
& + G\left[\left(\frac{\partial w}{\partial r}\right)\cos(nr) + \left(\frac{\partial w}{r\partial\theta} - \frac{w}{r}\right)\cos(n\theta) + \left(\frac{\partial w}{\partial z}\right)\cos(nz)\right] \\
& + G\left[\left(\frac{\partial u}{\partial z}\right)\cos(nr) + \left(\frac{\partial v}{\partial z}\right)\cos(n\theta) + \left(\frac{\partial w}{\partial z}\right)\cos(nz)\right],
\end{aligned} \tag{20}
$$

where n – the normal to the pipe wall, $\beta = \frac{\partial u}{\partial r} + \frac{\partial v}{r\partial\theta} + \frac{u}{r} + \frac{\partial w}{\partial z},\ \lambda = \frac{2\mu G}{1-2\mu}$.

Shearing modulus G of the sediment and Poisson's coefficient μ depend on the wave propagation rate in the sediment and can be approximately calculated [4].

In our case (see Fig. 5) $\cos(rn) = 1,\ \cos(\theta n) = 0,\ \cos(zn) = 0.$

Then equations (18)–(20) can be rewritten

$$p_{rn} = \lambda\beta + 2G\left(\frac{\partial u}{\partial r}\right), \tag{21}$$

$$p_{\theta n} = G\left(\frac{\partial v}{\partial r} - \frac{v}{r} + \frac{\partial u}{r\partial\theta}\right), \tag{22}$$

$$p_{zn} = G\left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z}\right). \tag{23}$$

Then the total pressure

$$p_r = \sqrt{p_{rn}^2 + p_{\theta n}^2 + p_{zn}^2}. \tag{24}$$

Substituting into equations (21)–(23) the values (4)–(6) of test displacements calculated by [3], the pressure on the pipe construction due to test displacement under the action of test impact load at any point of the pipe cavity can be calculated. Besides, substituting into the same expressions the values (12)–(14) of real displacements calculated by [3], we can assess the influence of the pipe structure. This can be given by the following mathematical equations

$$p_r^{(t)} = q_i^{(t)}\eta_r, \tag{25}$$

$$p_r = q_i\eta_r, \tag{26}$$

where $\eta_r = f_r(G, \mu, U, V, W)$ (see [3]) and index (t) shows that the operation of the test displacement calculation is being used.

Eventually, making use of expressions (25) and (26), test and real pressures can be calculated and constrains on pipe structure steadiness can be introduced

$$p_r^{(t)} = \sigma_{r\ allow}^{(t)}, \tag{27}$$

$$p_r = \sigma_{r\ allow}, \tag{28}$$

where $\sigma_{r\ allow}^{(t)}, \sigma_{r\ allow}$ – test and real allowable stress of pipe structure respectively.

On the θ and z axes we can do an analogical procedure.

3.5 Numerical example

The problem simulated numerically is sketched in Fig. 3 and Fig. 5. In this simplified model we calculating only displacement of centre of a segment in the cross-section k. For example, radius $R_1 = 19.5\,\mathrm{cm}$, $R_2 = 22.5\,\mathrm{cm}$ and length of pipe $L = 500\,\mathrm{cm}$. Let we know the physical-mechanical parameters of the material: sediment density $\rho = 800\,\mathrm{kg/m}^3$, shear modulus $G = 20\,\mathrm{MPa}$ and Poisson's coefficient $\mu = 0.4$. Let's suppose that shock impact is received after an explosion of a spherical ammonite charge of $1\,\mathrm{kg}$ mass in the pipe at the position in the cylindrical system: $r_0 = 20\,\mathrm{cm}$, $\theta_0 = 60^\circ$ and $z_0 = 20\,\mathrm{cm}$ and we calculating the theoretical and real displacements in the cross-section $z_k = 10\,\mathrm{cm}$, $r_1 = 20\,\mathrm{cm}$ for segments $[A_{1,1}]_k, \ldots, [A_{1,24}]_k$ by equations (4)–(6), (8) and (12)–(15) according [3].

Obtained test summed displacements $[s_{1,1}]_k^t, \ldots, [s_{1,24}]_k^t$ of the centre of a segments $[A_{1,1}]_k, \ldots, [A_{1,24}]_k$ is presented in Fig. 6a and real summed displacements $[s_{1,1}]_k, \ldots, [s_{1,24}]_k$ in Fig. 6b. Substituting into equations (21)–(23) cal-

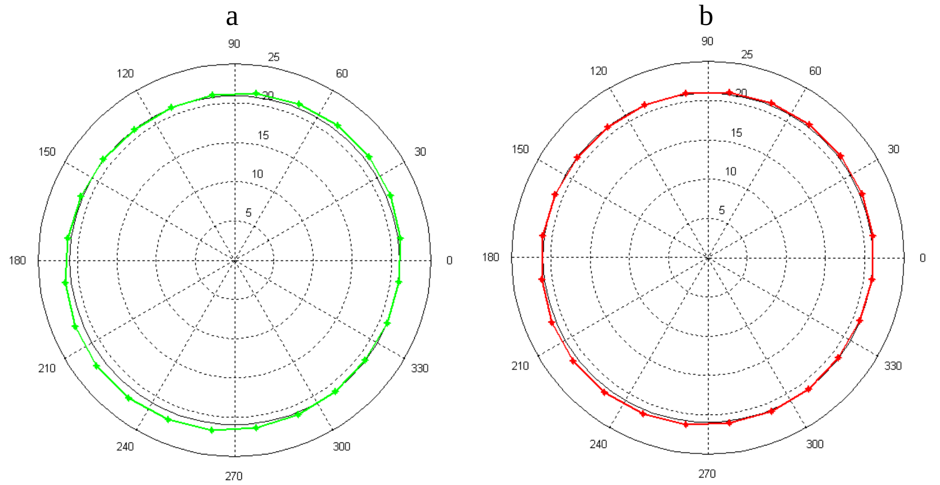

Fig. 6. Distribution of summed displacement of segments $[A_{1,1}]_k$, ..., $[A_{1,24}]_k$: a) test displacement $[s_{1,1}]_k^t$, ..., $[s_{1,24}]_k^t$; b) real displacements $[s_{1,1}]_k$, ..., $[s_{1,24}]_k$.

culated values of test and real displacements of segments $[A_{1,1}]_k, \ldots, [A_{1,24}]_k$ we calculating the total pressure on the pipe construction due to test and real displacement under the action by (24)–(26). Obtained distribution of the total pressure on the pipe construction is presented in Fig. 7.

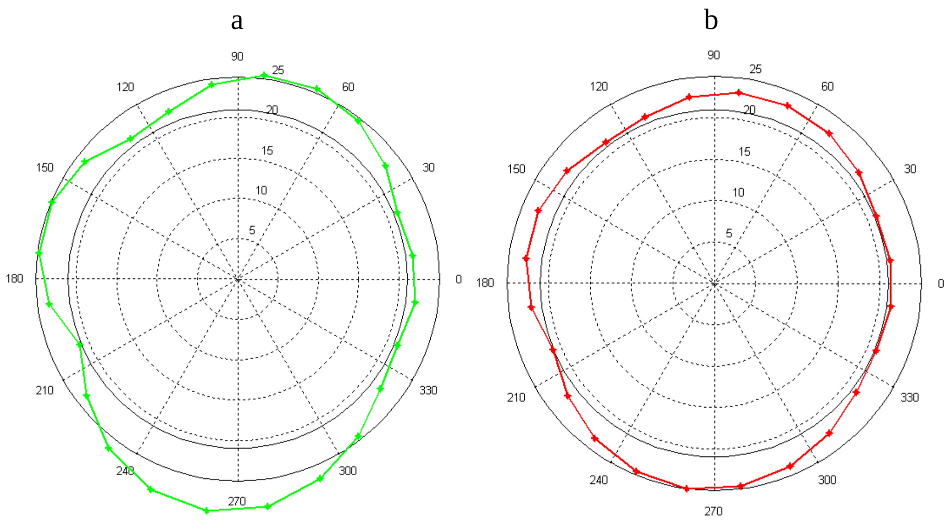

Fig. 7. Distribution of the total pressure on the pipe construction due to test and real displacement of segments $[A_{1,1}]_k, \ldots, [A_{1,24}]_k$: a) under test displacements $[s_{1,1}]_k^t, \ldots, [s_{1,24}]_k^t$; b) under real displacements $[s_{1,1}]_k, \ldots, [s_{1,24}]_k$.

3.6 Finality optimization procedure

The mathematical procedures discussed above allows the evaluation of the influence of the dynamic processes on the pipe structure.

Because of this we have to imitate optimized impact load in the segment and in relevance to this evaluate the increase of the pressure on the pipe wall from the displacements of all segment, which appeared according to equation (26). The strength of the pipe is checked according to the equation (28). If the conditions of stability are not performed it is necessary to adjust the size of the impact load and the period or the place of imitation and to repeat the whole procedure of calculation again. So, we create the algorithm of optimization procedures (see Fig 8). In this way multi-criteria problem of optimization according to the conditions of limitation and the aim function can be formulated, which we can introduce in a standard form. For example as the following [5]:

$$\text{To find} \quad \min \Phi(s_{ij}^{(t)}, s_{ij}, p_r, I, t). \tag{29}$$

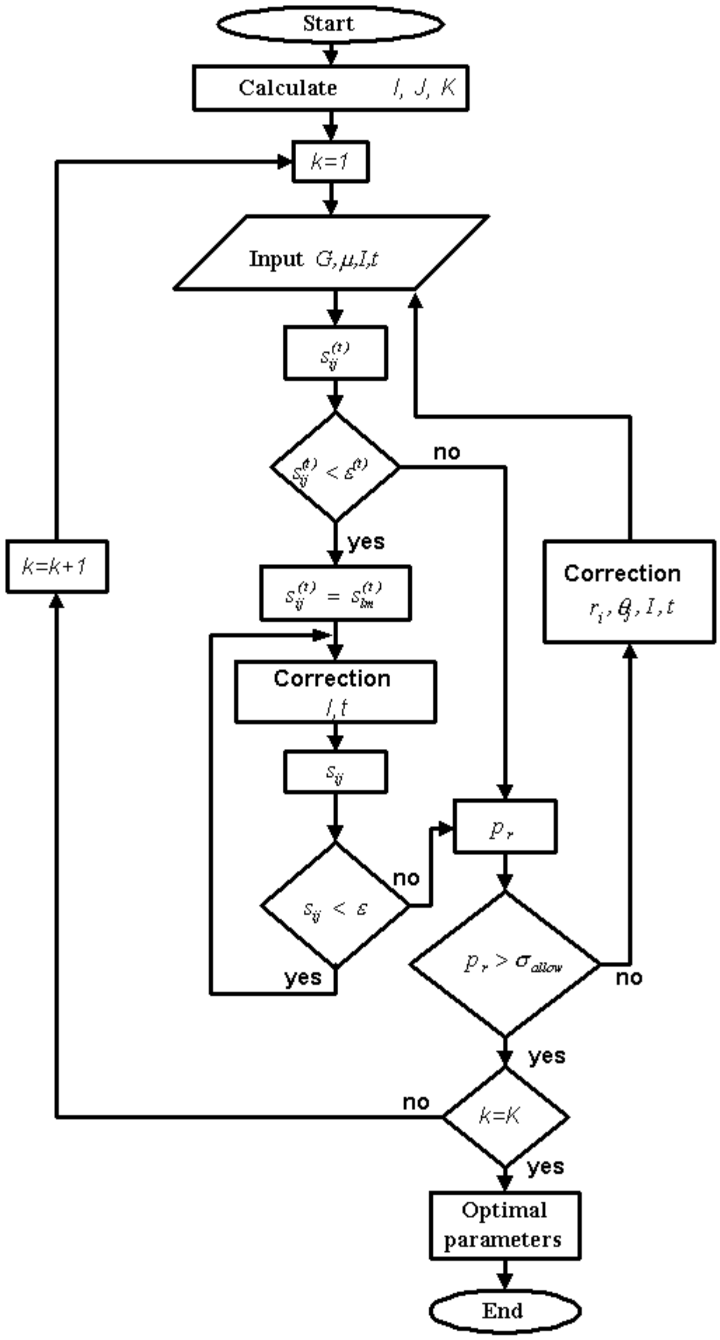

Fig 8. The algorithm of optimization procedures.

When the limitations exist:

$$[s_{ij}^{(t)}]_k \geq \varepsilon_k^{(t)}, \quad [s_{ij}]_k \geq \varepsilon, \quad p_r < \sigma_{allow}, \quad t \leq t_{kr}, \quad I \leq I_{allow}. \tag{30}$$

Such a problem can be solved while using the standard method of optimization with the LP search [6].

In this way we can optimize the dynamic processes, which occurred due to the impact load effect in the material of solid sediment and define the parameters, which ensure the optimum of the imitative processes in the model. Due to this we can use the given optimized parameters in the real system, i.e. after initiating shock impact effects in the defined places of the pipe volume and eliminate the sediment.

4 Conclusions

Developed and presented optimization method enables:

- to model and develop new ways of pipeline cleaning;
- to calculate approximate sediment displacements that appeared through shock impact at a specific place of the pipe and to estimate the influence of solidified sediment displacements on the pipe structure;
- to establish functional dependence between shock impact, pipe structure and sediment displacements.

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