Voronoi Analysis of a Soccer Game

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Received: 16.06.2004
Accepted: 30.08.2004

Abstract. This paper describes a Voronoi analysis method to analyze a soccer game. It is important for us to know the quantitative assessment of contribution done by a player or a team in the game as an individual or collective behavior. The mean numbers of vertices are reported to be 5–6, which is a little less than those of a perfect random system. Voronoi polygons areas can be used in evaluating the dominance of a team over the other. By introducing an excess Voronoi area, we can draw some fruitful results to appraise a player or a team rather quantitatively.

Keywords: Voronoi analysis, image processing.

1 Introduction

In 1908, Voronoi [1] found a way of partitioning all space amongst a collection of points using specially constructed polygons. A Voronoi diagram is one of the most important structures in computational geometry [2]. It has information about what is close to what. More precisely, each point is surrounded by a unique limiting convex polygon such that all points within a point's polygon are closer to this point than all the other points.

Voronoi analysis was widely used in characterizing a structure of the soft condensed matter [3]. If Voronoi polygons are constructed around a point, such as in a crystal, all the area in the unit cell will be apportioned to the points.Voronoi construction quantify a packing with Voronoi polygons.

In this paper, we will check whether this method is useful in analyzing a computer soccer game or not. In fact we have already reported some interesting results of a soccer game, in particular fractal behaviors of a ball [4].

2 Voronoi cell analysis

Here we will consider the Voronoi analysis only in a 2-D system. The Voronoi cell is defined as follows: Consider a set of coplanar points P. For each point $\overrightarrow{x_i}$ in the set P, we can draw a boundary enclosing all the intermediate points lying closer to $\overrightarrow{x_i}$ than to any other points in the set P. Such a boundary is called a Voronoi polygon or Voronoi cell, and the set of all Voronoi polygons for a given point set P is called a Voronoi diagram of the point set P.

A Voronoi polygon can be determined by the following procedures:

1. Draw lines to connect a given point to all nearby points.
2. At midpoints and normal to these lines, draw new lines.
3. The smallest area enclosed in this way is called a Voronoi polygon of a given point.

The Voronoi cell associated with a single point is a constructed polygonal area for which all the points contained within the area are nearest to the associated point. The space is partitionized into a set of polygons with all points that are closer to a particular point than to any others belonging to its polygon. Points can be regarded as adjacent if their polygons share a common line, thus we define the neighborhood of a point uniquely. Each point is surrounded by a single convex polygon and allocated space within it. A large area implies a cavity. In condensed matter physics, the polygons themselves are considerable interest since the interactions between molecules can influence their geometrical properties in the molecular system [3]. Note that the mean number of vertices for random system is just 6 [5].

Given a set of coplanar points P, a Delaunay triangulation is a set of lines connecting each point to its natural neighbors. In other words, it is a set of triangles such that no points are contained in any triangle's circumcircle. Thus it is dual to the Voronoi diagram.

3 Results

As we pointed out in earlier paper [4], generally speaking, we actually have enormous difficulties to get the coordinates of the moving players and a ball from a real-world game, so we must get around. If we play the electronic soccer game in the computer, fortunately, we feel that it seems to be quite similar with a real-world soccer game. The electronic soccer game which we adopted in our analysis is FIFA Soccer 2003 by EA Sports. In this game, if we set the RADAR, we have the small inset which shows the motions of all the players and the ball. We captured these small insets to make them into a video file. We measured the locations of all the players and the ball by using standard image/video processing techniques [6]. The formation of starting positions is as follows: A team F has a 4-4-2 formation, and the opponent team S has a 4-3-3 formation. The Voronoi diagram for their starting formation is shown in Fig. 1. The numbers in the figure

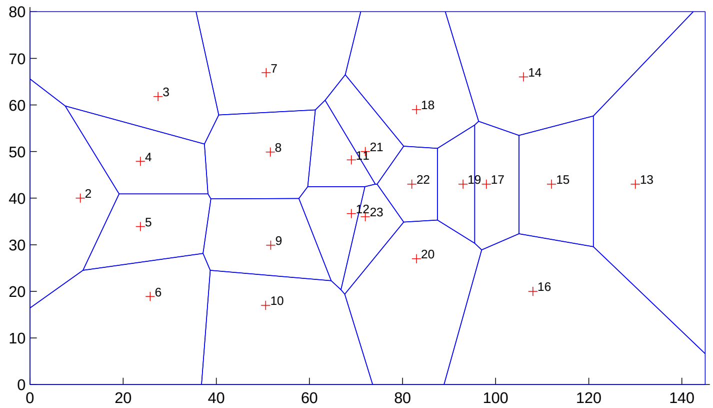

Fig. 1. The Voronoi diagram for starting formation. The numbers in the figure represent the players. For example, 2 and 13 stand for a goalkeeper for each team, respectively. Number 1, which corresponds to the ball, is deliberately omitted here.

stand for players and a ball. For example, 2 and 13 represent a goalkeeper of each team, respectively. Inner Voronoi polygons are determined by a standard method [5]. Since the playing ground has an outer boundary, to construct outer Voronoi polygons, however, we should take this fact into consideration. The boundary point at the boundary of the rectangular soccer field is determined by drawing a line starting from a vertex with equal distant from adjacent outer points to a point with equal distant from the same adjacent outer points. Thus the outer Voronoi polygons may have a somewhat different meaning from the ones with standard Voronoi polygons. Notice a symmetry along the direction of the touch line since the formation itself has a symmetry in configuration.

Now we will describe the state of affairs. The discrete time is counted in the unit of a frame. During the time interval from time step $t = 1$ to $t = 325$, team S gets a goal. At $t = 812$, team F gets a goal. At $t = 2358$, team F gets a second goal. At $t = 3075$, the first half is over. At $t = 3698$, team F gets a third goal. At $t = 4012$, team F gets a fourth goal. At $t = 5979$, the game is over.

A typical Voronoi diagram in our soccer game is shown in Fig. 2. Notice that the symmetry is broken in this situation. The calculated results for the 1st half are summarized in Table 1. Object 1 corresponds to the ball. The ball has mean vertices of 5.23 and has its standard deviation of 1.11. Also it has the smallest Voronoi area of 188.23. The relatively large standard deviation indicates that the distribution of Voronoi areas are rather broad. As shown in Table 1, the average number of vertices range within 5–6. These values are comparable to the value of 6 for a random point set. The area covered by a Voronoi cell can be interpreted as the allotted responsible area of the numbered player or a control area of corresponding player. In general, outer Voronoi polygons have larger areas than the inner Voronoi polygons and the ball has the smallest area. Also, of course, goalkeepers have the largest area in their team, as expected.

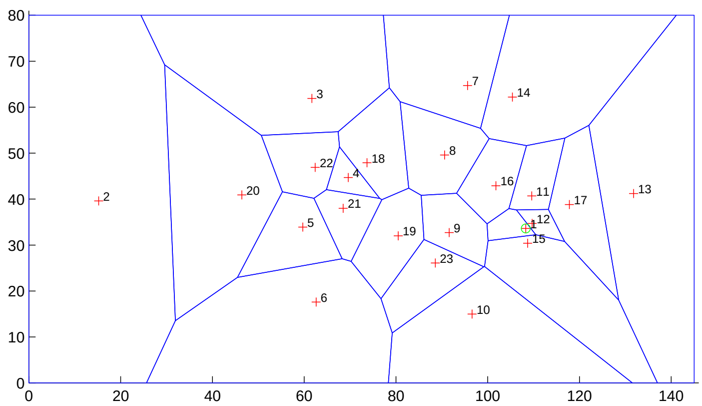

Fig. 2. A typical Voronoi diagram. The symbols are the same as in Fig. 1 except that Number 1, which corresponds to the ball, is included.

Player 21 has the broadest distribution in Voronoi areas and player 7 the narrowest. This can be explained as follows: Player 21 is involved in the vigorous activity and waiting for the ball since it is a foremost forwarder. Likewise, player 7 steadily involved in the game since it is a member of middle fielder. These facts are reflected in the values.

Table 1. Calculated results of mean numbers of vertices and Voronoi polygon areas in the 1st half of the game. Their standard deviations are also shown. Areas are measured in a unit of pixels

Object	Number of vertices	Area	Object	Number of vertices	Area
1	5.23 ± 1.11	188.23 ± 135.79	13	5.24 ± 1.05	1254.10 ± 830.58
2	6.08 ± 0.97	1904.60 ± 1020.50	14	5.85 ± 1.22	721.45 ± 451.93
3	5.57 ± 0.99	983.95 ± 477.99	15	5.17 ± 1.09	457.30 ± 412.15
4	5.01 ± 1.01	332.87 ± 206.01	16	5.52 ± 1.30	562.07 ± 469.45
5	5.20 ± 1.07	469.86 ± 448.64	17	5.45 ± 1.10	392.10 ± 357.54
6	5.40 ± 1.14	749.71 ± 502.87	18	5.47 ± 1.29	272.72 ± 197.15
7	5.47 ± 0.98	636.92 ± 285.44	19	5.72 ± 1.30	257.13 ± 239.88
8	5.74 ± 1.25	194.28 ± 113.06	20	5.45 ± 1.18	283.21 ± 208.17
9	5.77 ± 1.28	200.82 ± 112.80	21	5.45 ± 1.23	260.19 ± 284.70
10	5.25 ± 1.10	410.94 ± 314.15	22	5.64 ± 1.38	272.61 ± 203.39
11	5.43 ± 1.14	246.30 ± 186.71	23	5.29 ± 1.17	179.76 ± 133.26
12	5.53 ± 1.22	368.89 ± 324.78			

If we compare total Voronoi areas of the two teams concerned in a game, we can determine a “dominant ratio” of a team over the other team. If a team dominate the other team in a given period, then the team’s Voronoi area is larger than that of the other team. This fact is confirmed, as shown in Table 2. In this meaning, we can say that team S dominate team F, for example, in the period of frames 1–1581 with a ratio of areas, i.e. $6126.9/5285.7 = 1.16$. In the 1st half, we have a ratio of 1.39. In the 2nd half, on the other, we have a ratio of 0.95. Thus we can say that team S dominate in the 1st half and team F, on the other hand, dominate in the 2nd half. If we divide the 1st half into several smaller periods, we can see the effect more in detail. This is also appended to the lower part of Table 2. During the time period from 1 to 325, team S dominate team F with a ratio of areas of 2.5. In fact, team S scores a goal during this period. Furthermore, in a smaller period than this period, i.e. during the time period from $t = 233$ to $t = 325$ (team S scores a goal during this period), team S dominate team F with a further higher ratio of 4.3. On the other hand, during the time period from

$t = 326$ to $t = 517$, team F dominate team S with a ratio of 1.3. This is related with the fact that team F made a counterattack. So the team's Voronoi areas were smoothed out in this time period. Since they did not score a goal during the time period from $t = 518$ to $t = 2358$, the ratio becomes closer to the value of 1. This period is a state of lull and they made a tedious offensive and defensive battle.

Table 2. Shows the total Voronoi areas of each team. First column denotes number of frame from start to end. Second column is the total Voronoi area for team S and third column for team F

Frames	Team S	Team F
1–1581	6126.9	5285.7
1582–2358	7366.8	4035.1
2359–3074	6393.0	5032.3
3075–4730	5439.7	6028.0
4731–5979	5919.8	5915.9
1–233	8127.9	3194.0
233–325	9332.0	2192.3
326–517	4891.9	6549.3
518–2358	6423.5	4985.3

To see more specific contribution of each player, we calculated the Voronoi areas for their starting formation, the Voronoi diagram of which is already shown in Fig. 1. These areas will be used for reference. Now we subtracted the area of the starting formation from the value of average area of the corresponding player. This subtracted area is defined as an excess Voronoi area. The calculated results are summarized in Table 3.

From Table 3, we can draw the following conclusions.

1. The goalkeeper's excess area of team S is significantly greater than that of team F. Thus we can deduce that team S is dominant in the game with a ratio of 5.3.

2. From the fact that left members of team S have positive excess areas and right members of team S, on the other hand, have negative excess areas, we can get the fact that team F mostly takes a left attack route.

3. Again from the fact that left members of team F have negative excess areas

and center members of team F have positive excess areas, we can get the fact that team S takes a side attack route.

4. Since the excess area of attack members of team S is larger than that of team F, team S dominate the attack in the period. In other words, the attack members of team S have more chances of engagement in the game.

Table 3. Calculated excess areas in the period from $t = 1$ to $t = 1592$, which is shown in 4th column. Averaged Voronoi areas are in 2nd column. Reference Voronoi areas for starting formation are in 3rd column

Object	Averaged area	Reference area	Excess area	Object	Averaged area	Reference area	Excess area
2	1642.6	569.3	1073.3	13	1448.4	1245.6	202.8
3	1006.7	881.4	125.4	14	881.5	932.3	−50.7
4	341.7	337.4	4.3	15	543.1	393.0	150.0
5	332.2	323.5	8.7	16	554.8	1291.9	−737.1
6	773.3	929.6	−156.3	17	349.6	233.6	116.0
7	748.5	637.3	111.2	18	226.7	604.5	−377.9
8	235.3	404.2	−168.9	19	282.3	163.0	119.4
9	232.4	378.3	−145.8	20	269.5	729.8	−460.3
10	421.1	750.2	−329.1	21	325.4	167.9	157.5
11	238.8	152.5	86.3	22	331.0	161.2	169.8
12	234.9	177.4	57.6	23	180.3	136.4	43.9

4 Discussions and conclusions

We presented here a method of Voronoi analysis in analyzing a soccer game. Even if a team dominate the opponent team in the Voronoi analysis, as a matter of fact, the team may not win the game. This fact is usually experienced in the game. But, in most cases, to score a goal, the team should have a larger area ratio.

This analysis method can be used in accessing a player or a team in an individual or collective level. If a player or a team has a stronger power or potential, it may have a larger Voronoi area than its average value.

If a team has a stronger teamwork than the opponent team, it has a stronger ability for a group of players to work together.

As a defence strategy, one usually takes two choices: One is zone defence and the other is man-to-man defence. The former will have smaller excess Voronoi areas and the latter will have more fluctuating excess Voronoi areas.

Furthermore, if we collect the cases of scoring a goal, we will classify the characteristics of the scoring positions. By doing so, we can increase the chance of winning.

We can also use these results to improve teamwork or team's organizing ability.

Acknowledgments

The author is grateful to the Kyonggi University for sponsoring the present research work under promoting research grant in the year 2002. Also thanks to Prof. Je-Young Choi for helpful discussions.

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